

ON EXISTENCE OF SOLUTIONS TO DEGENERATE NONLINEAR OPTIMIZATION PROBLEMS

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Abstract

We investigate the existence of the solution to the following problem

$$\min \varphi(x) \text{ subject to } G(x) = 0,$$

where $\varphi : X \rightarrow \mathbb{R}$, $G : X \rightarrow Y$ and X, Y are Banach spaces. The question of existence is considered in a neighborhood of such point x_0 that the Hessian of the Lagrange function is degenerate. There was obtained an approximation for the distance of solution x^* to the initial point x_0 .

Keywords: Lagrange function, necessary condition of optimality, p -regularity, contracting mapping, p -factor operator.

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