Comparison of transverse mode instability in Yb, Tm and Ho doped bent high-power fiber amplifiers

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S: miembro de SEDOPTICA / SEDOPTICA member

Received: 08/10/2021

Accepted: 02/12/2021

DOI: 10.7149/OPA.54.4.51071

ABSTRACT:

In this paper, the transverse mode instability of three different high power silica fiber amplifiers doped with rare earth ions, Ytterbium, Thulium and Holmium is compared. Mode instability in these amplifiers has been investigated using the normalized propagation constant model in straight and bent fibers. The behavior of three amplifiers with different signal and pump wavelengths, though with the same V -number, refractive index of cladding, core diameter, cladding diameter and the thermophysical constants, were analyzed. In this model, we investigated the role of fiber amplifiers parameters, and the result was higher mode instability threshold for Holmium amplifier, which used at lower frequency and for Thulium amplifier was lower mode instability threshold due to high quantum defect heating, in comparison with Ytterbium amplifier. Nevertheless, bending in Thulium and Holmium amplifiers with higher signal wavelength (in compare of Ytterbium), does not have significant effect on increasing the mode instability threshold. To analyze the effect of higher order modes on mode instability, we considered two different V -numbers in our simulations.

Key words: Ytterbium doped fiber amplifier; Thulium doped fiber amplifier; Holmium doped fiber amplifier; mode instability; bent fiber; normalized propagation constant.

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1. Introduction

Silica fibers (glass fibers) due to linewidth spectral coverage and their output power are one of the preferable active mediums since 1964 [1, 2]. Fiber lasers with about 2 micrometers wavelength are harmless for the eyes and they are favorable to research centers for remote sensing, medical, meteorological and military lidars usages [3-6]. Fiber lasers with about 1 micrometer wavelength have got many applications in industrial, medical and military fields because of their high beam quality and power [1, 7].

Due to the large surface-to-volume ratio, fiber amplifiers have better cooling potential than conventional rod amplifiers and therefore high power and good beam quality is achievable [8, 9]. In case of powers higher than the threshold, increasing the power of the amplifier was limited due to the heat generated in the fiber [10-12].

In amplifiers with powers up to the threshold, the seed signal's high percentage is LP_{01} , and this amount changes after the threshold, and higher percentage is allocated to higher order modes at the output of the amplifier [13]. The interference of fundamental transverse mode with higher-order transverse modes, along the fiber, with the heat generated by the quantum defect heating effect creates a heat-grating pattern along the fiber. On the other hand, according to the thermo-optical properties, the refractive index is a function of temperature. Therefore, this created thermal pattern forms the refractive index grating along the fiber [14]. The experimental results of transverse mode instability show that just LP_{01} and LP_{11} modes are involved in large mode areas fibers, except for fibers with very large V -numbers [15]. LP_{11} mode has high gain comparing the other higher order modes, so LP_{01} and LP_{11} modes have the largest share in the signal radiation pattern [16]. The heat source pattern generated from the quantum defect heating effect creates pattern of the traveling temperature wave along the fiber, and according to the thermo-optical effect, thermal grating is formed, which causes generation of refractive index grating along the fiber. This thermal grating has the appropriate period for energy transfer between fundamental modes with higher-order transverse modes. This process is called the effect of transverse mode instability, which has been observed in higher powers according to recent reports [17, 18].

Nonlinear processes as stimulated Brillouin scattering (SBS), stimulated Raman scattering (SRS), and stimulated thermal Rayleigh scattering (STRS) are the limitations in achieving a high power in fiber amplifiers [19-22]. Fibers with symmetric bend compensated claddings are means to reducing mode instability effect in high-power amplifiers and lasers. Using fibers with low numerical aperture (NA) or bent active amplifier with a diameter of about 10 μ m has been also reported as techniques for controlling modal instability [23-27].

There is another method to control the mode instability based on bi-directional pumping, which is presented by Chen Shi, Friedrich Moller etc. [28, 29]. Although this method optimally increases the mode instability threshold, due to the configuration of the opposite direction pumps in this arrangement, the possibility of optical damage to the pumping diodes increases, which can be resolved using an optical isolator in front of the pump to protect it from residual unabsorbed pumping.

