Numerical results of Monte Carlo code in lidar returns considering polarization of light and different phase functions

Resultados numéricos del programa Monte Carlo en retornos lidar considerando la polarización de la luz y diferentes funciones de fase

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ABSTRACT:

Monte Carlo method (MCM) is a useful tool to simulate and understand random process in the nature. Many of this process are difficult to solve by means of analytic expressions, due we do not know many of its variables involved in the process. The core of MCM is to generate random variables, which represent physical variables, through of its probability distribution function (PDF). Every random physical variable has a law of probability to obtain a certain value, within a specific interval. Specially, in the data obtained by lidar returns, we got information of the type of scatterer from the radiation backscatter to receiver. This portion of backscatter radiation is a little part of the function that describes the total scattering in 4π radians. This function is called "phase function" and has a particular way depending on size, refractive index, shape, etc. of the scatterer. In this work we present the numerical results to consider different phase functions in the simulation of lidar returns, through of MCM.

Key words: Monte Carlo Method, Phase Functions, Lidar Returns.

RESUMEN:

El método Monte Carlo es una herramienta útil para simular y entender procesos aleatorios en la naturaleza. Generalmente, muchos de estos procesos son difíciles de resolver por medio de expresiones analíticas, debido a que no conocemos la mayoría de las variables involucradas en el proceso. El núcleo de Monte Carlo consiste en generar variables aleatorias, que representan variables físicas (observables), a través de su función de distribución de probabilidad (FDP); esto es, cada observable está gobernada por una ley de probabilidad para obtener un determinado valor, dentro de un intervalo específico. Especialmente, en los datos obtenidos de retornos Lidar obtenemos información del dispersor a partir de la radiación retrodispersada al receptor. Esta porción de radiación retrodispersada es una pequeña parte de la función que describe el esparcimiento total en el espacio. Esta función es llamada la "función de fase" y tiene una dependencia particular del tamaño, índice de refracción y forma del dispersor. En este trabajo se presentan resultados numéricos al considerar diferentes funciones de fase, en la simulación de retornos Lidar, a través del método Monte Carlo.

Palabras clave: Método Monte Carlo, Funciones de Fase, Retornos Lidar.

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1. Introduction

The multiple scattering in lidar technique is a problem of radiative transfer which is possible to write as mathematical equation. It depends of physical parameters as speed of light, extinction coefficient, scattering coefficient and the phase function (scattering function). The radiative transfer can be visualized as a group of photons with different trajectories. Each of these photons have own polarization which is represented by means of Stokes vectors.

The Monte Carlo method (MCM) gives approximate solutions to a variety of mathematical problems by performing statistical sampling [1]. This statistical simulation is based on generating a series of random numbers that follow a probability distribution function (PDF). Therefore, while exists a big set of these random numbers, a result that resembles the problem should be obtained. A model based on statistical process can be obtained with this method. In this model, the random numbers that represent physical variables of the problem also have a PDF. The heart or core of MCM is the generation of random numbers that represents a physical variable and follow a PDF.

2. Defining the problem

In this section, the algorithm of modified MCM is described, that simulates the pass of photons through turbid media. This modification consists in adding some steps to the standard MCM in order to follow the light polarization.

The beginning consists in considering the light beam as a set of photons that have their own polarization, through Stokes vectors, and follow a fixed direction. In this step, the

constants of the problem are defined, as well as the coordinate system [2]. The meridian planes model [3] explains the polarization. Therefore, we define the reference plane of the output polarization. For instance, it could be the xz plane which implies the azimuthal angle equal to zero ($\phi = 0$). The initial polarization state is referred to this plane, and we take advantage that a laser beam is plane-polarized (P-state). It is represented by Stokes vector as $s_0 = (1, 1, 0, 0)$.

Figure 1 shows the MCM algorithm that takes into account the light polarization [3]. The photon begins to move. After a scattering event, the photon next position is calculated. The extinction coefficient is useful to calculate the

Fig. 1. Flow chart of polarized Monte Carlo. The gray cells were added to the standard Monte Carlo program (white cells).

free path length between scattering events. With the new position is necessary to make the following question: is the photon inside of turbid medium? If the answer is yes, the photon continues hitting the scatterers every time with less energy or probability to have a scattering event. If it is not inside the medium, was the photon transmitted or reflected? Depending if the photon was transmitted or reflected, we have to increase the counter of transmission or reflection, respectively.

To continue the photon crashing inside the medium, it is necessary to generate the new scattering direction (scattering angle θ) and the free path length. This is done by means of the core of MCM. The most important function at this point is the scattering function called the “phase function”, which can be as complex as the one used by Ramella *et al* [3], or as simple as the Henyey-Greenstein function.

The phase function has physical parameters of the scatterer like shape, size, refractive index coefficient and wavelength of the incident radiation, according to the Mie theory [4,5].

