THE RYJÁČEK CLOSURE AND A FORBIDDEN SUBGRAPH

AKIRA SAITO

Department of Information Science
Nihon University
Sakurajosui 3-25-40
Setagaya-Ku, Tokyo 156-8550, Japan
e-mail: asaito@chs.nihon-u.ac.jp

AND

LIMING XIONG

School of Mathematics and Statistics
Beijing Institute of Technology
Beijing 100081, China
and
Beijing Key Laboratory on MCAACI
Beijing Institute of Technology
Beijing 100081, China
e-mail: lmxiong@bit.edu.cn

Abstract

The Ryjáček closure is a powerful tool in the study of Hamiltonian properties of claw-free graphs. Because of its usefulness, we may hope to use it in the classes of graphs defined by another forbidden subgraph. In this note, we give a negative answer to this hope, and show that the claw is the only forbidden subgraph that produces non-trivial results on Hamiltonicity by the use of the Ryjáček closure.

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