There are many reports of Yb, Tm and Ho doped high-power fiber amplifiers used to produce lasers with the wavelength about 1 to 2 micrometers. According to Yb-doped fiber absorption spectra diagram, the 1060 nm signal is produced by pump at 976 nm, as 2040 nm signal is produced by Tm-doped fiber 790 nm pump, and also Ho-doped fiber 1950 nm pump produces 2060 nm signal [13, 30, 31].

Most of earlier studies applied the fiber bending technique to raise the threshold of mode instability. In all of the afore-mentioned articles, the only reason and given analysis for applying this bending technique

is to increase the loss due to the fiber bending. No more detailed and perfect analysis is provided for applying bending technique. However, in this paper, we present a comprehensive model to analyze the mode instability in bent fibers, using the normalized propagation constant, which contains all the geometric parameters and fiber material. This is the difference between the present work and other references [32-34]. Using beam propagation method (BPM), in conjunction with the conformal mapping technique, bent fiber can be modeled by an equivalent straight fiber, in which a modified refractive index distribution is considered. [35- 37]. An additional change occurs to the physical refractive index of the fiber upon bending, due to stress-optic effects as a result of fiber bending [38- 39]. These two mentioned effects can be combined to define an equivalent bend radius. The second effect (bending-induced birefringence) is remarkable only in bent fibers with small radii, however the first effect is significant even in bent fibers with large radii. In this paper both the mentioned effects are considered in the bent fibers according to references. According to the Schermer model, in bent fibers with large radii, first effect is significant against the birefringence effect [34, 37].

In this paper, we used three rare earth ions doped fibers. The output signal wavelengths of Thulium and Holmium fibers are about twice as much as Ytterbium fiber signal wavelengths, and in all three fibers, pump wavelength differed. These fibers modal instability were modeled in two states of straight and bent fibers in the same conditions (V -number, Refractive index of cladding, Core diameter, Cladding diameter and the physical constants), and the behavior of each fiber was simulated independently at first and then their behavior was compared with each other. In order to investigate the effect of higher order modes on mode instability, we simulated in two V -numbers.

2- Theory

In the amplification process, due to the quantum defect heating effect, the temperature wave is generated and the interference between the pump and scattered beams leads to moving the temperature wave by absorbing heat along the fiber. This traveling temperature wave causes the frequency shift in the scattered beam. The interference pattern and modes become slightly different in frequency along the fiber amplifier. The traveling temperature wave using the thermal optics effect leads to an increase in the nonlinear coupling of the modes along the fiber. The deposited heat propagation is mostly directed in the fiber radius and the traveling temperature wave is directed along the fiber. The pattern of interference is formed along the fiber due to the interference of transverse fiber modes. It works as a longitudinal grating, which causes energy to be coupled from the fundamental mode (FM) to higher-order modes (HOM). This coupling of energy of modes is proportional to nonlinear coupling coefficient.

As the probability of the presence and amplification of different transverse modes within the fiber is correspondent with the normalized propagation constant, and the fiber used in this paper with $V=5.3348$, the propagation constant of the fundamental and first-order modes is much larger than the other modes. Therefore, just the fundamental and first-order modes are investigated in calculations. In this model, step-index, constant gain (g) and just LP_{11} among the higher order modes are selected to simplify the equations. Next time the simulation was run in V -numbers = 3.5565 as just LP_{11} exists in higher order modes in this state. By comparing the simulation results in two afore-mentioned normalized frequencies, the effect of higher order modes can be calculated. The nonlinear coupling coefficient as shown in equation (1), along the fiber is a fraction which converts g_{01} to g_{mn} . It depends on the physical parameters of the fiber, the quantum defect heat and the electric fields of the fiber modes.

