Conical refraction: Fundamentals and Applications

Refracción Cónica: Fundamentos y Aplicaciones

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Received / Recibido: 12/07/2019

Accepted / Aceptado: 23/09/2019

DOI: 10.7149/OPA.52.3.51027

ABSTRACT:

In 1832 Hamilton predicted that a collimated light beam propagating through a biaxial crystal parallel to one of its optic axes refracts as a slanted cone within the crystal and emerges as a hollow light cylinder, this optical effect being named as conical refraction (CR). The diffractive solution of CR presented by Belsky and Khapalyuk in 1978 and the corresponding reformulation carried out by Berry in 2004 rekindled this old and almost forgotten phenomenon. This article reviews the CR phenomenon and uses the singular properties of the CR beams to describe different applications of CR in the fields such as optical trapping and free-space optical communications.

Key words: Conical refraction, crystal optics, polarization, optical trapping, optical communications.

RESUMEN:

En 1832 Hamilton predijo que un haz colimado de luz propagándose a través de un cristal biáxico de manera paralela a uno de los ejes ópticos se refracta como un cono de luz en su interior y emerge como un cilindro hueco, siendo este fenómeno conocido como refracción cónica (CR). La solución basada en óptica difractiva de la CR fue presentada por Belsky y Khapalyuk en 1978 y reformulada por Berry en 2004. Este artículo, revisa el fenómeno de la CR y utiliza todas sus particularidades para describir diferentes aplicaciones de la CR en áreas como la captura óptica y las telecomunicaciones ópticas.

Palabras clave: Refracción cónica, cristalografía óptica, polarización, captura óptica, comunicaciones ópticas

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1. Introduction

Double refraction or birefringence is an optical phenomenon occurring in uniaxial crystals, where an input beam is decomposed into two beams with orthogonal linear polarizations. These beams known as ordinary and extraordinary beams. While the ordinary beam follows the usual laws of refraction, the extraordinary

one is laterally shifted with respect to the former due to a difference in the refractive indices for both polarizations. However, when the input beam to the uniaxial crystal propagates parallel to the optic axis, both beams overlap and they emerge from the crystal as a single light beam.

Biaxial crystals (BCs) possessing three different principal refractive indices (and therefore two optic axes), also exhibit double refraction. However, as Hamilton theoretically predicted in 1832, in this case, when a light beam propagates parallel to one of the two optic axes, it refracts as a slanted cone inside the crystal and emerges as a hollow cylinder yielding an annular transverse intensity pattern or light ring of internal conical refraction (CR) [1,2], see Fig. 1. Hamilton also predicted that a light cone passing through a BC along the biradial would refract as a needle within the crystal parallel to the biradial and emerge again as a light cone. He called this phenomenon as external CR. Soon afterwards Hamilton's prediction, Lloyd used aragonite and sunlight to report the first experimental observation of both internal and external CR [3]. He investigated the transition from double refraction to CR as the optic axes of the crystal and the direction of the beam approach each other, and described the polarization distribution along the ring. In what follows, we will concentrate in internal CR and we will refer to it simply as CR.

Fig. 1. Basic parameters in CR. R_0 is the geometric approximation of the CR light ring that can be calculated as $R_0 = l\alpha$, where l is the length of the BC and α is the semi-angle of the light cone or conicity. ρ_0 measures the ring radius in units of waist radius, w_0 , of the input beam. $\vec{G}$ is a vector that can be attached to the crystal to describe both the direction and the magnitude of the lateral displacement that the CR ring suffers. ϕ accounts for the orientation of the plane of optical axes of the crystal.

This article is based in my doctoral thesis [4] and, while keeping apart many contributions there presented, I will focus on the fundamental aspects and most appealing applications of internal CR that I developed during my PhD. First, we will introduce the CR phenomenon and explain its most characteristic features, including the generation of vector beams, optical bottle beams, and optical singularities. After that, all these elements will be used to demonstrate that CR is perfect for applications such as optical trapping of ultra-cold atoms, absorbing macroscopic particles, and free-space optical communications.

