A CHARACTERIZATION OF HYPERGRAPHS WITH LARGE DOMINATION NUMBER

MICHAEL A. HENNING^{1a} AND CHRISTIAN LÖWENSTEIN^{2a,b}

^a*Department of Pure and Applied Mathematics
University of Johannesburg
Auckland Park, 2006, South Africa*

^b*Institute of Optimization and Operations Research
Ulm University
Ulm 89081, Germany*

e-mail: mahenning@uj.ac.za
christian.loewenstein@uni-ulm.de

Abstract

Let $H = (V, E)$ be a hypergraph with vertex set V and edge set E . A dominating set in H is a subset of vertices $D \subseteq V$ such that for every vertex $v \in V \setminus D$ there exists an edge $e \in E$ for which $v \in e$ and $e \cap D \neq \emptyset$. The domination number $\gamma(H)$ is the minimum cardinality of a dominating set in H . It is known [Cs. Bujtás, M.A. Henning and Zs. Tuza, *Transversals and domination in uniform hypergraphs*, European J. Combin. **33** (2012) 62–71] that for $k \geq 5$, if H is a hypergraph of order n and size m with all edges of size at least k and with no isolated vertex, then $\gamma(H) \leq (n + \lfloor (k-3)/2 \rfloor m) / (\lfloor 3(k-1)/2 \rfloor)$. In this paper, we apply a recent result of the authors on hypergraphs with large transversal number [M.A. Henning and C. Löwenstein, *A characterization of hypergraphs that achieve equality in the Chvátal-McDiarmid Theorem*, Discrete Math. **323** (2014) 69–75] to characterize the hypergraphs achieving equality in this bound.

Keywords: domination, transversal, hypergraph.

2010 Mathematics Subject Classification: 05C69.

REFERENCES

- [1] B.D. Acharya, *Domination in hypergraphs*, AKCE Int. J. Graphs Comb. **4** (2007) 117–126.

¹Research supported in part by the South African National Research Foundation and the University of Johannesburg.

²Research supported by the Deutsche Forschungsgemeinschaft (GZ: LO 1758/1-1).

- [2] B.D. Acharya, *Domination in hypergraphs II. New directions*, in: Proc. Int. Conf. ICDM 2008, Mysore, India, 1–16.
- [3] B.D. Acharya and P. Gupta, *Weak edge-degree domination in hypergraphs*, Czechoslovak Math. J. **56** (131) (2006) 99–108.
doi:10.1007/s10587-006-0008-6
- [4] Cs. Bujtás, M.A. Henning and Zs. Tuza, *Transversals and domination in uniform hypergraphs*, European J. Combin. **33** (2012) 62–71.
doi:10.1016/j.ejc.2011.08.002
- [5] V. Chvátal and C. McDiarmid, *Small transversals in hypergraphs*, Combinatorica **12** (1992) 19–26.
doi:10.1007/BF01191201
- [6] M.A. Henning, I. Schiermeyer and A. Yeo, *A new bound on the domination number of graphs with minimum degree two*, Electron. J. Combin. **18** (2011) #P12.
- [7] M.A. Henning and A. Yeo, *Hypergraphs with large transversal number and with edge sizes at least three*, J. Graph Theory **59** (2008) 326–348.
doi:10.1002/jgt.20340
- [8] M.A. Henning and C. Löwenstein, *Hypergraphs with large domination number and edge sizes at least 3*, Discrete Applied Math. **160** (2012) 1757–1765.
doi:/10.1016/j.dam.2012.03.023
- [9] M.A. Henning and C. Löwenstein, *Hypergraphs with large transversal number and with edge sizes at least four*, Cent. Eur. J. Math. **10** (2012) 1133–1140.
doi:10.2478/s11533-012-0023-9
- [10] M.A. Henning and C. Löwenstein, *A characterization of hypergraphs that achieve equality in the Chvátal-McDiarmid Theorem*, Discrete Math. **323** (2014) 69–75.
doi:10.1016/j.disc.2014.01.014
- [11] T. Honjo, K. Kawarabayashi and A. Nakamoto, *Dominating sets in triangulations on surfaces*, J. Graph Theory **63** (2010) 17–30.
doi:10.1002/jgt.20401
- [12] B.K. Jose and Zs. Tuza, *Hypergraph domination and strong independence*, Appl. Anal. Discrete Math. **3** (2009) 237–358.
doi:10.2298/AADM0902347J
- [13] F.C. Lai and G.J. Chang, *An upper bound for the transversal numbers of 4-uniform hypergraphs*, J. Combin. Theory Ser. B **50** (1990) 129–133.
doi:10.1016/0095-8956(90)90101-5
- [14] C. Löwenstein and D. Rautenbach, *Domination in graphs of minimum degree at least two and large girth*, Graphs Combin. **24** (2008) 37–46.
doi:10.1007/s00373-007-0770-8

Received 18 December 2014

Revised 9 August 2015

Accepted 9 August 2015