Nonlinear coupling coefficient (χ) for straight fiber amplifiers is as follows [13]:

$$g_{01}\chi_{mn} = g_{01}(\chi_{mn}^r + i\chi_{mn}^i) = g_{01} \sum_{i=1}^{\infty} \frac{2\left(\frac{2\Omega}{\Gamma_{ml}} - i\right)}{1 + \left(\frac{2\Omega}{\Gamma_{ml}}\right)^2} \chi_{mnl} = \frac{2\pi k_0 k_T}{\rho C} \left(\frac{\lambda_s}{\lambda_p} - 1\right) \sum_{i=1}^{\infty} \frac{2\left(\frac{2\Omega}{\Gamma_{ml}} - i\right) \int_0^d g f_{01}(r) f_{mn}(r) T_{ml}(r) r dr \int_0^b f_{01}(r) f_{mn}(r) T_{ml}(r) r dr}{1 + \left(\frac{2\Omega}{\Gamma_{ml}}\right)^2 N_{01} N_{mn} \Gamma_{ml} \int_0^b T_{ml}^2(r) r dr} \quad (1)$$

Where, $f_{mn}(r)$ is the electric field in a high order mode, m and n represent azimuthal and radial modes order, respectively; $f_{01}(r)$ reveals the field in the fundamental mode, Γ_{ml} denotes the heat damping factor, $T_{ml}(r)$ shows the spatial temperature mode, N_{mn} and N_{01} are normalization factors in HOM and FM, respectively. The parameter g represents gain also g_{01} stands for the gain coefficient corresponding to the fundamental mode. The spatial temperature mode number of the heat transfer equation is l .

$\Omega/2\pi$ represents frequency separation between LP_{01} and LP_{11} mode. K_0 and $2d$ are the wavenumber in the vacuum and doped-area diameters. The other parameters and physical constants are available in Table 1.

Table.1. The physical constants and the parameters used in comparing Yb, Tm and To-doped fiber amplifiers with similar specifications [13, 30, 31].

Coefficient	Yb		Tm		Ho	
V-number	3.5565	5.3348	3.55652	5.3348	3.55652	5.3348
Core diameter (2a)	20 μm	30 μm	20 μm	30 μm	20 μm	30 μm
Cladding diameter (2b)	400 μm	400 μm	400 μm	400 μm	400 μm	400 μm
Thermo- optics coefficient (K_T)	$1.1 \times 10^{-5} K^{-1}$		$1.1 \times 10^{-5} K^{-1}$		$1.1 \times 10^{-5} K^{-1}$	
Density (ρ)	$2.2 \times 10^3 kg/m^3$		$2.2 \times 10^3 kg/m^3$		$2.2 \times 10^3 kg/m^3$	
Specific heat (C)	741 J/kg/K		741 J/kg/K		741 J/kg/K	
Thermal conductivity (κ)	1.38 W/m/K		1.38 W/m/K		1.38 W/m/K	
Thermal diffusivity ($D = \kappa/\rho C$)	$8.46 \times 10^{-7} m^2/s$		$8.46 \times 10^{-7} m^2/s$		$8.46 \times 10^{-7} m^2/s$	
Refractive index of Silica (n_{co})	1.45000		1.45335		1.45345	
Refractive index of cladding (n_{cl})	1.4488		1.4488		1.4488	
NA	0.06		0.1154		0.1166	
Signal wavelength (λ_s)	1060 nm		2040 nm		2060 nm	
Pump wavelength (λ_p)	976 nm		790 nm		1950 nm	
Gain coefficient (g)	$6.9 m^{-1}$		$9.1 m^{-1}$		$9.2 m^{-1}$	

According to Equation 1, by reducing the quantum defect, the value of the nonlinear coupling coefficient decreases, and the value of χ can be changed by changing the pattern of transverse mode interference. When the fiber is bent, the normalized propagation constant decreases comparing with the straight fiber and consequently the effective refractive index (n_{eff}) and propagation constant decrease as well. According to Schermer report [34], the propagation modes in bent fiber are obtained based on the normalized propagation constant (b) parameter in a completely analytical method. The parameter b in different transverse modes includes all the parameters related to the fiber geometry (Core diameter, Cladding diameter, bend radius and etc.) and material (core refractive index, cladding refractive index and etc.). Therefore, in this paper, to investigate the mode instability in bent fibers, we used the normalized propagation constant as the base of calculations, which is completely different from the methods used in previous studies [11, 25, 31, and 33].