When is taking into account the polarization in MCM it is necessary to do three operations with two rotation matrices and the Mueller matrix. All together define the phase matrix.

3. Probability distribution functions

To understand a PDF, let us explain the following example: if we have a dice with six faces and we do the experiment of throwing it away, we have six possible numbers, one to six, to obtain for each time we do the experiment. Doing the experiment one hundred times it is obtained the same value of random numbers. Plotting these numbers, frequency versus value, the PDF or histogram is obtained. The more times the experiment is done, a bigger set of random numbers are obtained. Specifically, this kind of experiment has a Gaussian or normal PDF.

In the scattering theory, the “phase function” describes the spatial scattering shape that light is thrown away (plotting magnitude vs scattering angle) and is not related with the phase of the scattered electromagnetic wave.

The first time that was used the term “phase function” was in astrophysics, for the scattering of stellar particles.

Equations (1) to (3) present three phase functions, useful to simulate the light scattering:

$$p(\theta) = \frac{|f_{11}|^2 + |f_{22}|^2}{2}, \quad (1)$$

$$p(\theta, \phi) = S_{11}(\theta)I_0 + S_{12}(\theta) \times [Q_0 \cos(2\phi) + U_0 \sin(2\phi)], \quad (2)$$

$$p(\cos\theta) = \frac{1 - g^2}{2(1 + g^2 - 2g\cos\theta)^{3/2}}. \quad (3)$$

Bruscaglioni used Eq. (1) in [2]. It was made by the first two diagonal elements of Mueller matrix, where f_{11} , f_{22} are the scattering amplitude functions which depend of scattering angle θ . Equation (2) was used by Ramella-Roman *et al* in [3]. This function depends of the polar and azimuthal angles (θ, ϕ) , respectively. Besides it is a function of two elements of the Stokes vector Q_0 and U_0 . Finally, the Henyey-Greenstein function, Eq. (3), is used in many fields, where g is anisotropy parameter.

Figure 2 shows the plots of Eqs. (1) and (2), left and right, respectively. These plots are not normalized with the purpose to see all the direction lobules. Equation (1) is better observed in a rectangular plot because it has a big forward lobule; besides the bars represent the histogram of random numbers generated by MCM. In the inset we can observe the results from Eq. (2) in spherical plot, corresponding to a spherical scatterer.

4. Numerical results

Figure 3 represents the physical situation to simulate with MCM: the turbid media is a fog of water droplets like spherical scatterers. Three cases could be possible. The photon is transmitted or reflected from the turbid media or it could be absorbed.

To present the results, we have implemented the variable $\delta = (t - t_0)/t_0 = (l - L)/L$, where t_0 and L are the time and length of the ballistic trajectory of the photon to arrive to the photodetector, and t and l are the time and

Fig. 2. The red line is the plot of Eq. (1); besides the histogram of the random numbers generated by MCM. The inset shows the spherical plot of Eq. (2) to spherical scatterer according to the Mie theory.

Fig. 3. Photons striking with water droplets like a turbid media. The photons can be absorbed, transmitted or reflected from the turbid media.

length of any other trajectory (generic trajectory). To calculate t and t_0 we need the speed of light (c) inside the scattering medium ($t = l/c$, $t_0 = L/c$). In a complete trajectory, ballistic or not, we have taken into account the distances that there are between the laser-medium and medium-photodetector.

We have presented these results in histograms, since these plots give us a general insight of how the photons are scattered inside

the medium, as well as the percentage of the photons that reach the photodetector in backward and forward directions.

We defined four types of δ -delta variables: δ_{Refl} , δ_{Trans} , for photons that came out of medium, reflected and transmitted; $\delta_{ReflDetec}$, $\delta_{TransDetec}$ for photons that besides came out of medium, they went into the field of view of photodetector. These values are shown in the next Table I.

TABLE I
Percentage of photons that left of the medium and reach the photodetector.

Phase Function	$\delta Refle$	$\delta Trans$	$\delta RefleDetec$	$\delta TransDetec$
Equation 1	2.3%	87.5%	0.29%	17%
Equation 2	1.8%	90%	0.45%	17%
Equation 3	1.3%	88%	0.27%	18%

Fig. 4. (a) Histogram of transmitted photons; (b) Histogram of photons that arrive to the photodetector; (c) Histogram of reflected photons.

Figure 4 shows the histograms of these δ -delta variables only for the phase function in Eq. (2). The histogram for $\delta RefleDetec$ is not presented due to the negligible values.

5. Conclusions

In this work we have developed a Monte Carlo program to simulate the pass of photons through a turbid media, considering the polarization of the incident light. We have changed the phase

function according to Eqs. (1) to (3). The phase function is the most important part in the code since is where the parameters of the scatterer are taken into account. In this way, the histograms show the percentage of transmitted and reflected photons. These results will change if we change the phase function.

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