NICHE HYPERGRAPHS

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Abstract

If $D = (V, A)$ is a digraph, its *niche hypergraph* $N\mathcal{H}(D) = (V, \mathcal{E})$ has the edge set $\mathcal{E} = \{e \subseteq V \mid |e| \geq 2 \wedge \exists v \in V : e = N_D^-(v) \vee e = N_D^+(v)\}$. Niche hypergraphs generalize the well-known niche graphs (see [11]) and are closely related to competition hypergraphs (see [40]) as well as double competition hypergraphs (see [33]). We present several properties of niche hypergraphs of acyclic digraphs.

Keywords: niche hypergraph, niche number.

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REFERENCES

- [1] C.A. Anderson, *Loop and cyclic niche graphs*, Linear Algebra Appl. **217** (1995) 5–13.
doi:10.1016/0024-3795(94)00154-6
- [2] C.A. Anderson, K.F. Jones, J.R. Lundgren and S. Seager, *A suggestion for new niche numbers of graphs*, in: Proc. 22-nd Southeastern Conf. on Combinatorics, Graph Theory and Computing, Baton Rouge, USA (Util. Math. Publ. Inc., 1991) 23–32.

- [3] C.A. Anderson, J.R. Lundgren, S. Bowser and C. Cable, *Niche graphs and unit interval graphs*, Congr. Numer. **93** (1993) 83–90.
- [4] J. Bang-Jensen and G. Gutin, *Digraphs: Theory, Algorithms and Applications* (Springer, London, 2001).
- [5] C. Berge, *Graphs and Hypergraphs* (North-Holland, Amsterdam, 1973).
- [6] S. Bowser and C. Cable, *Cliques and niche graphs*, Congr. Numer. **76** (1990) 151–156.
- [7] S. Bowser and C. Cable, *Some recent results on niche graphs*, Discrete Appl. Math. **30** (1991) 101–108.
doi:10.1016/0166-218X(91)90036-V
- [8] S. Bowser and C. Cable, *The niche category of dense graphs*, Ars Combin. **59** (2001) 289–297.
- [9] S. Bowser and C. Cable, *The niche category of sparse graphs*, Ars Combin. **66** (2003) 179–192.
- [10] S. Bowser, C. Cable and J.R. Lundgren, *Niche graphs and mixed pair graphs of tournaments*, J. Graph Theory **31** (1999) 319–332.
doi:10.1002/(SICI)1097-0118(199908)31:4<319::AID-JGT7>3.0.CO;2-S
- [11] C. Cable, K.F. Jones, J.R. Lundgren and S. Seager, *Niche graphs*, Discrete Appl. Math. **23** (1989) 231–241.
doi:10.1016/0166-218X(89)90015-2
- [12] J.E. Cohen, *Interval graphs and food webs: a finding and a problem* (RAND Corp. Document 17696-PR, Santa Monica, CA, 1968).
- [13] M. Cozzens, *Food webs, competition graphs and habitat formation*, Math. Model. Nat. Phenom. **6** (2011) 22–38.
doi:10.1051/mmnp/20116602
- [14] P.C. Fishburn and W.V. Gehrlein, *Niche numbers*, J. Graph Theory **16** (1992) 131–139.
doi:10.1002/jgt.3190160204
- [15] W.V. Gehrlein and P.C. Fishburn, *The smallest graphs with niche number three*, Comput. Math. Appl. **27** (1994) 53–57.
doi:10.1016/0898-1221(94)90054-X
- [16] W.V. Gehrlein and P.C. Fishburn, *Niche number four*, Comput. Math. Appl. **32** (1996) 51–54.
doi:10.1016/0898-1221(96)00176-9
- [17] S.-R. Kim, *The competition number and its variants*, in: J. Gimbel, J.W. Kennedy and L.V. Quintas (Ed(s)), *Quo Vadis, Graph Theory?* Ann. Discrete Math. **55** (1993) 313–326.
doi:10.1016/s0167-5060(08)70396-0