The normalized propagation constant is defined according to Equation $b = [n_{eff}^2 - n_{cl}^2] / [n_{co}^2 - n_{cl}^2]$ which varies between 0 and 1. If the effective refractive index tends toward n_{cl} , b value is 0, and if it tends

toward n_{co} , b value is 1. The normalized propagation constant of bent glass fibers can be obtained by the following equation [34]:

$$b_{bmn} \cong b_{mn} - 1.575 \frac{a^* n_{cl}^2}{R * NA^2} \quad (2)$$

Where, R is the bend radius, b_{mn} and b_{bmn} are the normalized propagation constant at the straight and bent fibers consequently. On the other hand, the relationship between the normalized propagation constant of the straight fiber in the fundamental mode and the higher- order mode, with the normalized frequency (V) is obtained as follows [40]:

$$V = \frac{1}{\sqrt{1-b_{01}}} \left(\tan^{-1} \sqrt{\frac{b_{01}}{1-b_{01}}} + \frac{\pi}{4} \right) \quad (3)$$

$$V = \frac{1}{\sqrt{1-b_{mn}}} \left(\tan^{-1} \sqrt{\frac{b_{mn}}{1-b_{mn}}} + \frac{2\pi}{3} \right)$$

In three types of fibers investigated in this study, in V - numbers =5.3348, 3.5565 and $a=15,10\mu m$, the numerical aperture (NA) and the core refractive index are obtained for each fiber:

$$NA = \frac{V}{ak_0} \quad (4)$$

$$n_{co} = \sqrt{n_{cl}^2 + NA^2} \quad (5)$$

The refractive index cladding(n_{cl}) is taken as 1.4488, for all three fibers. Therefore, according to Equations 6, the effective refractive index of three types of fibers in the state of straight and bent is obtained:

$$n_{effmn} = [b_{mn}NA^2 + n_{cl}^2]^{1/2} \quad (6)$$

Variations of the modes propagation constant due to bending affected the modes field and their value changes in both fiber cladding and core. In the bent fibers, fluctuations in the modes field change the nonlinear coupling coefficient. The effective refractive index of each mode is obtained using the normalized propagation constant of the modes according to Eq. (6). The modes field for straight fiber can be found from Equation 7:

$$\begin{aligned} f_{mn}(r) &= \frac{J_m(ak_0^2(n_{co}^2 - n_{effmn}^2)r)}{J_m(a^2k_0^2(n_{co}^2 - n_{effmn}^2))} \quad a \geq r \geq 0 \\ f_{mn}(r) &= \frac{K_m(ak_0^2(n_{effmn}^2 - n_{cl}^2)r)}{K_m(a^2k_0^2(n_{effmn}^2 - n_{cl}^2))} \quad r > a \end{aligned} \quad (7)$$

The modes field for bent fiber can be calculated by using bending parameters into Eq. (7) as well. Where, J_m represents Bessel functions of the first kind and K_m denotes the modified Bessel function of the second kind.

3- Modeling

3. a. Yb-doped fiber amplifiers

Yb-doped fibers used in this paper are 30/400, and 20/400, $a=d$ and $n_{cl}=1.4488$. To validate the results in the first part of the model, the fiber parameters are selected according to Dong's article [13]. The second part of the model is generalized to bent fiber. First, the real part of nonlinear coupling coefficient(χ_{mn}^r) plotted according to the frequency separation between modes, then the peak of χ_{mn}^r is obtained in both straight and bent states used in calculations. In this model, the fibers are bent with the radius of 0.1 m, 0.2 m and 0.5 m, and the results are compared with the straight fibers.

In Fig. 1(a), the real part of χ_{mn} is plotted according to the frequency separation between LP_{01} and LP_{11} modes in V -number=5.3348. $\Omega / 2\pi$ at the peak of $\chi_{mn}^r(\frac{\Omega}{2\pi})$ in Yb straight fiber is 3400 Hz. As a result of bending the fiber, the frequency separation between LP_{01} and LP_{11} modes increases compared to the straight fiber. As the bending radius increases, the frequency separation between modes decreases toward the amount of the straight fiber. According to Fig. 1(a), when bending radius are 0.1, 0.2 and 0.5 m, the values of $\frac{\Omega}{2\pi} = 6000, 4500$ and 3750 Hz are obtained, respectively. Therefore; As $\frac{\Omega}{2\pi}$ increases, the probability of mode coupling decreases and as a result mode instability decreases as well.

