THE DEPRESSION OF A GRAPH AND k -KERNELS

MARK SCHURCH AND CHRISTINE MYNHARDT

*Department of Mathematics and Statistics
University of Victoria, P.O. Box 3060 STN CSC
Victoria, BC, Canada V8W 3R4*

e-mail: mschurch@uvic.ca
mynhardt@math.uvic.ca

Abstract

An *edge ordering* of a graph G is an injection $f : E(G) \rightarrow \mathbb{R}$, the set of real numbers. A path in G for which the edge ordering f increases along its edge sequence is called an f -*ascent*; an f -ascent is *maximal* if it is not contained in a longer f -ascent. The *depression* of G is the smallest integer k such that any edge ordering f has a maximal f -ascent of length at most k . A k -kernel of a graph G is a set of vertices $U \subseteq V(G)$ such that for any edge ordering f of G there exists a maximal f -ascent of length at most k which neither starts nor ends in U . Identifying a k -kernel of a graph G enables one to construct an infinite family of graphs from G which have depression at most k . We discuss various results related to the concept of k -kernels, including an improved upper bound for the depression of trees.

Keywords: edge ordering of a graph, increasing path, monotone path, depression.

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