SOLUTIONS OF SOME $L(2, 1)$ -COLORING RELATED OPEN PROBLEMS

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Abstract

An $L(2, 1)$ -coloring (or *labeling*) of a graph G is a vertex coloring $f : V(G) \rightarrow \mathbb{Z}^+ \cup \{0\}$ such that $|f(u) - f(v)| \geq 2$ for all edges uv of G , and $|f(u) - f(v)| \geq 1$ if $d(u, v) = 2$, where $d(u, v)$ is the distance between vertices u and v in G . The *span of an $L(2, 1)$ -coloring* is the maximum color (or label) assigned by it. The *span of a graph G* is the smallest integer λ such that there exists an $L(2, 1)$ -coloring of G with span λ . An $L(2, 1)$ -coloring of a graph with span equal to the span of the graph is called a *span coloring*. For an $L(2, 1)$ -coloring f of a graph G with span k , an integer h is a *hole* in f if $h \in (0, k)$ and there is no vertex v in G such that $f(v) = h$. A *no-hole coloring* is an $L(2, 1)$ -coloring with no hole in it. An $L(2, 1)$ -coloring is *irreducible* if color of none of the vertices in the graph can be decreased to yield another $L(2, 1)$ -coloring of the same graph. A graph G is *inh-colorable* if there exists an irreducible no-hole coloring of G . Most of the results obtained in this paper are answers to some problems asked by Laskar *et al.* [5]. These problems are mainly about relationship between the span and maximum no-hole span of a graph, lower inh-span and upper inh-span of a graph, and the maximum number of holes and minimum number of holes in a span coloring of a graph. We also give some sufficient conditions for a tree and an unicyclic graph to have inh-span $\Delta + 1$.

Keywords: $L(2, 1)$ -coloring, span of a graph, no-hole coloring, irreducible coloring, unicyclic graph.

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REFERENCES

- [1] P.C. Fishburn and F.S. Roberts, *No-hole $L(2, 1)$ -colorings*, Discrete Appl. Math. **130** (2003) 513–519.
doi:10.1016/S0166-218X(03)00329-9
- [2] J.P. Georges, D.W. Mauro and M.A. Whittlesey, *Relating path coverings to vertex labellings with a condition at distance two*, Discrete Math. **135** (1994) 103–111.
doi:10.1016/0012-365X(93)E0098-O
- [3] J.R. Griggs and R.K. Yeh, *Labelling graphs with a condition at distance 2*, SIAM J. Discrete Math. **5** (1992) 586–595.
doi:10.1137/0405048
- [4] R. Laskar and G. Eyabi, *Holes in $L(2, 1)$ -coloring on certain classes of graphs*, AKCE Int. J. Graphs Comb. **6** (2009) 329–339.
- [5] R.C. Laskar, J. Jacob and J. Lyle, *Variations of graph coloring, domination and combinations of both: a brief survey*, Advances in Discrete Mathematics and Applications, Ramanujan Mathematical Society Lecture Notes Series **13** (2010) 133–152.
- [6] R.C. Laskar, G.L. Matthews, B. Novick and J. Villalpando, *On irreducible no-hole $L(2, 1)$ -coloring of trees*, Networks **53** (2009) 206–211.
doi:10.1002/net.20286
- [7] R.C. Laskar and J.J. Villalpando, *Irreducibility of $L(2, 1)$ -coloring and inh-colorability of unicyclic and hex graphs*, Util. Math. **69** (2006) 65–83.
- [8] W.F. Wang, *The $L(2, 1)$ -labelling of trees*, Discrete Appl. Math. **154** (2006) 598–603.
doi:10.1016/j.dam.2005.09.007
- [9] D.B. West, Introduction to Graph Theory (New Delhi, Prentice-Hall, 2003).
- [10] M.Q. Zhai, C.H. Lu and J.L. Shu, *A note on $L(2, 1)$ -labelling of trees*, Acta Math. Appl. Sin. Engl. Ser. **28** (2012) 395–400.
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