2. Fundamentals of CR

As stated above, CR occurs when a collimated light beam with waist radius w_0 propagates through a BC parallel to one of its two optic axes. In this case, the beam refracts as a slanted cone within the crystal and, as a consequence, the center of the emerging light ring appears displaced with respect to the original input beam position, see Fig. 2. This displacement can be accounted for by a characteristic vector $\vec{G} = R_0(\cos \phi_G, \sin \phi_G)$ of the BC giving both the magnitude and direction of this displacement. $R_0 = l\alpha$ gives the radius of the CR light ring, where l is the length of the BC and α is its conicity, or half-angle of the light cone, which depends on the three refractive indices of the BC; ϕ_G gives the orientation of the plane containing the two optic axes of the crystal [2,5,6]. Nevertheless, collimated beams are only geometrical approximations of physical beams, since real light beams are always focused up to some extent. The diffractive theory of CR describing the transformation of focused light beams was firstly presented by Belsky and Khapalyuk [7] and later on reformulated by Berry [2,8]. According to this description, the electric field $\vec{D}$ after the BC for a circularly-symmetric and uniformly polarized light beam $E(r, Z)$ is given by the following equations:

$$\vec{D} = \begin{pmatrix} B_0 + B_1 \cos(\varphi) & B_1 \sin(\varphi) \\ B_1 \sin(\varphi) & B_0 - B_1 \cos(\varphi) \end{pmatrix} \vec{e}_0 \quad (1)$$

$$B_1(r, z, \rho_0) = \frac{1}{2\pi} \int_0^\infty k a(k) \exp(-i k^2 z / 2n) J_1(kr) \sin(k\rho_0) dk \quad (2)$$

$$B_0(r, z, \rho_0) = \frac{1}{2\pi} \int_0^\infty k a(k) \exp(-i k^2 z / 2n) J_0(kr) \cos(k\rho_0) dk \quad (3)$$

where $a(k) = \int_0^\infty k E(r, z=0) J_0(kr) dk$ is the Hankel transform of order 0 [for a Gaussian beam, $a(k) \propto \exp(-k^2/4)$], $J_\nu(x)$ is the Bessel function of order ν , and $\mathbf{e}_0$ describes the input polarization. In Eqs. (1)-(3), n is the mean refractive index of the BC, (r, ϕ, z) are the polar coordinates normalized to the beam waist w_0 and the Rayleigh range $z_R = \pi w_0^2/\lambda$, and $\rho_0 \equiv R_0/w_0$. For $\rho_0 \gg 1$, i. e. for CR ring radii much larger than the waist radius of the input Gaussian beam, and for a Gaussian input beam the CR ring is divided into two light rings by a dark ring known as Poggendorff ring [8,9]. The energy distribution of these two rings is approximately 1:4 (inner ring : outer ring), while the state of polarization of the rings is linear at every point and every two diametrically opposite points are orthogonally polarized. As a consequence, the complete CR rings are only visible for circularly or randomly polarized input beams [see Fig. 1(a)], while an azimuthally crescent pattern is obtained for linearly polarized input beams, as shown in Fig. 1(b). Furthermore, the Poggendorff ring is sharpest at the otherwise expected focal plane of the Gaussian beam in the absence of the BC. As the imaging plane is moved far away from this plane, the CR rings become wider and, after a certain distance $z_{\text{Raman}} = \pm \sqrt{4/3} \rho_0 z_R$, most of the beam's energy becomes concentrated at two symmetrically placed axial spots known as Raman spots [10], see Fig. 1(c). Closer inspection of the CR beam evolution shows that indeed the CR beam is formed by the interference of two conical waves with opposite vertices, as shown in Figs. 1(d) and (e) [11].

