ON ORDER PRIME DIVISOR GRAPHS OF FINITE GROUPS

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Abstract

The order prime divisor graph $\mathcal{PD}(G)$ of a finite group G is a simple graph whose vertex set is G and two vertices $a, b \in G$ are adjacent if and only if either $ab = e$ or $o(ab)$ is some prime number, where e is the identity element of the group G and $o(x)$ denotes the order of an element $x \in G$. In this paper, we establish the necessary and sufficient condition for the completeness of order prime divisor graph $\mathcal{PD}(G)$ of a group G . Concentrating on the graph $\mathcal{PD}(D_n)$, we investigate several properties like degrees, girth, regularity, Eulerianity, Hamiltonicity, planarity etc. We characterize some graph theoretic properties of $\mathcal{PD}(\mathbb{Z}_n)$, $\mathcal{PD}(S_n)$, $\mathcal{PD}(A_n)$.

Keywords: group, dihedral group, complete graph, Eulerian graph, regular graph, planar graph, order prime divisor graph.

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REFERENCES

- [1] J. Bosák, The Graphs of Semigroups, in: *Theory of Graphs and Application* (Academic Press, New York, 1964) 119–125.
- [2] B. Csákány and G. Pollák, *The graph of subgroups of a finite group*, Czechoslovak Math. J. **19** (1969) 241–247.
<https://doi.org/10.21136/CMJ.1969.100891>

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- [3] I. Chakrabarty, S. Ghosh, T.K. Mukherjee and M.K. Sen, *Intersection graphs of ideals of rings*, Discrete Math. **309** (2009) 5381–5392.
<https://doi.org/10.1016/j.disc.2008.11.034>
- [4] I. Chakrabarty, S. Ghosh and M.K. Sen, *Undirected power graphs of semigroups*, Semigroup Forum **78** (2009) 410–426.
<https://doi.org/10.1007/s00233-008-9132-y>
- [5] D.S. Dummit and R.M. Foote, Abstract Algebra, Third Edition (John Wiley and Sons, Inc., New York, 2004).
- [6] A.V. Kelarev and S.J. Quinn, *A combinatorial property and power graphs of groups*, Contributions to General Algebra **12** (2000) 229–235.
- [7] M. Sattanathan and R. Kala, *An introduction to order prime graph*, Int. J. Contemp. Math. Sciences **4** (2009) 467–474.
<https://doi.org/10.1111/j.1439-0507.1967.tb02798.x>
- [8] D.B. West, Introduction to Graph Theory, Second Edition (Pearson India Education Services Pvt. Ltd., 2017).
- [9] B. Zelinka, *Intersection graphs of finite Abelian groups*, Czech. Math. J. **25** (1975) 171–174.
<https://doi.org/10.21136/CMJ.1975.101307>

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