Approximate determination of the maximum power stored in Stokes lines generated in silica fibers

Determinación aproximada de la potencia máxima almacenada en líneas Stokes generadas en fibras de sílice

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ABSTRACT:

This work presents a methodology for estimating the maximum power stored in 1, 2,... N orders Stokes. Analysis of the solution of equations that govern the stimulated Raman scattering is essential for the development of a novel mathematical relationship, which estimates with great approximation the maximum power stored in Stokes lines. The numerical simulations were tested experimentally for 9.6 km LEAF fiber, where it was found the theoretical maximum pumping power for the first Stokes was 2.4 W, while approximately 2.43 W was found experimentally. In other hand, for the second Stokes power was 4.695 W theoretically, while 4.1 W was found experimentally. The inaccuracy of estimation of optical conversion efficiency of the first Stokes was approximately 0.5%. While the second Stokes, the estimation error of the optical conversion efficiency was around 2.3%. Those results prove the consistency of the equations here reported.

Key words: Optical Conversion Efficiency, Raman Amplifiers, Raman Fiber Lasers.

RESUMEN:

Este trabajo presenta una metodología para estimar la máxima potencia almacenada en líneas Stokes de 1, 2, ..., hasta orden N . El análisis de la solución de ecuaciones que gobiernan el esparramamiento Raman estimulado es fundamental para el desarrollo de una relación matemática novedosa, que estima con gran aproximación la máxima potencia almacenada en líneas Stokes. Simulaciones numéricas y experimentales fueron realizadas para 9.6 km de fibra LEAF, de la simulación se encontró que la potencia máxima de bombeo para el primer Stokes fue 2.4 W, mientras que experimentalmente fue 2.43 W. Por otro lado, para el segundo Stokes la potencia de bombeo simulada fue 4.695 W, mientras que 4.1 W fue encontrado experimentalmente. El error de estimación de la eficiencia de conversión óptica del primer Stokes fue aproximadamente 0.5%, mientras que para el segundo Stokes, el error de estimación de la eficiencia de conversión óptica fue de alrededor de 2.3%. Estos resultados prueban la consistencia de las ecuaciones aquí reportadas.

Palabras clave: Eficiencia de Conversión Óptica, Amplificadores Raman, Láseres Raman de Fibras.

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1. Introduction

Stimulated Raman scattering (SRS) in optical fiber is a technique that has enabled generation of fiber lasers and Raman amplifiers. This is partially because optical fibers are an ideal medium that can facilitate the Raman scattering. Raman scattering is a nonlinear effect that occurs when a monochromatic light beam propagates along the optical fiber. The majority of the beam power is transmitted without change, but a small quantity (typically 10^{-6}) is scattered with a new frequency [1]. For high coupled pump power, the scattered light can grow rapidly with most of the coupled pump power transformed in a Stokes component. This process is called SRS and it is the platform for the development of Raman lasers and Raman amplifiers. As Stokes component grows quickly, the pump power decreases until it is unable to transfer energy. Now, the Stokes can be high enough to produce the following Stokes by SRS, the process is repeated again to generated the next Stokes. The SRS in the spectral region from 1.1-1.7 μm is of great importance in developing of several applications such as optical fiber communications, material processing, signal processing, laser spectroscopy and medicine [2]. Equations for the behaviour of the Raman amplification have been described [3,4] and solutions have been proposed [3,5]. Despite these advances, there are parameters such as maximum power stored in the Stokes signals, critical power for Raman lasers, and other parameters that have not been described.

This paper describes the maximum power stored in Stokes lines produced in a Raman amplifier. From existing theory, a formula has been developed, which is verified with experimental results.

