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THE SATURATION NUMBER FOR THE LENGTH OF DEGREE MONOTONE PATHS

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Abstract

A degree monotone path in a graph G is a path P such that the sequence of degrees of the vertices in the order in which they appear on P is monotonic. The length (number of vertices) of the longest degree monotone path in G is denoted by mp(G). This parameter, inspired by the well-known Erdős-Szekeres theorem, has been studied by the authors in two earlier papers. Here we consider a saturation problem for the parameter mp(G). We call G saturated if, for every edge e added to G, mp(G + e) > mp(G), and we define h(n, k) to be the least possible number of edges in a saturated graph G on n vertices with mp(G) < k, while $mp(G + e) \ge k$ for every new edge e.

We obtain linear lower and upper bounds for h(n, k), we determine exactly the values of h(n, k) for k = 3 and 4, and we present constructions of saturated graphs.

Keywords: paths, degrees, saturation.

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