LOOSENESS AND INDEPENDENCE NUMBER OF TRIANGULATIONS ON CLOSED SURFACES

ATSUHIRO NAKAMOTO, SEIYA NEGAMI

Graduate School of Environment and Information Science
Yokohama National University

79-7 Tokiwadai, Hodogaya-Ku, Yokohama 240-8501, Japan

e-mail: nakamoto@ynu.ac.jp, negami@ynu.ac.jp

KYOJI OHBA

Yonago National College of Technology
Yonago, Tottori 683-8502, Japan

e-mail: ooba@yonago-k.ac.jp

AND

YUSUKE SUZUKI

Department of Mathematics
Niigata University

8050 Ikarashi 2-no-cho, Nishi-ku, Niigata, 950-2181, Japan

e-mail: y-suzuki@math.sc.niigata-u.ac.jp

Abstract

The *looseness* of a triangulation G on a closed surface F^2 , denoted by $\xi(G)$, is defined as the minimum number k such that for any surjection $c : V(G) \rightarrow \{1, 2, \dots, k+3\}$, there is a face uvw of G with $c(u), c(v)$ and $c(w)$ all distinct. We shall bound $\xi(G)$ for triangulations G on closed surfaces by the independence number of G denoted by $\alpha(G)$. In particular, for a triangulation G on the sphere, we have

$$\xi(G) \leq \frac{11\alpha(G) - 10}{6}$$

and this bound is sharp. For a triangulation G on a non-spherical surface F^2 , we have

$$\xi(G) \leq 2\alpha(G) + l(F^2) - 2,$$

where $l(F^2) = \lfloor (2 - \chi(F^2))/2 \rfloor$ with Euler characteristic $\chi(F^2)$.

Keywords: triangulations, closed surfaces, looseness, k -loosely tight, independence number.

2010 Mathematics Subject Classification: 05C10.

REFERENCES

- [1] K. Appel, W. Haken and J. Koch, *Every planar map is four colorable*, Illinois J. Math. **21** (1977) 429–567.
- [2] J.L. Arocha, J. Bracho and V. Neumann-Lara, *On the minimum size of tight hypergraphs*, J. Graph Theory **16** (1992) 319–326.
doi:10.1002/jgt.3190160405
- [3] J.L. Arocha, J. Bracho and V. Neumann-Lara, *Tight and untight triangulations surfaces by complete graphs*, J. Combin. Theory Ser. B **63** (1995) 185–199.
doi:10.1006/jctb.1995.1015
- [4] J. Czap, S. Jendrol', F. Kardoš and J. Miškuf, *Looseness of plane graphs*, Graphs Combin. **27** (2011) 73–85.
doi:10.1007/s00373-010-0961-6
- [5] S. Negami and T. Midorikawa, *Loosely-tightness of triangulations of closed surfaces*, Sci. Rep. Yokohama Nat. Univ., Sec. I **43** (1996) 25–41.
- [6] S. Negami, *Looseness ranges of traingulations on closed surfaces*, Discrete Math. **303** (2005) 167–174.
doi:10.1016/j.disc.2005.01.010
- [7] G. Ringel, *Map Color Theorem* (Springer-Verlag, Berlin Heidelberg, 1974).
doi:10.1007/978-3-642-65759-7
- [8] T. Tanuma, *One-loosely tight triangulations on closed surfaces*, Yokohama Math. J. **47** (1999) 203–211.

Received 13 April 2015

Revised 8 September 2015

Accepted 8 September 2015