DECOMPOSABILITY OF ABSTRACT AND PATH-INDUCED CONVEXITIES IN HYPERGRAPHS

FRANCESCO MARIO MALVESTUTO

AND

MARINA MOSCARINI

*Department of Informatics
Sapienza University of Rome
Via Salaria 113, 00198 Roma, Italy*

e-mail: malvestuto@di.uniroma1.it
moscarini@di.uniroma1.it

Abstract

An *abstract convexity space* on a connected hypergraph H with vertex set $V(H)$ is a family C of subsets of $V(H)$ (to be called the *convex sets* of H) such that: (i) C contains the empty set and $V(H)$, (ii) C is closed under intersection, and (iii) every set in C is connected in H . A convex set X of H is a *minimal vertex convex separator* of H if there exist two vertices of H that are separated by X and are not separated by any convex set that is a proper subset of X . A nonempty subset X of $V(H)$ is a *cluster* of H if in H every two vertices in X are not separated by any convex set. The *cluster hypergraph* of H is the hypergraph with vertex set $V(H)$ whose edges are the maximal clusters of H . A convexity space on H is called *decomposable* if it satisfies the following three properties:

- (C1) the cluster hypergraph of H is acyclic,
- (C2) every edge of the cluster hypergraph of H is convex,
- (C3) for every nonempty proper subset X of $V(H)$, a vertex v does not belong to the convex hull of X if and only if v is separated from X in H by a convex cluster.

It is known that the *monophonic convexity* (i.e., the convexity induced by the set of chordless paths) on a connected hypergraph is decomposable.

In this paper we first provide two characterizations of decomposable convexities and then, after introducing the notion of a *hereditary path family* in a connected hypergraph H , we show that the convexity space on H induced

by any hereditary path family containing all chordless paths (such as the families of simple paths and of all paths) is decomposable.

Keywords: convex hull, hypergraph convexity, path-induced convexity, convex geometry.

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REFERENCES

- [1] C. Beeri, R. Fagin, D. Maier and M. Yannakakis, *On the desirability of acyclic database schemes*, J. ACM **30** (1983) 479–513.
doi:10.1145/2402.322389
- [2] M. Changat and J. Mathew, *On triangle path convexity in graphs*, Discrete Math. **206** (1999) 91–95.
doi:10.1016/S0012-365X(98)00394-X
- [3] M. Changat, H.M. Mulder and G. Sierksma, *Convexities related to path properties on graphs*, Discrete Math. **290** (2005) 117–131.
doi:10.1016/j.disc.2003.07.014
- [4] R. Diestel, *Graph Decompositions: A Study in Infinity Graph Theory* (Clarendon Press, Oxford, 1990).
- [5] P. Duchet, *Convexity in combinatorial structures*, in: Proceedings of the 14th Winter School on Abstract Analysis, Frolík, Souček and Fabián (Eds), (Circolo Matematico di Palermo, Palermo 1987), Serie II **14** 261–293
- [6] P. Duchet, *Convex sets in graphs II: minimal path convexity*, J. Combin. Theory Ser. B **44** (1988) 307–316.
doi:10.1016/0095-8956(88)90039-1
- [7] P. Duchet, *Discrete convexity: retractions, morphisms and the partition problem*, in: Proceedings of the Conference on Graph Connections, Balakrishnan, Mulder and Vijayakumar (Ed(s)), (Allied Publishers, New Delhi, 1999) 10–18.
- [8] M. Farber and R.E. Jamison, *Convexity in graphs and hypergraphs*, SIAM J. Algebraic Discrete Methods **7** (1986) 433–444.
doi:10.1137/0607049
- [9] H.-G. Leimer, *Optimal decomposition by clique separators*, Discrete Math. **113** (1993) 99–123.
doi:10.1016/0012-365X(93)90510-Z
- [10] F.M. Malvestuto, *Canonical and monophonic convexities in hypergraphs*, Discrete Math. **309** (2009) 4287–4298.
doi:10.1016/j.disc.2009.01.003
- [11] F.M. Malvestuto, *Decomposable convexities in graphs and hypergraphs*, ISRN Combinatorics **2013** Article ID 453808.
doi:10.1155/2013/453808

- [12] F.M. Malvestuto, M. Mezzini and M. Moscarini, *Equivalence between hypergraph convexities* ISRN Discrete Mathematics **2011** Article ID 806193.
doi:10.5402/2011/806193
- [13] R.E. Tarjan, *Decomposition by clique separators*, Discrete Math. **55** (1985) 221–232.
doi:10.1016/0012-365X(85)90051-2
- [14] M. Van de Vel, *Theory of Convex Structures* (North-Holland Publishing Co., Amsterdam, 1993).
- [15] S. Whitesides, *An Algorithm for finding clique cut-sets*, Inform. Process. Lett. **12** (1981) 31–32.
doi:10.1016/0020-0190(81)90072-7

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