Fig. 2. Transverse intensity patterns of the CR beam for $\rho_0 \gg 1$, obtained for a Gaussian input beam with (a) circular polarization and (b) horizontal linear polarization. The blue-double arrows depict the polarization distribution on the CR rings. (c)-(e): intensity distributions in the xz plane for (c) the complete CR beam and (d,e) the two conical waves that compose it.

Note that the evolution of the CR beam depends strongly on the ρ_0 parameter. Although the regime $\rho_0 \gg 1$ covers most of the works on the field, for $\rho_0 \approx 1$ the CR intensity distribution becomes substantially different. In particular, for $\rho_0 < 2.5$ the two bright rings are no longer distinguishable. In Fig. 3 we show the beam evolution and both the radial and the transverse intensity distribution at the focal plane for $\rho_0 = [1.5, 1.0, 0.5]$. As it can be seen, for $\rho_0 = 1.5$ the inner CR ring collapses into a central spot, at $\rho_0 = 1$ only a single (doughnut-like) ring is found, and for $\rho_0 = 0.5$ the ring disappears. The characteristics of the CR beam for such low values of ρ_0 are investigated in detail in Refs. [12-14].

Fig. 3. Intensity variation (a) along the radial direction ρ at the focal plane $Z = 0$ and (b) along the axial direction z at the beam center ($\rho = 0$) for CR vector beams obtained using $\rho_0 = 1.50$ (blue-solid line), $\rho_0 = 1.0$ (red-dashed line) and $\rho_0 = 0.50$ (black-dotted line). The corresponding intensity distribution in the (z, x) plane are shown in figures (c)-(e), while the transverse intensity distribution at the focal plane are shown in figures (f)-(h).

The above-discussed features of CR beams are valid only for cylindrically symmetric and uniformly polarized beams. However, the CR diffractive theory can be modified to include more involved polarizations, as well as propagation out of the optic axes, and input beams with non-symmetric transverse intensity profiles [15]. This formalism has served us, for instance, to investigate the transformation of beams with radial and azimuthal polarizations [16] and also to demonstrate a type of healing of truncated beams by the CR phenomenon [17].

Finally, it is worth mentioning the action of arranging multiple BCs in cascade with aligned optic axes, also known as cascaded CR. In this case, the CR ring after a crystal splits into two new rings (each of which with the characteristic Poggendorff splitting) after passing through subsequent crystals [18]. As a consequence, for a cascade of N crystals, the light pattern after the last crystal is composed by 2^{N-1} concentric light rings, whose intensities depend on the mutual orientation the optic axes planes of the crystals. Although observing cascaded CR is relatively simple, it is computationally very demanding to simulate the phenomenon for cascades with more than 2 crystals. For this reason, we developed a simple theoretical formalism based in splitting of wave-vectors after passing through BCs that allows predicting the general properties (intensity and polarization distribution under the geometric approximation) for an indefinitely big cascade of BCs [19,20].

In the forthcoming sections, all these properties of CR beams mentioned above are used to demonstrate applications of the phenomenon in the fields of optical trapping and free-space optical communications.

3. An optical vault for absorbing particles

The double-cone structure formed by the CR beam for $\rho_0 \gg 1$ is a type of optical bottle beam [21]. Optical bottles are beams possessing a low (ideally null) intensity region surrounded entirely by light. These beams are perfect candidates for trapping of refractive particles, ultra-cold atoms, or absorbing particles, by means of optical forces isolating the samples in the low intensity region of the bottle [22,23]. The problem with an ideal bottle beam is that the higher is the difference between dark and bright intensity regions, the more difficult is for the particles to penetrate the light walls. Once an optical bottle is formed it actually prevents particles from entering it. To cope with this issue, one straightforward solution consists in turning on the bottle beam when the particles already float in the region where the trap will be formed. Another and much more convenient choice would be to design a bottle in such a way that it could be partially opened and closed so it could be loaded and unloaded with particles as required.

