RADIO GRACEFUL HAMMING GRAPHS

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Abstract

For $k \in \mathbb{Z}_+$ and G a simple, connected graph, a k -radio labeling $f : V(G) \rightarrow \mathbb{Z}_+$ of G requires all pairs of distinct vertices u and v to satisfy $|f(u) - f(v)| \geq k + 1 - d(u, v)$. We consider k -radio labelings of G when $k = \text{diam}(G)$. In this setting, f is injective; if f is also surjective onto $\{1, 2, \dots, |V(G)|\}$, then f is a *consecutive radio labeling*. Graphs that can be labeled with such a labeling are called *radio graceful*. In this paper, we give two results on the existence of radio graceful Hamming graphs. The main result shows that the Cartesian product of t copies of a complete graph is radio graceful for certain t . Graphs of this form provide infinitely many examples of radio graceful graphs of arbitrary diameter. We also show that these graphs are not radio graceful for large t .

Keywords: radio labeling, radio graceful graph, Hamming graph.

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