IMPROVED SUFFICIENT CONDITIONS FOR HAMILTONIAN PROPERTIES

JENS-P. BODE¹, ANIKA FRICKE²

AND

ARNFRIED KEMNITZ¹

¹*Computational Mathematics*
Technische Universität Braunschweig
38092 Braunschweig, Germany

²*Zentrum für erfolgreiches Lehren und Lernen*
Ostfalia Hochschule für angewandte Wissenschaften
38302 Wolfenbüttel, Germany

e-mail: jp.bode@tu-bs.de
Anika.Fricke@ostfalia.de
a.kemnitz@tu-bs.de

Abstract

In 1980 Bondy [2] proved that a $(k+s)$ -connected graph of order $n \geq 3$ is traceable ($s = -1$) or Hamiltonian ($s = 0$) or Hamiltonian-connected ($s = 1$) if the degree sum of every set of $k+1$ pairwise nonadjacent vertices is at least $((k+1)(n+s-1)+1)/2$. It is shown in [1] that one can allow exceptional $(k+1)$ -sets violating this condition and still implying the considered Hamiltonian property. In this note we generalize this result for $s = -1$ and $s = 0$ and graphs that fulfill a certain connectivity condition.

Keywords: Hamiltonian, traceable, Hamiltonian-connected.

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