We have demonstrated that such a design is indeed possible by reconfiguring the optical bottle formed by a CR beam in real-time and by trapping airborne particles with the photophoretic force [10], in collaboration with the group of Prof. Krokowski at the Australian National University. The latter is a thermal force induced by optical beams that works as follows: when a gas-suspended particle is illuminated, light is diffracted, refracted and reflected by the particle. Additionally, if the particle is not transparent, it absorbs part of the energy of the light field and its temperature rises up. As a consequence, the illuminated particle radiates heat to the environment and the molecules of the surrounding gas increase their kinetic energy. This kinetic energy increase by the gas molecules manifests as linear momentum exchange with the illuminated particle that typically pushes it away from the light source. However, if the particle is illuminated symmetrically, it receives the same amount of linear momentum in all directions and it remains stable in the three dimensions (3D), see Fig. 4(a). For instance, the photophoretic force has been used to generate optical tractor beams [24] and to design a volumetric display [25].

Our trapping scheme is based on the possibility of opening and closing in real time the CR optical bottle for well-defined CR rings by tailoring the input beam polarization. One of the main advantages of using the CR phenomenon for photophoretic trapping is that the required experimental set-up is rather simple, since only a half-waveplate, a quarter-waveplate, a focusing lens, and a BC is needed to generate the 3D trapping potential, see top image in Fig. 4. For a CP input beam, the CR optical bottle is completely closed in 3D and absorbing particles falling down inside the bottle are pushed away from the bottle's walls. In this [video](#) one can see particles falling down due to the action of gravity and being repelled by the light walls of the CR rings. Note that, although there is a certain probability for the particles to penetrate the bottle, such probability is very low. In contrast, a LP input beam impinging the crystal results in the loss of perfect cylindrical symmetry of the CR optical bottle. From the 3D point of view, this means the formation of a hole in one of the sides of the otherwise perfect bottle beam, see Fig. 4(b).

The angular position of the null-intensity region can be varied by rotating the azimuth of the linear polarization of the input beam. If the latter coincides with the azimuth from the bottom part of the CR rings

at the focal plane, a null intensity point at their top is generated. This allows increasing the trapping efficiency, since absorbing particles find a stable point at the bottom part of the CR rings, due to the gravity field, as shown in this [video](#). Once a particle has been loaded inside the bottle, the latter can be closed by modifying the state of polarization of the input beam from linear to circular, thus isolating the particle from the environment. Finally, reverting the state of polarization of the input beam to linear would enable opening a hole in the trap at a desired angular location to unload the particles, see sequence (c)-(f) in Fig. 4 and this [video](#).

As trapping particles, glass shells covered with a thin layer of carbon (around 200nm thick) in order to enhance light absorption were used. The external diameter of the shells ranged from a few to tens of micrometers. Because of gravity they were all located at the bottom of the bottle. The trapping was generally very robust, with the particles resting on the lower “wall”, although other stable trapping positions were also possible, see Fig. 4(h). With respect to particle statistics, the average size of the trapped particles normalized to the dimensions of the trap was 0.23. With the optical bottle used in our experiments, we were able to trap particles with sizes up to 100 μ m.

In summary, with this work we demonstrated the efficient loading and unloading of absorbing particles by reconfiguring the optical bottle generated with the CR phenomenon by means of a simple and affordable set-up.

Fig.4. Top: experimental set-up. HWP: half-waveplate; QWP: quarter-waveplate; FL: focusing lens; IL: illuminating lens. (a) Action of the photophoretic force over an absorbing particle. If the particle is illuminated symmetrically it can be trapped in a 3D region of space. (b) 3D structure of the CR beam for LP illumination. (c)-(f): Experimentally recorded sequence of images illustrating the unloading process of a trapped glass sphere from the optical bottle. (g) Statistics of the trapped particles size normalized to the radial dimensions of the trap. (h) Experimental images of hollow glass shells of different size trapped inside the bottle beam. Particles diameter has been measured to be in the range 20-50 μ m. Note that simultaneous trapping of two glass spheres is also possible. Here $\rho_0 = 12$, being the CR ring radius $R_0 \approx 200\mu$ m.

