A FAN-TYPE HEAVY PAIR OF SUBGRAPHS FOR PANCYCLICITY OF 2-CONNECTED GRAPHS

WOJCIECH WIDEL

AGH University of Science and Technology
Faculty of Applied Mathematics
Department of Discrete Mathematics
al. A. Mickiewicza 30, 30-059 Kraków, Poland
e-mail: widel@agh.edu.pl

Abstract

Let G be a graph on n vertices and let H be a given graph. We say that G is pancyclic, if it contains cycles of all lengths from 3 up to n , and that it is H - f_1 -heavy, if for every induced subgraph K of G isomorphic to H and every two vertices $u, v \in V(K)$, $d_K(u, v) = 2$ implies $\min\{d_G(u), d_G(v)\} \geq \frac{n+1}{2}$. In this paper we prove that every 2-connected $\{K_{1,3}, P_5\}$ - f_1 -heavy graph is pancyclic. This result completes the answer to the problem of finding f_1 -heavy pairs of subgraphs implying pancyclicity of 2-connected graphs.

Keywords: cycle, Fan-type heavy subgraph, Hamilton cycle, pancyclicity.

2010 Mathematics Subject Classification: 05C38, 05C45.

REFERENCES

- [1] P. Bedrossian, Forbidden subgraph and Minimum Degree Conditions for Hamiltonicity, PhD Thesis (Memphis State University, USA, 1991).
- [2] P. Bedrossian, G. Chen and R.H. Schelp, *A generalization of Fan's condition for Hamiltonicity, pancyclicity and Hamiltonian connectedness*, Discrete Math. **115** (1993) 39–59.
doi:10.1016/0012-365X(93)90476-A
- [3] A. Benhocine and A.P. Wojda, *The Geng-Hua Fan conditions for pancyclic or Hamilton-connected graphs*, J. Combin. Theory Ser. B **58** (1987) 167–180.
doi:10.1016/0095-8956(87)90038-4
- [4] J.A. Bondy, *Pancyclic graphs I*, J. Combin. Theory Ser. B **11** (1971) 80–84.
doi:10.1016/0095-8956(71)90016-5

- [5] J.A. Bondy and U.S.R. Murty, *Graph Theory with Applications* (Macmillan London and Elsevier, 1976).
- [6] G. Fan, *New sufficient conditions for cycles in graphs*, J. Combin. Theory Ser. B **37** (1984) 221–227.
doi:0.1016/0095-8956(84)90054-6
- [7] M. Ferrara, M.S. Jacobson and A. Harris, *Cycle lengths in Hamiltonian graphs with a pair of vertices having large degree sum*, Graphs Combin. **26** (2010) 215–223.
doi:10.1007/s00373-010-0915-z
- [8] B. Ning, *Pairs of Fan-type heavy subgraphs for pancyclicity of 2-connected graphs*, Australas. J. Combin. **58** (2014) 127–136.
- [9] B. Ning and S. Zhang, *Ore- and Fan-type heavy subgraphs for Hamiltonicity of 2-connected graphs*, Discrete Math. **313** (2013) 1715–1725.
doi:10.1016/j.disc.2013.04.023
- [10] E.F. Schmeichel and S.L. Hakimi, *A cycle structure theorem for Hamiltonian graphs*, J. Combin. Theory Ser. B **45** (1988) 99–107.
doi:10.1016/0095-8956(88)90058-5

Received 10 December 2014

Revised 28 May 2015

Accepted 28 May 2015