SOME REMARKS ON THE STRUCTURE OF STRONG k -TRANSITIVE DIGRAPHS¹

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Abstract

A digraph D is k -transitive if the existence of a directed path $(v_0, v_1, \dots, v_k)$, of length k implies that $(v_0, v_k) \in A(D)$. Clearly, a 2-transitive digraph is a transitive digraph in the usual sense. Transitive digraphs have been characterized as compositions of complete digraphs on an acyclic transitive digraph. Also, strong 3 and 4-transitive digraphs have been characterized.

In this work we analyze the structure of strong k -transitive digraphs having a cycle of length at least k . We show that in most cases, such digraphs are complete digraphs or cycle extensions. Also, the obtained results are used to prove some particular cases of the Laborde-Payan-Xuong Conjecture.

Keywords: digraph, transitive digraph, k -transitive digraph, quasi-transitive digraph, k -quasi-transitive digraph, Laborde-Payan-Xuong Conjecture.

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