(a) $V=5.3348$

(b) $V=3.5565$

Fig. 1. The real part of nonlinear coupling coefficient for LP_{01} – LP_{11} versus the frequency separation between modes of straight Yb-doped fibers in comparison with the bent Yb-doped fibers for $R=0.1, 0.2$ and 0.5 m. in (a) V -number=5.3348 & (b) V -number=3.5565.

The nonlinear coupling coefficients amplitude (χ_{mnl}) depends on how the heat is distributed in the fiber. In Fig. 2(a), χ_{mnl} is plotted to the spatial temperature mode number (l) for different bendings in V -number=5.3348. When the fiber is bent, the peak of nonlinear coupling coefficients amplitude declines and moves to larger spatial temperature mode number. The spatial temperature mode number increasing leads to the damping factor(Γ_{ml}) increase and then it reduces the nonlinear coupling coefficient.

(a) $V=5.3348$

(b) $V=3.5565$

Fig. 2. The comparison of nonlinear coupling coefficient amplitude χ_{mnl} of bent ($R=0.1, 0.2$ and 0.5 m) and straight Yb-doped fiber amplifiers versus l . in (a) V -number=5.3348 & (b) V -number= 3.5565.

According to Eq. (8), the mode instability threshold power decreases when $P_{11}(0)/P_{01}(0)$ and the gain increase. The mode instability threshold power is plotted to gain in Fig. 3.

$$P_{01}^{th} \approx \frac{1}{\chi_{mn}^r} \ln \left(x \frac{P_{01}(0)}{P_{mn}(0)} \right) e^{1.25/e g_{01} L} \quad (8)$$

Based on the above equation, the threshold is defined when the power in the LP_{11} mode reaches a certain fraction of the LP_{01} mode at the output, i.e., $x = P_{mn}(L) / P_{01}(L)$. As shown in Fig. 3, the power threshold of the mode instability is increased in the bent fiber and as the bending radius increases, the value of P_{01}^{th} declines and the mode instability occurs in the fiber amplifier earlier.

Next, all the simulation steps are repeated for Yb-doped fiber (20/400) in V -number=3.5565. In Fig. 1(b), χ_{mn}^r is plotted to the frequency of mode separation in both straight and bent states. Frequency separation at the peak of χ_{mn}^r for straight state is 6000 Hz and for bending state with radius of 0.1, 0.2 and 0.5 m are 7000, 6500, 6250 Hz, respectively. Fig. 2(b) shows χ_{mnl} to the spatial temperature mode number, in which the peak of χ_{mnl} declines in smaller V -number. Fig. 3(b) shows that the mode instability threshold power increases with V -number decreasing.

(a) $V=5.3348$

(b) $V= 3.5565$

Fig. 3. Comparing the threshold powers of straight and bent ($R=0.1, 0.2$ and 0.5 m) Yb-doped fibers at $x= 1\%$, $L=1$ m and $P_{01}(0)/P_{11}(0)= 10^5$. in (a) V -number=5.3348 & (b) V -number= 3.5565.

3. b. Tm-doped fiber amplifiers

Tm-doped fibers used here are 30/400 and 20/400, in which $a=d$, $n_{cl}=1.4488$ and the pump and signal wavelengths are 790 and 2040 nm, respectively. According to Table. (1), the physical constants are selected like Yb-doped fiber constants. Using Eq. (6), the numerical aperture of Tm-doped fiber obtained 0.1155. In Fig. 4, the nonlinear coupling coefficient is plotted to the separation frequency of LP_{01} and LP_{11} modes in Tm-doped fiber. The peak of real part of χ_{mn} occurs at 3350 Hz, in straight Tm-doped fiber with V -number=5.3348 as shown in Fig. 4(a). $\Omega/2\pi$ of the modes in bending state with radius of 0.1, 0.2 and 0.5 m reaches 3800, 3500 and 3400 Hz, respectively. According to Fig. 4(b), the peak of χ_{mn}^r in V -number=3.5565 for straight fiber is about 6100 Hz and for bending state with radius of 0.1, 0.2 and 0.5 m are 6500, 6300 and 6250 Hz respectively. The amount of χ_{mn}^r for Tm fiber is about 1.2 and extremely increased compared to Yb fiber.

