GRAPHS WITH LARGE GENERALIZED (EDGE-)CONNECTIVITY

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Abstract

The generalized k -connectivity $\kappa_k(G)$ of a graph G , introduced by Hager in 1985, is a nice generalization of the classical connectivity. Recently, as a natural counterpart, we proposed the concept of generalized k -edge-connectivity $\lambda_k(G)$. In this paper, graphs of order n such that $\kappa_k(G) = n - \frac{k}{2} - 1$ and $\lambda_k(G) = n - \frac{k}{2} - 1$ for even k are characterized.

Keywords: (edge-)connectivity, Steiner tree, internally disjoint trees, edge-disjoint trees, packing, generalized (edge-)connectivity.

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