4. Trapping of Bose-Einstein condensates with CR

Optical ring potentials (ORPs) with axial symmetry are considered as basic building blocks and the simplest nontrivial closed-loop circuits in atomtronics and atom interferometry [26,27]. Atoms can be trapped by means of the optical dipolar force in high or low intensity regions with red-detuned or blue-detuned light [23]. Blue-detuned ORPs are particularly interesting to trap single atoms and Bose-Einstein condensates (BECs) since they have ideally null scattering rate of photons, which preserves the coherence of matter-waves. Frequently used techniques to generate blue-detuned ORPs, which are based on spatial-light modulators (SLMs) and amplitude masks present some limitations such as power losses, non-perfect smoothness of the generated potentials and the required accurate control on the position and alignment of the optical elements being. As a consequence, these two techniques yield typically not null intensity minima. Therefore, producing ORPs with zero-intensity annular regions both along the radial and axial directions is a challenging task.

The Poggendorff dark ring (PDR) generated by the CR phenomenon offers a perfect scenario to trap BECs in a blue-detuned light ORP, as we demonstrated in Ref. [9]. Along the radial direction [green curve in Fig. 5(a)], the PDR is a null intensity point due to a π -phase difference between the electric field at the inner and outer CR rings [28]. The PDR is further enclosed by higher intensity regions along the axial (beam propagation) direction, as depicted by the pink curve Figs. 5(a). Therefore, the PDR is a null intensity region in 3D, i.e., it forms a toroidal dark region, as shown by the 2D density plot in Fig. 5(b). This makes the PDR a perfect optical geometry for atom trapping, in particular with blue-detuned light for atomtronic applications.

For alkaline atoms, the dipolar optical potential with a blue-detuned light field is given by [23]:

$$U(\mathbf{r}) = -I(\mathbf{r})\tilde{U}_0 \quad (4)$$

$$\tilde{U}_0 = \frac{\pi c^2}{2} \left[\frac{\Gamma_{D_2}}{\omega_{D_2}^3} \left(\frac{2}{\omega_{D_2} - \omega_L} \right) + \frac{\Gamma_{D_1}}{\omega_{D_1}^3} \left(\frac{1}{\omega_{D_1} - \omega_L} \right) \right], \quad (5)$$

where c is light's velocity in vacuum, Γ_{D_i} and ω_{D_i} ($i=1,2$) are, respectively, the natural line width and frequency of the D_i line of the atomic specie used, and ω_L is the frequency of the input beam. From Eqs.(4) and (5), it can be seen that, from the optical point of view, the relevant parameters are the trapping frequency ω_L and the light intensity distribution $I(\mathbf{r})$. In our case $I(\mathbf{r})$ is given by Eqs.(1)-(3). However, from such equations it is not straightforward to obtain the trapping frequencies under the harmonic approximation. For this reason, it is interesting to obtain analytical expressions for the intensity distribution near the PDR. Under the approximation $\rho_0 \rightarrow \infty$, the amplitude of the CR light field at the PDR can be analytically described by the Kummer confluent hyper-geometric function ${}_1F_1(a; b; z)$ [8]:

$$f(\tilde{r}, z) = \frac{C}{(1+iz)^{\frac{3}{4}}} \left[\Gamma\left(\frac{3}{4}\right) {}_1F_1\left(\frac{3}{4}; \frac{1}{2}; -\frac{\tilde{r}^2}{1+iz}\right) + \frac{2\tilde{r}}{(1+iz)^{\frac{1}{2}}} \Gamma\left(\frac{5}{4}\right) {}_1F_1\left(\frac{5}{4}; \frac{3}{2}; -\frac{\tilde{r}^2}{1+iz}\right) \right], \quad (6)$$

where $\Gamma(x)$ is the Gamma function, $C = \sqrt{P/2\pi^2 w_0^2 \rho_0}$, and $\tilde{r} = (r - R_0)/w_0$. As Eq. (6) is analytic, it allows obtaining the harmonic trapping frequencies at the PDR:

$$\omega_{r,z} \sqrt{\frac{A_{r,z} \tilde{U}_0 P}{m \pi^2 w_0^4 \rho_0}}, \quad (7)$$

where $A_r = 4.63$, $A_z = 0.64$, and m is the atomic mass.

