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EXTREMAL GRAPHS FOR A BOUND ON THE ROMAN DOMINATION NUMBER

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Abstract

A Roman dominating function on a graph $G = (V, E)$ is a function $f{:}V(G) \longrightarrow \{0, 1, 2\}$ such that every vertex u for which $f(u) = 0$ is adjacent to at least one vertex v with $f(v) = 2$. The weight of a Roman dominating function is the value $w(f) = \sum_{u \in V(G)} f(u)$. The minimum weight of a Roman dominating function on a graph G is called the Roman domination number of G, denoted by $\gamma_R(G)$. In 2009, Chambers, Kinnersley, Prince and West proved that for any graph G with n vertices and maximum degree Δ, $\gamma_R(G) \leq n + 1 - \Delta$. In this paper, we give a characterization of graphs attaining the previous bound including trees, regular and semiregular graphs. Moreover, we prove that the problem of deciding whether $\gamma_R(G) = n + 1 - \Delta$ is *co*-$\mathcal{NP}$-complete. Finally, we provide a characterization of extremal graphs of a Nordhaus–Gaddum bound for $\gamma_R(G) + \gamma_R(\overline{G})$, where $\overline{G}$ is the complement graph of G.

Keywords: Roman domination, Roman domination number, Nordhaus-Gaddum inequalities.

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