THE 1, 2, 3-CONJECTURE AND 1, 2-CONJECTURE FOR SPARSE GRAPHS

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Abstract

The 1, 2, 3-Conjecture states that the edges of a graph without isolated edges can be labeled from $\{1, 2, 3\}$ so that the sums of labels at adjacent vertices are distinct. The 1, 2-Conjecture states that if vertices also receive labels and the vertex label is added to the sum of its incident edge labels, then adjacent vertices can be distinguished using only $\{1, 2\}$. We show that various configurations cannot occur in minimal counterexamples to these conjectures. Discharging then confirms the conjectures for graphs with maximum average degree less than $8/3$. The conjectures are already confirmed for larger families, but the structure theorems and reducibility results are of independent interest.

Keywords: 1, 2, 3-Conjecture, 1, 2-Conjecture, reducible configuration, discharging method.

2010 Mathematics Subject Classification: 05C15, 05C22, 05C78.

¹Research supported in part by National Science Foundation grant DMS 09-01276.

²Research supported in part by National Security Agency grant H98230-10-1-0363.

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Received 29 March 2013

Revised 13 November 2013

Accepted 13 November 2013