KERNELS BY MONOCHROMATIC PATHS AND COLOR-PERFECT DIGRAPHS

HORTENSIA GALEANA-SÁNCHEZ¹ AND ROCÍO SÁNCHEZ-LÓPEZ

*Instituto de Matemáticas UNAM, Ciudad Universitaria
Circuito Exterior 04510, México, D.F., México*

e-mail: hgaleana@matem.unam.mx
usagitsukinomx@yahoo.com.mx

Abstract

For a digraph D , $V(D)$ and $A(D)$ will denote the sets of vertices and arcs of D respectively. In an arc-colored digraph, a subset K of $V(D)$ is said to be *kernel by monochromatic paths* (mp-kernel) if (1) for any two different vertices x, y in N there is no monochromatic directed path between them (N is mp-independent) and (2) for each vertex u in $V(D) \setminus N$ there exists $v \in N$ such that there is a monochromatic directed path from u to v in D (N is mp-absorbent). If every arc in D has a different color, then a kernel by monochromatic paths is said to be a *kernel*. Two associated digraphs to an arc-colored digraph are the closure and the color-class digraph $\mathcal{C}_C(D)$. In this paper we will approach an mp-kernel via the closure of induced subdigraphs of D which have the property of having few colors in their arcs with respect to D . We will introduce the concept of color-perfect digraph and we are going to prove that if D is an arc-colored digraph such that D is a quasi color-perfect digraph and $\mathcal{C}_C(D)$ is not strong, then D has an mp-kernel. Previous interesting results are generalized, as for example Richardson's Theorem.

Keywords: kernel, kernel perfect digraph, kernel by monochromatic paths, color-class digraph, quasi color-perfect digraph, color-perfect digraph.

2010 Mathematics Subject Classification: 05C20, 05C38, 05C69.

REFERENCES

- [1] P. Arpin and V. Linek, *Reachability problems in edge-colored digraphs*, Discrete Math. **307** (2007) 2276–2289.
doi:10.1016/j.disc.2006.09.042

¹Research supported by UNAM-DGAPA-PAPIIT IN106613-2 and CONACyT 219840.

- [2] J. Bang-Jensen and G. Gutin, *Digraphs: Theory, Algorithms and Applications* (Springer, London, 2000).
- [3] C. Berge, *Graphs* (North-Holland, Amsterdam, 1989).
- [4] C. Berge and P. Duchet, *Perfect graphs and kernels*, Bull. Inst. Math. Acad. Sin. **16** (1988) 263–274.
- [5] C. Berge and P. Duchet, *Recent problems and results about kernels in directed graphs*, Discrete Math. **86** (1990) 27–31.
doi:10.1016/0012-365X(90)90346-J
- [6] E. Boros and V. Gurvich, *Perfect graphs, kernels and cores of cooperative games*, RUTCOR Research Report **12** (Rutgers University, April 2003).
- [7] V. Chvátal, *On the computational complexity of finding a kernel*, Report CRM300, Centre de Recherches Mathématiques (Université de Montréal, 1973).
- [8] P. Duchet, *Graphes noyau-parfaits*, Ann. Discrete Math. **9** (1980) 93–101.
doi:10.1016/S0167-5060(08)70041-4
- [9] A.S. Fraenkel, *Planar Kernel and Grundy with $d \leq 3$, $d_{out} \leq 2$, $d_{in} \leq 2$, are NP-complete*, Discrete Appl. Math. **3** (1981) 257–262.
doi:10.1016/0166-218X(81)90003-2
- [10] A.S. Fraenkel, *Combinatorial games: Selected bibliography with a succinct gourmet introduction*, Electron J. Combin. **14** (2009) DS2.
- [11] H. Galeana-Sánchez and V. Neumann-Lara, *On kernels and semikernels of digraphs*, Discrete Math. **48** (1984) 67–76.
doi:10.1016/0012-365X(84)90131-6
- [12] H. Galeana-Sánchez, *On monochromatic paths and monochromatic cycles in edge coloured tournaments*, Discrete Math. **156** (1996) 103–112.
doi:10.1016/0012-365X(95)00036-V
- [13] H. Galeana-Sánchez, *Kernels in edge coloured digraphs*, Discrete Math. **184** (1998) 87–99.
doi:10.1016/S0012-365X(97)00162-3
- [14] H. Galeana-Sánchez, *Kernels by monochromatic paths and the color-class digraph*, Discuss. Math. Graph Theory **31** (2011) 273–281.
doi:10.7151/dmgt.1544
- [15] H. Galeana-Sánchez and R. Rojas-Monroy, *A counterexample to a conjecture on edge-coloured tournaments*, Discrete Math. **282** (2004) 275–276.
doi:10.1016/j.disc.2003.11.015
- [16] S. Minggang, *On monochromatic paths in m -coloured tournaments*, J. Combin. Theory Ser. B **45** (1988) 108–111.
doi:10.1016/0095-8956(88)90059-7
- [17] J. von Neumann and O. Morgenstern, *Theory of Games and Economic Behavior* (Princeton University Press, Princeton, 1944).

- [18] V. Neumann-Lara, *Seminúcleos de una digráfica*, An. Inst. Mat. **11** (1971) 55–62.
- [19] M. Richardson, *Solutions of irreflexive relations*, Ann. of Math. (2) **58** (1953) 573–590.
doi:10.2307/1969755
- [20] M. Richardson, *Extensions theorems for solutions of irreflexive relations*, Proc. Natl. Acad. Sci. USA **39** (1953) 649–655.
doi:10.1073/pnas.39.7.649
- [21] B. Sands, N. Sauer and R. Woodrow, *On monochromatic paths in edge coloured digraphs*, J. Combin. Theory Ser. B **33** (1982) 271–275.
doi:10.1016/0095-8956(82)90047-8

Received 18 August 2014

Revised 23 June 2015

Accepted 23 June 2015