(a) $V=5.3348$

(b) $V= 3.5565$

Fig. 4. The real part of nonlinear coupling coefficient for LP_{01} – LP_{11} versus the frequency separation between modes of straight Tm-doped fibers in comparison with the bent Tm-doped fibers for $R=0.1, 0.2$ and 0.5 m. in (a) V -number=5.3348 & (b) V -number= 3.5565.

The behavior of χ_{mnl} to l is shown in Fig. 5. With Tm-doped fiber bending, in V -number=5.3348, the peak of nonlinear coupling coefficient moves towards the large l as shown in Fig. 5(a), and therefore the peak of χ_{mnl} declines. The χ_{mnl} diagram is plotted in a smaller V in Fig. 5(b), and comparing to the diagram in Fig. 5(a), the χ_{mnl} value is reduced and the peak of χ_{mnl} is gently sloped toward the larger l . The spatial

temperature mode number for the peak χ_{mnl} moves to larger l for smaller core radius or smaller V -number to account for the smaller active area. Also comparing Figures 5(a) and 5(b) shows that fiber bending has lower effect on mode instability in fibers with smaller V -number.

(a) $V=5.3348$

(b) $V= 3.5565$

Fig. 5. The comparison of nonlinear coupling coefficient amplitude χ_{mnl} of bent ($R=0.1, 0.2$ and 0.5 m) and straight Tm-doped fiber amplifiers versus l . in (a) V -number= 5.334 & (b) V -number= 3.5565 .

Fig. 6(a) shows that the mode instability in Tm-doped fibers occurs at lower powers compared to Yb-doped fibers. Fig. 6(b) shows the threshold power in V -number= 3.5565 , and the threshold power varies slightly while fiber bending compared to Fig. 6(a).

(a) $V=5.3348$

(b) $V= 3.5565$

Fig. 6. Comparing the threshold powers of straight and bent($R=0.1, 0.2$ and 0.5 m) Tm-doped fibers at $x=1\%$, $L=1$ m and $P_{01}(0)/P_{11}(0)=10^5$. in (a) V -number=5.334 & (b) V -number= 3.5565.

3. c. Ho-doped fiber amplifiers

Ho-doped fibers 30/400 and 20/400 with the pump and signal wavelength of 1950 and 2060 nm used in this section, respectively. As the pump and signal wavelength are close together, Ho-doped fiber is selected. At first glance, by reducing the quantum defect heating, the effects of modal instability on this fiber is expected to be weak, or in other words, Ho-doped fiber will show better resistance to this effect. The value of separation frequency LP_{01} and LP_{11} is obtained for the straight and bent fiber modes in Fig. 7.

(a) $V=5.3348$

(b) $V= 3.5565$

Fig. 7. The real part of nonlinear coupling coefficient for $LP_{01}-LP_{11}$ versus the frequency separation between modes of straight Ho-doped fibers in comparison with the bent Ho-doped fibers for $R= 0.1, 0.2$ and 0.5 m. in (a) V -number=5.3348 & (b) V -number= 3.5565.

Fig. 7(a) shows that the diagram of the nonlinear coupling coefficient and the frequency between the modes differ slightly in the straight and bent Ho-doped fiber amplifiers in V -number=5.3348. The value of $\frac{\Omega}{2\pi} \chi^r$ is 3350 Hz for straight fiber, and 3750, 3500 and 3400 Hz for bent fibers with bending radius of 0.1, 0.2 and 0.5 m, respectively. According to Fig. 7(b), the value of $\frac{\Omega}{2\pi} \chi^r$ for straight fiber in V -number=3.5565 is about 6000 Hz and for the bending radius of 0.1, 0.2 and 0.5 m, $\Omega/2\pi$ is 6400, 6200 and 6100 Hz, respectively.

The χ_{mnl} to l in V -numbers=5.3348, 3.5565 is plotted in Fig. 8, the peak of χ_{mnl} of Ho-doped fiber is much lower than the rest of the fibers, but the bending of the fiber does not have much effect on shifting the peak χ_{mnl} to large l .

(a) $V=5.3348$

(b) $V= 3.5565$

Fig. 8. The comparison of nonlinear coupling coefficient amplitude χ_{mnl} of bent ($R=0.1, 0.2$ and 0.5 m) and straight Ho-doped fiber amplifiers versus l . in (a) V -number= 5.334 & (b) V -number= 3.5565 .

