NOTE

CHARACTERIZATION OF LINE-CONSISTENT SIGNED GRAPHS

DANIEL C. SLILATY

*Wright State University
Dayton, OH 45435-0001, USA*

e-mail: daniel.slilaty@wright.edu

AND

THOMAS ZASLAVSKY

*Binghamton University (SUNY)
Binghamton, NY 13902-6000, USA*

e-mail: zaslav@math.binghamton.edu

Abstract

The line graph of a graph with signed edges carries vertex signs. A vertex-signed graph is *consistent* if every circle (cycle, circuit) has positive vertex-sign product. Acharya, Acharya, and Sinha recently characterized *line-consistent* signed graphs, i.e., edge-signed graphs whose line graphs, with the naturally induced vertex signature, are consistent. Their proof applies Hoede's relatively difficult characterization of consistent vertex-signed graphs. We give a simple proof that does not depend on Hoede's theorem as well as a structural description of line-consistent signed graphs.

Keywords: line-consistent signed graph, line graph, consistent vertex-signed graph, consistent marked graph.

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REFERENCES

- [1] B.D. Acharya, *A characterization of consistent marked graphs*, Nat. Acad. Sci. Letters (India) **6** (1983) 431–440.
- [2] B.D. Acharya, *Some further properties of consistent marked graphs*, Indian J. Pure Appl. Math. **15** (1984) 837–842.
- [3] B.D. Acharya, M. Acharya and D. Sinha, *Cycle-compatible signed line graphs*, Indian J. Math. **50** (2008) 407–414.

- [4] B.D. Acharya, M. Acharya and D. Sinha, *Characterization of a signed graph whose signed line graph is S-consistent*, Bull. Malaysian Math. Sci. Soc. (2) **32** (2009) 335–341.
- [5] M. Acharya, $\times$ -line signed graphs, J. Combin. Math. Combin. Comput. **69** (2009) 103–111.
- [6] M. Behzad and G. Chartrand, *Line-coloring of signed graphs*, Elem. Math. **24** (1969) 49–52.
- [7] L.W. Beineke and F. Harary, *Consistent graphs with signed points*, Riv. Mat. Sci. Econom. Social. **1** (1978) 81–88.
doi:10.1002/jgt.3190160104
- [8] F. Harary, *On the notion of balance of a signed graph*, Michigan Math. J. **2** (1953–54) 143–146 and addendum preceding p. 1.
- [9] C. Hoede, *A characterization of consistent marked graphs*, J. Graph Theory **16** (1992) 17–23.
doi:10.1002/jgt.3190160104
- [10] M. Joglekar, N. Shah and A.A. Diwan, *Balanced group labeled graphs*, Discrete Math. **312** (2012) 1542–1549.
doi:10.1016/j.disc.2011.09.021
- [11] S.B. Rao, *Characterizations of harmonious marked graphs and consistent nets*, J. Comb. Inf. Syst. Sci. **9** (1984) 97–112.
- [12] F.S. Roberts and S. Xu, *Characterizations of consistent marked graphs*, Discrete Appl. Math. **127** (2003) 357–371.
doi:10.1016/S0166-218X(02)00254-8
- [13] T. Zaslavsky, *Matrices in the theory of signed simple graphs*, Advances in Discrete Mathematics and Applications: Mysore, 2008, B.D. Acharya, G.O.H. Katona, and J. Nešetřil, Eds., Ramanujan Math. Soc., Mysore, India (2010) 207–229.
- [14] T. Zaslavsky, *Consistency in the naturally vertex-signed line graph of a signed graph*, Bull. Malaysian Math. Sci. Soc., to appear.

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