CHARACTERIZATIONS OF GRAPHS HAVING LARGE PROPER CONNECTION NUMBERS

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Abstract

Let G be an edge-colored connected graph. A path P is a proper path in G if no two adjacent edges of P are colored the same. If P is a proper $u - v$ path of length $d(u, v)$, then P is a proper $u - v$ geodesic. An edge coloring c is a proper-path coloring of a connected graph G if every pair u, v of distinct vertices of G are connected by a proper $u - v$ path in G , and c is a strong proper-path coloring if every two vertices u and v are connected by a proper $u - v$ geodesic in G . The minimum number of colors required for a proper-path coloring or strong proper-path coloring of G is called the proper connection number $\text{pc}(G)$ or strong proper connection number $\text{spc}(G)$ of G , respectively. If G is a nontrivial connected graph of size m , then $\text{pc}(G) \leq \text{spc}(G) \leq m$ and $\text{pc}(G) = m$ or $\text{spc}(G) = m$ if and only if G is the star of size m . In this paper, we determine all connected graphs G of size m for which $\text{pc}(G)$ or $\text{spc}(G)$ is $m - 1, m - 2$ or $m - 3$.

Keywords: edge coloring, proper-path coloring, strong proper-path coloring.

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