- [18] S.-J. Kim, S.-R. Kim and Y. Rho, *On CCE graphs of doubly partial orders*, Discrete Appl. Math. **155** (2007) 971–978.
doi:10.1016/j.dam.2006.09.013
- [19] S.-R. Kim, J.Y. Lee, B. Park, W.J. Park and Y. Sano, *The niche graphs of doubly partial orders*, Congr. Numer. **195** (2009) 19–32.
- [20] S.-R. Kim, B. Park and Y. Sano, *The competition number of the complement of a cycle*, Discrete Appl. Math. **161** (2013) 1755–1760.
doi:10.1016/j.dam.2011.10.034
- [21] S.-R. Kim, J.Y. Lee, B. Park and Y. Sano, *The competition hypergraphs of doubly partial orders*, Discrete Appl. Math. **165** (2014) 185–191.
doi:10.1016/j.dam.2012.05.024
- [22] S.-R. Kim, J.Y. Lee, B. Park and Y. Sano, *A generalization of Opsut’s result on the competition numbers of line graphs*, Discrete Appl. Math. **181** (2015) 152–159.
doi:10.1016/j.dam.2014.10.014
- [23] J. Kuhl, *Transversals and competition numbers of complete multipartite graphs*, Discrete Appl. Math. **161** (2013) 435–440.
doi:10.1016/j.dam.2012.09.012
- [24] B.-J. Li and G.J. Chang, *Competition numbers of complete r -partite graphs*, Discrete Appl. Math. **160** (2012) 2271–2276.
doi:10.1016/j.dam.2012.05.005
- [25] J. Lu and Y. Wu, *Two minimal forbidden subgraphs for double competition graphs of posets of dimension at most two*, Appl. Math. Lett. **22** (2009) 841–845.
doi:10.1016/j.aml.2008.06.046
- [26] J.R. Lundgren, *Food webs, competition graphs, competition-common enemy graphs and niche graphs*, in: F. Roberts (Ed(s)), Applications of Combinatorics and Graph Theory to the Biological and Social Sciences (IMA **17**, Springer, New York, 1989) 221–243.
doi:10.1007/978-1-4684-6381-1_9
- [27] B.D. McKey, P. Schweitzer and P. Schweitzer, *Competition numbers, quasi line graphs, and holes*, SIAM J. Discrete Math. **28** (2014) 77–91.
doi:10.1137/110856277
- [28] B. Park and Y. Sano, *On the hypercompetition numbers of hypergraphs*, Ars Combin. **100** (2011) 151–159.
- [29] B. Park and Y. Sano, *The competition numbers of ternary Hamming graphs*, Appl. Math. Lett. **24** (2011) 1608–1613.
doi:/10.1016/j.aml.2011.04.012
- [30] B. Park and Y. Sano, *The competition number of a generalized line graph is at most two*, Discrete Math. Theor. Comput. Sci. **14** (2012) 1–10.
- [31] B. Park and S.-R. Kim, *On Opsut’s conjecture for hypercompetition numbers of hypergraphs*, Discrete Appl. Math. **160** (2012) 2286–2293.
doi:10.1016/j.dam.2012.05.009

- [32] J. Park and Y. Sano, *The niche graphs of interval orders*, Discuss. Math. Graph Theory **34** (2014) 353–359.
doi:10.7151/dmgt.1741
- [33] J. Park and Y. Sano, *The double competition hypergraph of a digraph*, Discrete Appl. Math. **195** (2015) 110–113.
doi:10.1016/j.dam.2014.04.001
- [34] Y. Sano, *The competition-common enemy graphs of digraphs satisfying conditions $C(p)$ and $C'(p)$* , Congr. Numer. **202** (2010) 187–194.
- [35] Y. Sano, *On the hypercompetition numbers of hypergraphs with maximum degree at most two*, Discuss. Math. Graph Theory **35** (2015) 595–598.
doi:10.7151/dmgt.1826
- [36] S. Seager, *Niche graph properties of trees*, in: Proc. 22-nd Southeastern Conf. on Combinatorics, Graph Theory and Computing, Baton Rouge, USA (Util. Math. Publ. Inc., 1991) 149–155.
- [37] S. Seager, *Relaxations of niche graphs*, Congr. Numer. **96** (1993) 205–214.
- [38] S. Seager, *Cyclic niche graphs and grids*, Ars Combin. **49** (1998) 21–32.
- [39] D.D. Scott, *The competition-common enemy graph of a digraph*, Discrete Appl. Math. **17** (1987) 269–280.
doi:10.1016/0166-218X(87)90030-8
- [40] M. Sonntag and H.-M. Teichert, *Competition hypergraphs*, Discrete Appl. Math. **143** (2004) 324–329.
doi:10.1016/j.dam.2004.02.010
- [41] M. Sonntag and H.-M. Teichert, *Competition hypergraphs of digraphs with certain properties I. Strong connectedness*, Discuss. Math. Graph Theory **28** (2008) 5–21.
doi:10.7151/dmgt.1388
- [42] M. Sonntag and H.-M. Teichert, *Competition hypergraphs of digraphs with certain properties II. Hamiltonicity*, Discuss. Math. Graph Theory **28** (2008) 23–34.
doi:10.7151/dmgt.1389
- [43] M. Sonntag and H.-M. Teichert, *Competition hypergraphs of products of digraphs*, Graphs Combin. **25** (2009) 611–624.
doi:10.1007/s00373-005-0868-9
- [44] Y. Wu and J. Lu, *Dimension-2 poset competition numbers and dimension-2 poset double competition numbers*, Discrete Appl. Math. **158** (2010) 706–717.
doi:10.1016/j.dam.2009.12.001

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