BOUNDS ON THE DISJUNCTIVE TOTAL DOMINATION NUMBER OF A TREE

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Abstract

Let G be a graph with no isolated vertex. In this paper, we study a parameter that is a relaxation of arguably the most important domination parameter, namely the total domination number, $\gamma_t(G)$. A set S of vertices in G is a disjunctive total dominating set of G if every vertex is adjacent to a vertex of S or has at least two vertices in S at distance 2 from it. The disjunctive total domination number, $\gamma_t^d(G)$, is the minimum cardinality of such a set. We observe that $\gamma_t^d(G) \leq \gamma_t(G)$. A leaf of G is a vertex of degree 1, while a support vertex of G is a vertex adjacent to a leaf. We show that if T is a tree of order n with ℓ leaves and s support vertices, then $2(n-\ell+3)/5 \leq \gamma_t^d(T) \leq (n+s-1)/2$ and we characterize the families of trees which attain these bounds. For every tree T , we show have $\gamma_t(T)/\gamma_t^d(T) < 2$ and this bound is asymptotically tight.

Keywords: total domination, disjunctive total domination, trees.

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REFERENCES

- [1] D. Archdeacon, J. Ellis-Monaghan, D. Fischer, D. Froncek, P.C.B. Lam, S. Seager, B. Wei and R. Yuster, *Some remarks on domination*, J. Graph Theory **46** (2004) 207–210.
doi:10.1002/jgt.20000
- [2] R.C. Brigham, J.R. Carrington and R.P. Vitray, *Connected graphs with maximum total domination number*, J. Combin. Math. Combin. Comput. **34** (2000) 81–96.
- [3] V. Chvátal and C. McDiarmid, *Small transversals in hypergraphs*, Combinatorica **12** (1992) 19–26.
doi:10.1007/BF01191201
- [4] M. Chellali and T.W. Haynes, *Total and paired-domination numbers of a tree*, AKCE Int. J. Graphs Comb. **1** (2004) 69–75.
- [5] M. Chellali and T.W. Haynes, *A note on the total domination number of a tree*, J. Combin. Math. Combin. Comput. **58** (2006) 189–193.
- [6] F. Chung, *Graph theory in the information age*, Notices Amer. Math. Soc. **57** (2010) 726–732.
- [7] E.J. Cockayne, R.M. Dawes, and S.T. Hedetniemi, *Total domination in graphs*, Networks **10** (1980) 211–219.
doi:10.1002/net.3230100304
- [8] W. Goddard, M.A. Henning and C.A. McPillan, *The disjunctive domination number of a graph*, Quaest. Math. **37** (2014) 547–561.
doi:10.2989/16073606.2014.894688
- [9] M.A. Henning, *A survey of selected recent results on total domination in graphs*, Discrete Math. **309** (2009) 32–63.
doi:10.1016/j.disc.2007.12.044
- [10] M.A. Henning, *Graphs with large total domination number*, J. Graph Theory **35** (2000) 21–45.
doi:10.1002/1097-0118(200009)35:1<21::AID-JGT3>3.0.CO;2-F
- [11] M.A. Henning and V. Naicker, *Disjunctive total domination in graphs*, J. Comb. Optim., to appear.
doi:10.1007/s10878-014-9811-4
- [12] M.A. Henning and V. Naicker, *Graphs with large disjunctive total domination number*, Discrete Math. Theoret. Comput. Sci. **17** (2015) 255–282.
- [13] M.A. Henning and A. Yeo, *Total Domination in Graphs* (Springer Monographs in Mathematics, 2013).
doi:10.1007/978-1-4614-6525-6
- [14] Zs. Tuza, *Covering all cliques of a graph*, Discrete Math. **86** (1990) 117–126.
doi:10.1016/0012-365X(90)90354-K

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