PRODUCTS OF DIGRAPHS AND THEIR COMPETITION GRAPHS

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Abstract

If $D = (V, A)$ is a digraph, its *competition graph (with loops)* $CG^l(D)$ has the vertex set V and $\{u, v\} \subseteq V$ is an edge of $CG^l(D)$ if and only if there is a vertex $w \in V$ such that $(u, w), (v, w) \in A$. In $CG^l(D)$, loops $\{v\}$ are allowed only if v is the only predecessor of a certain vertex $w \in V$. For several products $D_1 \circ D_2$ of digraphs D_1 and D_2 , we investigate the relations between the competition graphs of the factors D_1, D_2 and the competition graph of their product $D_1 \circ D_2$.

Keywords: competition graph, product of digraphs.

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