2. Theory

In order to find a mathematical relationship that predicts the maximum power stored in the Stokes signal is necessary to solve the equations that describe the forward propagation of the pumping power and Stokes line in a single mode fiber given by [3,6]:

$$\frac{dP_P}{dz} = -\alpha_P P_P - \frac{\nu_P}{\nu_S} \frac{g_r}{A_{eff}} P_P P_F, \quad (1)$$

$$\frac{dP_F}{dz} = -\alpha_S P_F + \frac{g_r}{A_{eff}} P_P P_F, \quad (2)$$

where P_P and P_F , ν_P and ν_S , α_P and α_S are powers, frequencies and losses coefficients of pump power and Stokes signal, respectively. A_{eff} is effective area, and g_r is the Raman gain coefficient of the Stokes signal.

It can be assumed that $\alpha = \alpha_P = \alpha_S$ because the attenuation curve of the silica fiber is approximately constant in the transparency window. An analytical solution of the equations that govern the SRS can be obtained by the following procedure: first dividing Eq. (1) by Eq. (2), and then adding Eqs. (1) and (2). The resulting differential equations are easily solved

and their solutions are evaluated in all the fiber from $z = 0$ to $z = L$, as shown below [5]:

$$\ln\left(\frac{P_F}{P_{F0}} \frac{P_{P0}}{P_P}\right) = \frac{g_r}{A_{eff}} \frac{v_P}{v_S} \frac{1}{\alpha} \left[\left(\frac{v_S}{v_P} P_{P0} + P_{F0} \right) - \left(\frac{v_S}{v_P} P_P + P_F \right) \right], \quad (3)$$

and

$$\frac{v_S}{v_P} P_P + P_F = \left(\frac{v_S}{v_P} P_{P0} + P_{F0} \right) \exp(-\alpha L), \quad (4)$$

where P_{P0} and P_P are the input and output powers, P_{F0} and P_F are initial and output Stokes signal. L is the fiber length. On the other hand, a millionth of the input pump power (P_{P0}) is transferred in initial Stokes signal (P_{F0}), i.e., $P_{F0} \approx 10^{-7} P_{P0}$. Therefore, the term $v_S/v_P P_{F0} + P_{F0} \approx v_S/v_P P_{F0}$ [1]. Substituting Eq. (4) in Eq. (3), it is obtained a general solution to differential Eqs. (1) and (2), given by;

$$\ln\left(\frac{P_F}{P_{F0}} \frac{P_{P0}}{P_P}\right) = \frac{g_r}{A_{eff}} P_{P0} L_{eff}, \quad (5)$$

where $L_{eff} = [(1 - \exp(-\alpha L))/\alpha]$ represents the interaction length of the pumping and the Stokes signal. This mathematical relationship provides two special points in the design of lasers and Raman amplifiers: Raman threshold and the maximum power stored in the first Stokes just before the generation of the second Stokes.

2.a. Raman threshold

Raman threshold has been regarded as a critical power, above this energy transfer to the first Stokes is very fast and below this multichannel communications systems operating efficiently since there is no energy transfer from one channel to another [1]. Raman threshold is defined as the power level, in which the Stokes signal and the residual power are equal, i.e., $P_P = P_F$. And taking into account that $\approx 10^{-6} P_{P0}$, Eq. (5) becomes:

$$16 \approx \frac{g_r}{A_{eff}} P_{cr} L_{eff}, \quad (6)$$

where P_{P0} , changed by the critical power P_{cr} to refer to this power that occurs the Raman threshold. This relationship has been previously reported [3] and used in several research

projects. At the point of the Raman threshold pump power transfer occurs very rapidly until it is exhausted. Getting the signal reaches its maximum energy storage and as a result causes the formation of the next Stokes.

