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**A NEW EXISTENCE RESULTS FOR FRACTIONAL
INTEGRO-DIFFERENTIAL EQUATIONS OF ORDER
 $\alpha \in (1, 2]$ WITH NONLOCAL CONDITIONS IN
BANACH SPACES**

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Abstract

In this manuscript, we examine a class of fractional integro-differential equations (abbreviated by, FIDEs) of order $\alpha \in (1, 2]$ with nonlocal conditions (abbreviated by, NLCs) in Banach spaces. In the beginning, a more

suitable idea for mild solution is presented and verified. Then, existence of mild solution is investigated by utilizing the fractional calculus, semigroup theory and fixed point theorems with the strongly continuous α -order cosine family. At last, an illustration is provided to exhibit the obtained results.

Keywords: fractional integro-differential equation, nonlocal condition, fractional cosine family, fixed point theorem.

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