Fig.5. (a) 1D plots of the CR beam intensity along the radial (green) and axial (pink) directions of the PDR. (b) 2D density plot of the CR beam intensity near the PDR demonstrating that it is a null intensity region in three dimensions. (c) Sketch of the experimental setup to trap BECs in a blue-detuned ORP based on CR. (d) and (e) show, respectively, the experimental and numerically calculated density distribution of a trapped ^{87}Rb BEC made of 25000 atoms in the CR ring after 30ms of free expansion from the top of the ring. Experimental parameters: $R_0=170\mu\text{m}$, $\omega_r = 2\pi \times (300 \pm 20)\text{Hz}$, $\omega_z = 2\pi \times (500 \pm 20)\text{Hz}$, $P=27\text{mW}$, $w_0=18\mu\text{m}$.

After characterizing the PDR in 3D, we applied it as a blue-detuned optical ring trap for ^{87}Rb BEC, in collaboration with the group of Prof. Birkel at the Technische Universität Darmstadt. The experimental setup is shown in Fig. 5(c). The CR ring potential ($\lambda=792.55\text{nm}$) is oriented horizontally, i.e., with the PDR being parallel to gravity. In our case, the confinement offered by the PDR in the vertical direction for the laser power available was insufficient to counteract the gravity force. As a consequence, and to ensure 2D dynamics of the BEC, vertical confinement was achieved by a horizontally oriented red-detuned light sheet ($\lambda=783.55\text{nm}$) generated by focusing a Gaussian beam with a cylindrical lens. Note that the same light sheet could be implemented with blue-detuned light to undergo optical levitation over the beam rather than trapping inside of it. Lenses $F_1 = 150\text{mm}$, $F_2=F_3=F_4 = 400\text{mm}$, $F_5 = 300\text{mm}$ were used to re-image the CR potential inside the vacuum chamber and to de-magnify it by a factor of 0.75. Half and quarter waveplates (HWP, QWP) control the polarization state of the incident illumination. Figs. 5(d) and (e) show, respectively, the experimental and numerically calculated atomic density distribution of a trapped ^{87}Rb BEC formed by 25000 atoms in the CR ring after 30ms of expansion along a CR PDR with a ring radius $R_0=170\mu\text{m}$ a radial trapping frequency $\omega_r = 2\pi \times (300 \pm 20)\text{Hz}$. Under these experimental conditions, the obtained trapping frequency using Eq. (7) was $\omega_r = 2\pi \times 265\text{Hz}$, which is in a reasonable agreement with the experimentally measured trapping frequency. The validity of the harmonic potential approximation has been further confirmed by studying the ground state of the BEC trapped at the PDR, showing deviations below the 0.7% between the PDR and the equivalent harmonic radial dark trap.

The reported results show the suitability of the PDR for trapping ultra-cold atoms with blue-detuned light, making this technique ideal for experiments where well-defined potentials and high intensity beams are required. The advantages of this technique are high efficiency power conversion, the high damage threshold of BCs, the broad spectral range at which BCs can be used, the use of other available bright and dark potentials based on CR [see Fig. 3], and the possible combination with microlens arrays or optical lattices.

5. A free-space optical communication multiplexer and de-multiplexer based in CR

In optical communications, different properties of a light field such as its intensity, wavelength, polarization, and OAM, can be used as information carriers such that the corresponding signals do not interfere with each other along the communication channel. Among these techniques, the problem with polarization multiplexing by means of a beam polarization splitter is that, for the monochromatic case, at most two orthogonally polarized signals can be transmitted without cross-talk.

