A NEIGHBORHOOD CONDITION FOR FRACTIONAL ID- $[A, B]$ -FACTOR-CRITICAL GRAPHS

SIZHONG ZHOU, FAN YANG

*School of Mathematics and Physics
Jiangsu University of Science and Technology
Mengxi Road 2, Zhenjiang, Jiangsu 212003, P.R. China*

e-mail: zsz_cumt@163.com
fanyang_just@163.com

AND

ZHIREN SUN

*School of Mathematical Sciences
Nanjing Normal University Nanjing
Jiangsu 210046, P.R. China*

e-mail: 05119@njnu.edu.cn

Abstract

Let G be a graph of order n , and let a and b be two integers with $1 \leq a \leq b$. Let $h : E(G) \rightarrow [0, 1]$ be a function. If $a \leq \sum_{e \ni x} h(e) \leq b$ holds for any $x \in V(G)$, then we call $G[F_h]$ a fractional $[a, b]$ -factor of G with indicator function h , where $F_h = \{e \in E(G) : h(e) > 0\}$. A graph G is fractional independent-set-deletable $[a, b]$ -factor-critical (in short, fractional ID- $[a, b]$ -factor-critical) if $G - I$ has a fractional $[a, b]$ -factor for every independent set I of G . In this paper, it is proved that if $n \geq \frac{(a+2b)(2a+2b-3)+1}{b}$, $\delta(G) \geq \frac{bn}{a+2b} + a$ and $|N_G(x) \cup N_G(y)| \geq \frac{(a+b)n}{a+2b}$ for any two nonadjacent vertices $x, y \in V(G)$, then G is fractional ID- $[a, b]$ -factor-critical. Furthermore, it is shown that this result is best possible in some sense.

Keywords: graph, minimum degree, neighborhood, fractional $[a, b]$ -factor, fractional ID- $[a, b]$ -factor-critical graph.

2010 Mathematics Subject Classification: 05C70, 05C72, 05C35.

REFERENCES

- [1] S. Zhou, Z. Sun and H. Liu, *A minimum degree condition for fractional ID - $[a, b]$ -factor-critical graphs*, Bull. Aust. Math. Soc. **86** (2012) 177–183.
doi:10.1017/S0004972711003467
- [2] H. Matsuda, *A neighborhood condition for graphs to have $[a, b]$ -factors*, Discrete Math. **224** (2000) 289–292.
doi:10.1016/S0012-365X(00)00140-0
- [3] J. Ekstein, P. Holub, T. Kaiser, L. Xiong and S. Zhang, *Star subdivisions and connected even factors in the square of a graph*, Discrete Math. **312** (2012) 2574–2578.
doi:10.1016/j.disc.2011.09.004
- [4] H. Lu, *Simplified existence theorems on all fractional $[a, b]$ -factors*, Discrete Appl. Math. **161** (2013) 2075–2078.
doi:10.1016/j.dam.2013.02.006
- [5] S. Zhou, *Independence number, connectivity and (a, b, k) -critical graphs*, Discrete Math. **309** (2009) 4144–4148.
doi:10.1016/j.disc.2008.12.013
- [6] S. Zhou, *A sufficient condition for graphs to be fractional (k, m) -deleted graphs*, Appl. Math. Lett. **24** (2011) 1533–1538.
doi:10.1016/j.aml.2011.03.041
- [7] S. Zhou and H. Liu, *Neighborhood conditions and fractional k -factors*, Bull. Malays. Math. Sci. Soc. **32** (2009) 37–45.
- [8] K. Kimura, *f -factors, complete-factors, and component-deleted subgraphs*, Discrete Math. **313** (2013) 1452–1463.
doi:10.1016/j.disc.2013.03.009
- [9] M. Kouider and Z. Lonc, *Stability number and $[a, b]$ -factors in graphs*, J. Graph Theory **46** (2004) 254–264.
doi:10.1002/jgt.20008
- [10] R. Chang, G. Liu and Y. Zhu, *Degree conditions of fractional ID - k -factor-critical graphs*, Bull. Malays. Math. Sci. Soc. **33** (2010) 355–360.
- [11] S. Zhou, Q. Bian and J. Wu, *A result on fractional ID - k -factor-critical graphs*, J. Combin. Math. Combin. Comput. **87** (2013) 229–236.
- [12] S. Zhou, *Binding numbers for fractional ID - k -factor-critical graphs*, Acta Math. Sin. Engl. Ser. **30** (2014) 181–186.
doi:10.1007/s10114-013-1396-9
- [13] G. Liu and L. Zhang, *Fractional (g, f) -factors of graphs*, Acta Math. Sci. Ser. B **21** (2001) 541–545.

Received 12 February 2015

Accepted 17 June 2015