A FINITE CHARACTERIZATION AND RECOGNITION
OF INTERSECTION GRAPHS OF HYPERGRAPHS WITH
RANK AT MOST 3 AND MULTIPLICITY AT MOST 2
IN THE CLASS OF THRESHOLD GRAPHS

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Abstract

We characterize the class L_3^2 of intersection graphs of hypergraphs with rank at most 3 and multiplicity at most 2 by means of a finite list of forbidden induced subgraphs in the class of threshold graphs. We also give an $O(n)$ -time algorithm for the recognition of graphs from L_3^2 in the class of threshold graphs, where n is the number of vertices of a tested graph.

Keywords: intersection graph, hypergraph rank, hypergraph multiplicity, forbidden induced subgraph, threshold graph.

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REFERENCES

- [1] L.W. Beineke, *Derived graphs and digraphs*, in: Beitrage zur Graphentheorie, H. Sachs, H.-J. Voss, H.-J. Walter (Ed(s)), (Leipzig, Teubner, 1968) 17–33.
- [2] J.C. Bermond and J.C. Meyer, *Graphs representatif des arêtes d'un multigraphe*, J. Math. Pures Appl. **52** (1973) 299–308.
- [3] V. Chvátal and P.L. Hammer, *Set-Packing and Threshold Graphs* (Comp. Sci. Dept. Univ. of Waterloo, Ontario, 1973).
- [4] D.G. Degiorgi and K. Simon, *A dynamic algorithm for line graph recognition*, Lect. Notes in Comput. Sci. **1017** (1995) 37–48.
doi:10.1007/3-540-60618-1_64
- [5] S. Földes and P.L. Hammer, *Split graphs having Dilworth number two*, Canad. J. Math. **29** (1977) 666–672.
doi:10.4153/CJM-1977-069-1
- [6] M.L. Gardner, *Forbidden configurations in intersection graphs of r -graphs*, Discrete Math. **31** (1980) 85–88.
doi:10.1016/0012-365X(80)90175-2
- [7] O.V. Glebova, Yu.M. Metelsky and P.V. Skums, *Krausz dimension and its generalizations in special graph classes*, Discrete Math. Theor. Comput. Sci. **15** (2013) 107–120.
- [8] P. Hliněný and J. Kratochvíl, *Computational complexity of the Krausz dimension of graphs*, Lect. Notes in Comput. Sci. **1335** (1997) 214–228.
doi:10.1007/BFb0024500
- [9] M.S. Jacobson, A.E. Kezdy and J. Lehel, *Recognizing intersection graphs of linear uniform hypergraphs*, Graphs Combin. **4** (1997) 359–367.
doi:10.1007/BF03353014
- [10] A.G. Levin and R.I. Tyshkevich, *Edge Hypergraphs*, Diskret. Mat. **5** (1993) 112–129, in Russian.
- [11] P.G.H. Lehot, *An optimal algorithm to detect a line graph and output its root graph*, J. Assoc. Comput. Mach. **21** (1974) 569–575.
doi:10.1145/321850.321853
- [12] Yu. Metelsky, *Split intersection graphs of hypergraphs with bounded rank*, Vestsi Nats. Akad. Navuk Belarusi. Ser. Fiz.-Mat. Navuk **3** (1997) 117–122, in Russian.
- [13] Yu. Metelsky and K.N. Shchamialiova, *Finite characterizability of intersection graphs of hypergraphs with bounded rank and multiplicity in the class of split graphs*, Vestn. Beloruss. Gos. Univ. Ser. 1 Fiz. Mat. Inform. **1** (2008) 102–105, in Russian.
- [14] Yu. Metelsky and R. Tyshkevich, *Line graphs of linear 3-uniform hypergraphs*, J. Graph Theory **25** (1997) 243–251.
doi:10.1002/(SICI)1097-0118(199708)25:4<243::AID-JGT1>3.0.CO;2-K

- [15] R.N. Naik, S.B. Rao, S.S. Shrikhande and N.M. Singhi, *Intersection graphs of k -uniform linear hypergraphs*, Ann. Discrete Math. **6** (1980) 275–279.
doi:10.1016/S0167-5060(08)70711-8
- [16] R.N. Naik, S.B. Rao, S.S. Shrikhande and N.M. Singhi, *Intersection graphs of k -uniform linear hypergraphs*, European J. Combin. **3** (1982) 159–172.
doi:10.1016/S0195-6698(82)80029-2
- [17] J. Naor and M.B. Novick, *An efficient reconstruction of a graph from its line graph in parallel*, J. Algorithms **11** (1990) 132–143.
doi:10.1016/0196-6774(90)90034-C
- [18] S. Poljak, V. Rödl and D. Turzik, *Complexity of representation of graphs by set systems*, Discrete Appl. Math. **3** (1981) 301–312.
doi:10.1016/0166-218X(81)90007-X
- [19] N.D. Roussopoulos, *A $\max\{m, n\}$ algorithm for determining the graph H from its line graph G* , Inform. Process. Lett. **2** (1973) 108–112.
doi:10.1016/0020-0190(73)90029-X
- [20] P.V. Skums, *Krausz decomposition in special classes of split graphs*, Vestn. Beloruss. Gos. Univ. Ser. 1 Fiz. Mat. Inform. **3** (2005) 96–100, in Russian.
- [21] P.V. Skums, S.V. Suzdal and R.I. Tyshkevich, *Edge intersection graphs of linear 3-uniform hypergraphs*, Discrete Math. **309** (2009) 3500–3517.
doi:10.1016/j.disc.2007.12.082
- [22] V.A. Tashkinov, *Characterization of intersection graphs of p -graphs*, in: Proc. 5th all-USSR Conference on Problems of Theoretical Cybernetics (Novosibirsk, 1980) 135–137, in Russian.

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