ON GENERALIZED DERIVATIONS AND COMMUTATIVITY OF ASSOCIATIVE RINGS

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Abstract

Let $\mathcal{R}$ be a ring with center $Z(\mathcal{R})$. A mapping $f : \mathcal{R} \rightarrow \mathcal{R}$ is said to be strong commutativity preserving (SCP) on $\mathcal{R}$ if $[f(x), f(y)] = [x, y]$ and is said to be strong anti-commutativity preserving (SACP) on $\mathcal{R}$ if $f(x) \circ f(y) = x \circ y$ for all $x, y \in \mathcal{R}$. In the present paper, we apply the standard theory of differential identities to characterize SCP and SACP derivations of prime and semiprime rings.

Keywords: generalized derivations, (semi)prime rings, generalized polynomial identities, Martindale ring of quotients.

2010 Mathematics Subject Classification: Primary 46J10, 16N20; Secondary 16N60, 16W25.

REFERENCES

- [1] S. Ali, V. De Filippis and M.S. Khan, *Pair of derivations of semiprime rings with applications to Banach algebras*, J. Algebra and Comp. Appl. **3** (2013) 01–13.

- [2] S. Ali and S. Huang, *On derivations in semiprime rings*, Algebra and Rep. Theory **15** (2012) 1023–1033.
doi:10.1007/s10468-011-9271-9
- [3] M. Ashraf and N. Rehman, *On commutativity of rings with derivations*, Results Math. **42** (2002) 3–8.
doi:10.1007/BF03323547
- [4] M. Ashraf, N. Rehman and M.A. Raza, *A note on commutativity of semiprime Banach algebras*, Beitr Algebra Geom. **57** (2016) 553–560.
doi:10.1007/s13366-015-0264-4
- [5] K.I. Beidar, W.S. Martindale III and A.V. Mikhalev, *Rings with generalized identities* (Pure and Applied Mathematics, 196 Marcel Dekker, New York, 1996).
- [6] K.I. Beidar, *Rings with generalized identities III*, Moscow Univ. Math. Bull. **33** (1978) 53–58.
- [7] H.E. Bell and M.N. Daif, *On commutativity and strong commutativity preserving maps*, Canad. Math. Bull. **37** (1994) 443–447.
doi:10.4153/CMB-1994-064-x
- [8] H.E. Bell and N. Rehman, *Generalized derivations with commutativity and anti-commutativity conditions*, Math. J. Okayama Univ. **49** (2007) 139–147.
- [9] J. Bergen, I.N. Herstein and J. W. Kerr, *Lie ideals and derivations of prime rings*, J. Algebra **71** (1981) 259–267.
doi:10.1016/0021-8693(81)90120-4
- [10] M. Bresar and C.R. Miers, *Strong commutativity preserving maps of semiprime rings*, Canad. Math. Bull. **37** (1994) 457–460.
doi:10.4153/CMB-1994-066-4
- [11] C.L. Chuang, *GPIs having coefficients in Utumi quotient rings*, Proc. Amer. Math. Soc. **103** (1988) 723–728.
doi:10.1090/S0002-9939-1988-0947646-4
- [12] C.L. Chuang, *Hypercentral derivations*, J. Algebra **166** (1994) 34–71.
doi:10.1006/jabr.1994.1140
- [13] C.L. Chuang, *The additive subgroup generated by a polynomial*, Israel J. Math. **59** (1987) 98–106.
doi:10.1007/BF02779669
- [14] Q. Deng and M. Ashraf, *On strong commutativity preserving mappings*, Result. Math. **30** (1996) 259–263.
doi:10.1007/BF03322194
- [15] T.S. Erickson, W.S. Martindale III and J. M. Osborn, *Prime nonassociative algebras*, Pacific J. Math. **60** (1975) 49–63.
doi:10.2140/pjm.1975.60.49

- [16] V. De Filippis and G. Scudo, *Strong commutativity and Engel condition preserving maps in prime and semiprime rings*, Linear and Multilinear Algebra **61** (2013) 917–938.
doi:10.1080/03081087.2012.716433
- [17] I.N. Herstein, *A note on derivations*, Canad. Math. Bull. **21** (1978) 369–370.
doi:10.4153/CMB-1978-065-x
- [18] I.N. Herstein, *Center-like elements in prime rings*, J. Algebra **60** (1979) 567–574.
doi:10.1016/0021-8693(79)90102-9
- [19] S. Huang, *Generalized derivations of prime and semiprime rings*, Taiwanese J. Math. **6** (2012) 771–776.
doi:10.11650/twjm/1500406614
- [20] S. Huang and B. Davvaz, *Generalized derivations of rings and Banach algebras*, Commun. Algebra **41** (2013) 1188–1194.
doi:10.1080/00927872.2011.642043
- [21] N. Jacobson, *Structure of rings* (Colloquium publications, vol. XXXVIII, Amer. Math. Soc. 190, Hope street, providence, R.I., 1956).
- [22] V.K. Kharchenko, *Differential identities of prime rings*, Algebra Logic **17** (1979) 155–168.
doi:10.1007/BF01670115
- [23] C. Lanski, *An Engel condition with derivation*, Proc. Amer. Math. Soc. **118** (1993) 731–734.
doi:10.2307/2160113
- [24] T.K. Lee, *Generalized derivations of left faithful rings*, Comm. Algebra **27** (1999) 4057–4073.
doi:10.1080/00927879908826682
- [25] T.K. Lee, *Semiprime rings with differential identities*, Bull. Inst. Math. Acad. Sin. **20** (1992) 27–38.
- [26] P.H. Lee and T.L. Wong, *Derivations cocentralizing Lie ideals*, Bull. Ins. Math. Acad. Sin. **23** (1995) 1–5.
- [27] C.K. Liu and P.K. Liao, *Strong commutativity preserving generalized derivations on Lie ideals*, Linear and Multilinear Algebra **59** (2011) 905–915.
doi:10.1080/03081087.2010.535819
- [28] J. Ma, X.W. Xu and F.W. Niu, *Strong commutativity preserving generalized derivations on semiprime rings*, Acta Math. Sin. **24** (2008) 1835–1842.
doi:10.1007/s10114-008-7445-0
- [29] W.S. Martindale III, *Prime rings satisfying a generalized polynomial identity*, J. Algebra **12** (1969) 576–584.
doi:10.1016/0021-8693(69)90029-5
- [30] L. Oukhtite, *Posner’s second theorem for Jordan ideals in rings with involutions*, Expo. Math. **29** (2011) 415–419.
doi:10.1016/j.exmath.2011.07.002

- [31] G.S. Sandhu and D. Kumar, *A note on derivations and Jordan ideals of prime rings*, AIMS Math. **2** (2017) 580–585.
doi:10.3934/Math.2017.4.580
- [32] X.W. Xu, *The Power Values Properties of Generalized Derivations* (Doctoral Thesis of Jilin University, Changchun, 2006).

Received 30 December 2018

Revised 10 October 2019

Accepted 24 January 2020