THE VERTEX-RAINBOW INDEX OF A GRAPH

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Abstract

The k -rainbow index $rx_k(G)$ of a connected graph G was introduced by Chartrand, Okamoto and Zhang in 2010. As a natural counterpart of the k -rainbow index, we introduce the concept of k -vertex-rainbow index $rvx_k(G)$ in this paper. In this paper, sharp upper and lower bounds of $rvx_k(G)$ are given for a connected graph G of order n , that is, $0 \leq rvx_k(G) \leq n - 2$. We obtain Nordhaus-Gaddum results for 3-vertex-rainbow index of a graph G of order n , and show that $rvx_3(G) + rvx_3(\overline{G}) = 4$ for $n = 4$ and $2 \leq rvx_3(G) + rvx_3(\overline{G}) \leq n - 1$ for $n \geq 5$. Let $t(n, k, \ell)$ denote the minimal size of a connected graph G of order n with $rvx_k(G) \leq \ell$, where $2 \leq \ell \leq n - 2$ and $2 \leq k \leq n$. Upper and lower bounds on $t(n, k, \ell)$ are also obtained.

Keywords: vertex-coloring, connectivity, vertex-rainbow S -tree, vertex-rainbow index, Nordhaus-Gaddum type.

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