THE k -RAINBOW BONDAGE NUMBER OF A DIGRAPH

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Abstract

Let $D = (V, A)$ be a finite and simple digraph. A k -rainbow dominating function (k RDF) of a digraph D is a function f from the vertex set V to the set of all subsets of the set $\{1, 2, \dots, k\}$ such that for any vertex $v \in V$ with $f(v) = \emptyset$ the condition $\bigcup_{u \in N^-(v)} f(u) = \{1, 2, \dots, k\}$ is fulfilled, where $N^-(v)$ is the set of in-neighbors of v . The *weight* of a k RDF f is the value $\omega(f) = \sum_{v \in V} |f(v)|$. The k -rainbow domination number of a digraph D , denoted by $\gamma_{rk}(D)$, is the minimum weight of a k RDF of D . The k -rainbow bondage number $b_{rk}(D)$ of a digraph D with maximum in-degree at least two, is the minimum cardinality of all sets $A' \subseteq A$ for which $\gamma_{rk}(D - A') > \gamma_{rk}(D)$. In this paper, we establish some bounds for the k -rainbow bondage number and determine the k -rainbow bondage number of several classes of digraphs.

Keywords: k -rainbow dominating function, k -rainbow domination number, k -rainbow bondage number, digraph.

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REFERENCES

- [1] J. Amjadi, A. Bahremandpour, S.M. Sheikholeslami and L. Volkmann, *The rainbow domination number of a digraph*, Kragujevac J. Math. **37** (2013) 257–268.
- [2] B. Brešar, M.A. Henning and D.F. Rall, *Rainbow domination in graphs*, Taiwanese J. Math. **12** (2008) 213–225.
- [3] B. Brešar and T.K. Šumenjak, *On the 2-rainbow domination in graphs*, Discrete Appl. Math. **155** (2007) 2394–2400.
doi:10.1016/j.dam.2007.07.018
- [4] G.J. Chang, J. Wu and X. Zhu, *Rainbow domination on trees*, Discrete Appl. Math. **158** (2010) 8–12.
doi:10.1016/j.dam.2009.08.010
- [5] Ch. Tong, X. Lin, Y. Yang and M.Luo, *2-rainbow domination of generalized Petersen graphs $P(n, 2)$* , Discrete Appl. Math. **157** (2009) 1932–1937.
doi:10.1016/j.dam.2009.01.020
- [6] N. Dehgard, S.M. Sheikholeslami and L. Volkmann, *The k -rainbow bondage number of a graph*, Discrete Appl. Math. **174** (2014) 133–139.
doi:10.1016/j.dam.2014.05.006
- [7] J.F. Fink, M.S. Jacobson, L.F. Kinch and J. Roberts, *The bondage number of a graph*, Discrete Math. **86** (1990) 47–57.
doi:10.1016/0012-365X(90)90348-L
- [8] T.W. Haynes, S.T. Hedetniemi and P.J. Slater, *Fundamentals of Domination in Graphs* (Marcel Dekker, Inc. New York, 1998).
- [9] D. Meierling, S.M. Sheikholeslami and L. Volkmann, *Nordhaus-Gaddum bounds on the k -rainbow domatic number of a graph*, Appl. Math. Lett. **24** (2011) 1758–1761.
doi:10.1016/j.aml.2011.04.046
- [10] S.M. Sheikholeslami and L. Volkmann, *The k -rainbow domatic number of a graph*, Discuss. Math. Graph Theory **32** (2012) 129–140.
doi:10.7151/dmgt.1591
- [11] D.B. West, *Introduction to Graph Theory* (Prentice-Hall, Inc., 2000).
- [12] Y. Wu and N. Jafari Rad, *Bounds on the 2-rainbow domination number of graphs*, Graphs Combin. **29** (2013) 1125–1133.
doi:10.1007/s00373-012-1158-y
- [13] G. Xu, *2-rainbow domination in generalized Petersen graphs $P(n, 3)$* , Discrete Appl. Math. **157** (2009) 2570–2573.
doi:10.1016/j.dam.2009.03.016

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