ORDERED REGULAR SEMIGROUPS WITH BIGGEST ASSOCIATES

T.S. BLYTH

Mathematical Institute
University of St Andrews, Scotland
e-mail: tsb@st-andrews.ac.uk

AND

M.H. ALMEIDA SANTOS

Centro de Matemática e Aplicações (CMA)
Departamento de Matemática, FCT
Universidade Nova de Lisboa, Portugal
e-mail: mhas@fct.unl.pt

Abstract

We investigate the class **BA** of ordered regular semigroups in which each element has a biggest associate $x^\dagger = \max\{y \mid xyx = x\}$. This class properly contains the class **PO** of principally ordered regular semigroups (in which there exists $x^\star = \max\{y \mid xyx \leq x\}$) and is properly contained in the class **BI** of ordered regular semigroups in which each element has a biggest inverse x° . We show that several basic properties of the unary operation $x \mapsto x^\star$ in **PO** extend to corresponding properties of the unary operation $x \mapsto x^\dagger$ in **BA**. We consider naturally ordered semigroups in **BA** and prove that those that are orthodox contain a biggest idempotent. We determine the structure of some such semigroups in terms of a principal left ideal and a principal right ideal. We also characterise the completely simple members of **BA**. Finally, we consider the naturally ordered semigroups in **BA** that do not have a biggest idempotent.

Keywords: regular semigroup, biggest associate, principally ordered, naturally ordered.

2010 Mathematics Subject Classification: 20M20, 20M17.

This work was partially supported by the Fundação para a Ciência e a Tecnologia (Portuguese Foundation for Science and Technology) through the project UID/MAT/00297/2013 (Centro de Matemática e Aplicações).

REFERENCES

- [1] T.S. Blyth, *Lattices and Ordered Algebraic Structures* (Springer, 2005).
doi:10.1007/b139095
- [2] T.S. Blyth, *On the endomorphism semigroup of an ordered set*, Glasgow Math. J. **37** (1995) 173–178.
doi:10.1017/S0017089500031074
- [3] T.S. Blyth and R. McFadden, *Naturally ordered regular semigroups with a greatest idempotent*, Proc. Roy. Soc. Edinburgh **91A** (1981) 107–122.
doi:10.1017/S0308210500012671
- [4] T.S. Blyth and G.A. Pinto, *Principally ordered regular semigroups*, Glasgow Math. J. **32** (1990) 349–364.
doi:10.1017/S0017089500009435
- [5] T.S. Blyth and M.H. Almeida Santos, *On weakly multiplicative inverse transversals*, Proc. Edinburgh Math. Soc. **37** (1993) 91–99.
doi:10.1017/s001309150001871x
- [6] T.S. Blyth and M.H. Almeida Santos, *On naturally ordered regular semigroups with biggest idempotents*, Communications in Algebra **21** (1993) 1761–1771.
doi:10.1080/00927879308824651
- [7] T.S. Blyth and M.H. Almeida Santos, *E-special ordered regular semigroups*, Communications in Algebra **43** (2015) 3294–3313.
doi:10.1080/00927872.2014.918987
- [8] R. Croisot, *Demi-groupes inversifs et demi-groupes réunions de demi-groupes simples*, Ann. Sci. École Norm. Sup. **70** (1953) 361–379.
doi:10.24033/asens.1015
- [9] P.A. Grillet, *Semigroups* (Marcel Dekker, New York, 1995).
doi:10.1201/9780203739938
- [10] R.D. Luce, *A note on boolean matrix theory*, Proc. Amer. Math. Soc. **3** (1952) 382–388.
doi:10.2307/2031888
- [11] D.B. McAlister, *Regular Rees matrix semigroups and regular Dubreil-Jacotin semigroups*, J. Australian Math. Soc. **31** (1981) 325–336.
doi:10.1017/S1446788700019467
- [12] K.S.S. Nambooripad, *The natural partial order on a regular semigroup*, Proc. Edinburgh Math. Soc. **23** (1980) 249–260.
doi:10.1017/S0013091500003801
- [13] T. Saito, *Naturally ordered regular semigroups with greatest inverses*, Proc. Edinburgh Math. Soc. **32** (1989) 33–39.
doi:10.1017/S001309150000688x

Received 23 May 2018

Revised 1 December 2018

Accepted 3 December 2018