In Fig. 9(a), Ho-doped fiber's P_{th} is about twice as high as Yb-doped fiber's, although the effect of the fiber bending on P_{th} of Ho is less than that of Yb fiber.

According to Fig. 9(b), Ho fiber bending threshold power with different radii varies slightly compared to Fig. 9(a).

(a) $V=5.3348$

(b) $V= 3.5565$

Fig. 9. Comparing the threshold powers of straight and bent($R=0.1, 0.2$ and 0.5 m) Ho-doped fibers at $x= 1\%$, $L=1$ m and $P_{01}(0)/P_{11}(0)= 10^{30}$. in (a) V -number= 5.3348 & (b) V -number= 3.5565 .

4. RESULTS AND DISCUSSION

Three type of fibers with different properties with V -numbers of 5.3348 and 3.5565 used to analyze the effect of mode instability in the previous sections. Yb-doped fiber amplifier has a signal wavelength of about $1 \mu\text{m}$ and its pump frequency is relatively close to the signal frequency. In Tm-doped fiber amplifier, signal wavelength is about $2 \mu\text{m}$ and the difference between pump wavelength and signal wavelength is approximately $1.2 \mu\text{m}$. Although Ho-doped fiber amplifier signal wavelength is about $2 \mu\text{m}$, due to better absorption spectrum, pump wavelength has the least difference with signal wavelength.

The basis of Dong method as well as the first stage of calculations is using an electric field inside the fiber; this method can be used by obtaining the electric field, whether the fiber is straight or bent. In this paper, we used bent fiber electric fields and the other parameters based on the electric field inside the bent fiber. Although the heat equation used in Dong's method is apparently related to straight fiber, considering that the dimensions of the fiber are about 30 microns, and the bending radius is at least about

10 cm, so bending with a radius of tens centimeters, cannot make a significant difference in the temperature distribution within the fiber. By increasing the bending radius of the fiber and evaluating the results in large bending radii, the computational results of this paper are completely consistent with Dong's, confirming the results of this paper.

As mentioned in the previous section, quantum defect heating is source of heat generation inside the fiber. The larger the difference between the pump and signal wavelength, the higher the heat generated by the quantum defect heating. This heat generates temperature gratings, and eventually leads to the traveling temperature wave along the fiber. On the other hand, according to the thermo-optical properties, the refractive index is a function of temperature. Therefore, this generated temperature pattern forms the refractive index grating along the fiber. Temperature gratings increase the effective refractive index of the modes [41] and thus, increase the normalized propagation constant of the modes. Using the equations and simulations presented in this model, the effective refractive index of LP_{11} mode increases more than LP_{01} mode, and as a result, the difference of the normalized propagation constant between the modes decreases, this result is consistent with Savolainen's results [42]. As the normalized propagation constant increases, the LP_{11} and LP_{01} modes are drawn to the core. Two factors "reducing the difference in normalized propagation constant between the LP_{11} and LP_{01} modes and drawing the modes to the core" increase the interference between the modes. It was obtained by simulating three fibers that increasing the signal wavelength or decreasing the V -number, the effective refractive index of the modes decreases, thus; the mode instability reduces which is consistent with the Shahzadi's results [43]. According to Figures. (2,5,8), the quantum defect heating in Tm-doped fiber is higher than the other fibers, and the nonlinear coupling coefficients amplitude is about ten times larger than Yb-doped fiber, and also the nonlinear coupling coefficients amplitude of Yb-doped fiber is about twice Ho-doped fiber.

