ON THE UNIQUENESS OF D -VERTEX MAGIC CONSTANT

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Abstract

Let $G = (V, E)$ be a graph of order n and let $D \subseteq \{0, 1, 2, 3, \dots\}$. For $v \in V$, let $N_D(v) = \{u \in V : d(u, v) \in D\}$. The graph G is said to be D -vertex magic if there exists a bijection $f : V(G) \rightarrow \{1, 2, \dots, n\}$ such that for all $v \in V$, $\sum_{u \in N_D(v)} f(u)$ is a constant, called D -vertex magic constant. O'Neal and Slater have proved the uniqueness of the D -vertex magic constant by showing that it can be determined by the D -neighborhood fractional domination number of the graph. In this paper we give a simple and elegant proof of this result. Using this result, we investigate the existence of distance magic labelings of complete r -partite graphs where $r \geq 4$.

Keywords: distance magic graph, D -vertex magic graph, magic constant, dominating function, fractional domination number.

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