In this sense, we proposed using cascaded CR as an alternative to increase the channel capacity of a polarization multiplexed free-space optical communication (FSOC) link [29]. The operation principle of the technique is explained in Fig. 6, where CR for a degenerated 2-crystal cascade, i.e., for the case of two cascaded identical crystals with opposite orientations of their $\mathbf{G}$ vectors, is introduced. The CR phenomenon transforms an ideally collimated CP light beam into a collimated hollow cylinder at the output facet of the first BC. As a consequence, a crystal with orientation of its $\mathbf{G}_1 = 90^\circ$ shifts the center of the CR ring a distance R_0 in the $+y$ direction, see Fig. 6(b). Let's call this transformation CR-forward transform. In contrast, a crystal with $\mathbf{G}_1 = 270^\circ$ shifts the CR ring a distance R_0 in the $-y$ direction, see Fig. 6(c), i.e., it provides a CR-backward transform. If both effects are combined, the output beam at the exit of the second BC is identical to the input beam entering into the first BC. In other words, the birefringent effects of both BCs cancel each other and the input beam remains invariant after passing through the degenerated 2-crystal cascade. The panel below Figs. 6(b) and (c), shows the transformation of a CP Gaussian (first row), a CP elliptical (second row) and a CP star-shaped (third row) input light beams as they pass through this arrangement. As it can be appreciated, the output beam emerging from the degenerate 2-crystal cascade reproduces the input beam. The forward-backward CR transform can be used for polarization multiplexing and de-multiplexing, as sketched in Fig. 6(a). One can consider that the light ring obtained behind the first BC is formed by an ideally infinite number of LP single beams placed along the ring, i.e., acting as a continuous de-multiplexer of the CP Gaussian input beam. These sub-beams can be filtered out by using, for instance, an angular amplitude mask (AAM) as shown in the inset of Fig. 6(a). The sectors of the CR ring filtered by the AAM can be used as communication signals. The second BC added in the degenerated cascaded configuration performs a backward transform and recombines, i.e. multiplexes, the signals back to a Gaussian-like beam that propagates 4m in free-space. Thus, the transmitter consists of the degenerated 2-crystal cascade and AAMs. Finally, a third BC performing a CR-forward transform combined with a CCD camera acts as a receiver to de-multiplex and read out the transmitted signals. Information encoding into the different selected signals could be performed by time varying the transmission coefficient for each sector of the mask. For further details of the system, see Ref. [29]. The bottom right panel in Fig. 6 shows the experimental results. As it can be observed in the third row, the technique served to independently modulate up to 12 sectors, which

constitutes an increase in one order of magnitude of the channel capacity of the FSOC link. Last row shows the intensity variation along the azimuthal direction of the corresponding de-multiplexed patterns. The intensity peaks of the received channels are perfectly distinguishable with respect to the background. Analysis on signal crosstalk revealed that the smaller the number of channels the smaller the crosstalk, while the thinner the open sectors, the smaller the crosstalk. In particular, for the 12 channels case shown in Fig. 6(g), the average crosstalk was less than 3%.

In summary, with this work we showed that cascaded CR can be used as a tool for increasing the number of signals that can be multiplexed with monochromatic light, which could potentially increase data bandwidth of FSOC systems.