According to Eq. (1), when the signal wavelength and pump wavelength increases, the nonlinear coupling coefficient declines; with the radius of the core or V -number decreasing, the modes field inside the fiber core modifies as well, and based on Eq. (7) and the behavior of Bessel function of the first kind (zero-order and one-order), LP_{01} field increases and LP_{11} field decreases. While Yb fiber is bent, LP_{01} mode field intensity increases and LP_{11} mode field intensity decreases; therefore, the nonlinear coupling coefficient declines. It can be concluded that the results of fiber bending are the same as the results of reducing the V -number or reducing the core radius of the fiber. In Figures. (1,4,7), the real part of χ_{mn} declines towards higher $\Omega/2\pi$ and as a result of bending, $\frac{\Omega \chi^r}{2\pi}$ in the amplifiers with higher signal wavelength increase slightly. This shows that in amplifiers with higher signal wavelength, bending has lower effect on controlling the mode instability. As fiber bending, the real part of χ_{mn} also decreases and the mode instability threshold increases according to Equation (8). In Tm fiber, $\frac{\Omega \chi^r}{2\pi}$ is about 10 times higher than Yb fiber, and the instability threshold is about 10 times lower than Yb fiber, and therefore it is difficult to take high power from Tm amplifier, and this is proved in most researches [44, 45]. According to the above equations and diagrams, fiber bending at higher wavelengths including Tm and Ho and amplifiers in lower V -number has insignificant effect on increasing the mode instability threshold.

One of the most important and effective parameters in bent fibers is NA parameter. If the wavelength increases in fixed V -number and core diameter, the NA value increases too. In larger NAs , the normalized propagation constant decreases less and it leads too less changes in the modes field; so the value of the nonlinear coupling coefficient was not altered significantly. It means that the bending of the fiber at higher wavelengths insignificantly influenced the mode instability. As it is observed in χ_{mn}^r diagrams, $\frac{\Omega \chi^r}{2\pi}$ in bending state with radius of 0.1 m in Yb, Tm and To fibers is 6000, 3800 and 3750 Hz, respectively and in the straight state, $\frac{\Omega \chi^r}{2\pi}$ for three fibers is 3400, 3300 and 3200 Hz, respectively. Therefore, in larger NAs , the $\frac{\Omega \chi^r}{2\pi}$ also changes slightly due to bending.

Fig. 10(a) shows the simulation of the normalized propagation constant of bend fiber versus NA parameter. In the straight state, V -number is 5.3348 and the normalized propagation constant for LP_{01} and LP_{11} modes are 0.8620 and 0.6720 respectively. According to Fig. 10(a), in $NA=0.06$, the value of

b_{01} in the bent state with radius of 0.1 is 0.724 with about 0.138 difference compared to the straight state, but in $NA = 0.1154$, b_{01} value is 0.825 and the difference is about 0.037 compared to the straight state. When the difference of the normalized propagation constant between modes is larger, the mode instability occurs at higher powers. Therefore, the bending of the fiber in larger NA s, insignificantly influenced on increasing the mode instability threshold.

(a) $V=5.3348$

(b) $V=3.5565$

Fig.10. Simulated $LP_{01}-LP_{11}$ normalized propagation constants of bent fibers($R=0.1$ m) versus NA in (a) V -number=5.3348 & (b) V -number= 3.5565.

According to Fig. 10(b), the value of b_{01} and b_{11} in the straight state in V -number= 3.5565 are 0.7380 and 0.3927 respectively. The difference of normalized propagation constant between modes is about 0.1790 in V -number=5.3348, which is about 0.3453 in V -number=3.5565. Therefore, by decreasing the V -number in the fiber, the difference of the normalized propagation constant between modes increases, and then, the χ decreases, and consequently the power threshold of mode instability increases. The results of Fig. 10 confirmed the theoretical and experimental results of Rumao Tao et al. article [46]. The result of decreasing the fiber core diameter or the V -number in reducing the mode instability is almost similar to the result of fiber bending. The difference of normalized propagation constant between modes declines in both two methods, decreasing the fiber V -number and the fiber bending method, although bending fiber is not a applicable method to reduce the mode instability after decreasing V -numbers.

5. Conclusion

In this paper, we investigated the mode instability of Yb, Tm and Ho-doped fibers using normalized propagation constant which was a creative method and simulated the bending behavior of all three fibers. For an accurate review of the mode instability, the simulation was performed in two V -numbers. The larger the difference between the signal and the pump wavelengths in the fiber, the higher the mode instability. We found that in the simulation process, the mode instability decreases with decreasing the bending radius. The impact of the bending fiber on mode instability decrease declines by increasing signal wavelength and in smaller the V -numbers. Also it shown that normalized propagation constant can be considered as an important factor in analyzing modal instability in fiber amplifiers.

Disclosures

This research did not receive any specific grant from funding agencies in the public, commercial, or not-for-profit sectors.