Fig. 6. (a) Sketch of the polarization de-multiplexing and multiplexing experimental set-up based on CR being used to increase the channel capacity of a FSOC system. The FSOC system is formed by the transmitter with its two identical biaxial crystals presenting opposite $\mathbf{G}$ vectors, a free space propagation distance of 4m, and the receiver with the third biaxial crystal. The top inset shows how polarization signals are selected by using an angular amplitude mask (AAM) that only allows passing some sectors of the CR ring. Green lines indicate the state of polarization. LP: linear polarizer. QWP: quarter waveplate. Bottom-left panel: principle of operation of the forward-backward CR transform provided by a degenerate 2-crystal cascade configuration formed by two BCs with the orientation of their characteristic vector being of (b) $\mathbf{G}_1 = 90^\circ$ and (c) $\mathbf{G}_2 = 270^\circ$. Images below show the experimental transverse intensity patterns of the input beam entering into the system (first column), the CR beam obtained at the ring plane after the first BC (second column) and the output beam emerging from the whole degenerate 2-crystal cascade (third column) for a CP Gaussian input beam (first row), a CP elliptical input beam (second row) and a CP star-shaped input beam (third row). (d)-(g) Experimental transverse intensity patterns (third row) and the corresponding integrated azimuthal intensities (fourth row) obtained by the receiver for multiplexer masks (first row) with 2 (d), 4 (e), 8 (f), and 12 (g) opened sectors. The second row shows the multiplexed beams at the entrance of the receiver.

6. Conclusions

In this manuscript, I have presented some of the results that I obtained during my PhD thesis, where I studied in detail both theoretically and experimentally the phenomenon of conical refraction, occurring in optical biaxial crystals. My aim has been to show that the physics behind the phenomenon of CR are very rich and that the singular properties of CR can be really useful in a wide variety of situations. For many years, this optical effect was considered as a curiosity and almost fell into oblivion until the last quarter of

the 20th century. One of the reasons for this is that crystal technology was not developed enough to offer high quality BCs cut, with high precision, perpendicular to one of its optic axes. This issue was solved during the last decades and, at the beginning of the 21st century, the community paid again attention to the CR phenomenon both experimentally and theoretically. The following main reasons make CR so attractive:

- When using a focused CP Gaussian input beam, the CR beam forms an optical bottle, i.e. it opens and closes itself symmetrically along its propagation leaving a central region without intensity.
- At the intermediate region of the optical bottle -the focal plane-, the transverse intensity pattern is formed by a pair of concentric bright rings split by a dark ring with null intensity known as Poggendorff ring.
- The state of polarization along the rings is linear and unique at every point in a way such that every two diametrically opposite points of the ring have orthogonal polarizations. This polarization distribution differs from the well-known radial and azimuthal polarization distributions.
- For a Gaussian input beam, by varying the parameter $\rho_0 \equiv R_0/w_0$ (where R_0 and w_0 are the CR ring radius and the beam waist radius, respectively) both the shape and polarization of the CR beam can be controlled, yielding to Laguerre-Gauss-like modes useful in many different applications.
- For an optical system formed by N BCs in cascade with aligned optic axis, it has been shown that the output beam is formed by up to 2^{N-1} concentric light rings each one with its Poggendorff splitting. In particular, for $N = 2$ identical crystals with opposite orientations, any input beam to the first crystal, emerges unchanged from the cascade.

These properties, which were investigated in detail during my PhD thesis, have allowed me to find applications of the phenomenon in different areas of science and technology. As an example, here I have discussed the reconfiguration of the CR optical bottle by tailoring the state of polarization of the input beam to generate optical vaults for macroscopic absorbing particles throughout the photophoretic force. Additionally, I have demonstrated that the Poggendorff dark ring forms a toroidal optical bottle, this is that it is a dark ring surrounded by regions of higher intensity in three dimensions. Then, I have shown an implementation of this light geometry as a blue-detuned optical potential to trap ultra-cold atoms experimentally. Finally, I have also shown that cascaded CR provides a useful scenario to improve by an order of magnitude polarization multiplexing for free-space optical communications.

I hope that these investigations will motivate the scientific community to take profit of the flexibility offered by the CR phenomenon, giving a second life to this unique effect.

Acknowledgements

The author is in debt with Dr. Jordi Mompart for his valuable support and excellent supervision during the PhD period, as well as he is thankful to Dr. Todor Kirilov and Dr. Yury V. Loiko for all their guidance and assistance. This work was funded through the fellowships: AP2010-2310 from the MICINN and the DAAD grant 91526836.