2.b. Scaling of the Raman gain

For this, we first rewrite Eqs. (1) and (2), and assume that the first Stokes take the role of the pump power and the Stokes signal take the role of the second Stokes. Consequently, we used the same procedure to determine Eq. (6), so that the Raman threshold for the second Stokes is given by:

$$16 \approx \frac{g_{r2}}{A_{eff2}} P_{cr2} L_{eff2}, \quad (7)$$

where g_{r2} and P_{cr2} are the Raman gain coefficient and the critical power for the second Stokes, respectively. $L_{eff2} = (1 - \exp(-\alpha_{s2} L_{eff}))/\alpha_{s2}$ is the effective fiber length of interaction between the first and second Stokes. Equating Eqs. (6) and (7), we obtain the general relationship for the scaling of the Raman gain coefficient given by:

$$g_{r2} = \frac{P_{cr}}{P_{cr2}} \frac{L_{eff}}{L_{eff2}} \frac{A_{eff2}}{A_{eff}} g_r. \quad (8)$$

It has been shown that the pumping power of a Gaussian beam is proportional to the wavelength [7], assuming the form of critical powers are Gaussian, we have that the ratio of the critical power can be approximated to the ratio of the wavelengths, so that Eq. (8) becomes:

$$g_{r2} = \frac{\lambda_P}{\lambda_S} \frac{L_{eff}}{L_{eff2}} \frac{A_{eff2}}{A_{eff}} g_r. \quad (9)$$

Equation (9) indicates that the Raman gain coefficient scales with the inverse of the Stokes wavelength, the effective fiber length of interaction between the first and second Stokes and the effective area.

2.c. Obtaining of the maximum power stored in the first Stokes

Raman lasers and amplifiers are widely studied in various optical fibers with silica matrix, and its use is common in telecommunications systems, also other fields of study are

investigated as medicine and military applications. However, there are still parameters to consider studying the most of these systems. Experimentally, it has shown about a millionth of the pump power propagating in an optical fiber is converted into Raman scattering, this paper proposes that the transfer of the first Stokes pumping will cease when the pump power is around a millionth of the Stokes signal. That is $P_p \approx 10^{-6} P_F$ under this proposal and taking into account the already known expression $P_{F0} \approx 10^{-6} P_{P0}$, Eq. (5) becomes;

$$30 \approx \frac{g_r}{A_{eff}} P_{Max}^1 L_{eff}, \quad (10)$$

where P_{P0} , was changed to the maximum power stored P_{Max}^1 to refer to this power which is stored the maximum amount of energy in the Stokes signal, i.e. the Stokes signal will stop growing and in their absence generate the second Stokes. Now to measure the power stored in the first Stokes, Eq. (10) is substituted into Eq. (4), resulting:

$$P_1^{Sto} = \frac{\nu_S}{\nu_P} \frac{30 A_{eff}}{g_r L_{eff}} e^{-\alpha L}, \quad (11)$$

where P_F has been changed to the maximum power stored, which represents the maximum cumulative power at the first Stokes signal as shown in Fig. 1.

Once the first Stokes reaches its maximum power level, instantly begins to generate the second Stokes. That is, the first Stokes acts as the

Fig. 1: The points represent maximum pumping power versus maximum power stored in the Stokes lines.

pump source that transfers its energy to the second Stokes, this dynamic ends when the second Stokes reaches its maximum power level. Following the same procedure to obtain Eq. (11), the second Stokes power reaches its maximum as shown in the next formula:

$$30 \approx \frac{g_{r2}}{A_{eff2}} P_{Max}^2 L_{eff2}, \quad (12)$$

where the parameter P_F has been changed P_{Max}^2 , to reference that is stored the maximum amount of power at the second Stokes, i.e. the Stokes signal is saturated and instead generates the third Stokes. Now the corresponding power stored in the second Stokes is given by:

$$P_2^{Sto} = \frac{\nu_{S2}}{\nu_S} \frac{30 A_{eff2}}{g_{r2} L_{eff2}} \exp(-\alpha_2 L_{eff}), \quad (13)$$

where P_{F2} has been changed by P_2^{Sto} , which represents the maximum power stored in the second Stokes signal as shown in Fig. 1. Now, the pump power coupled to the fiber to achieve maximum power at the second Stokes is given by the sum of the maximum storage powers ($P_{02}^{Max} = P_{Max}^1 + P_{Max}^2$) shown in Eqs. (10) and (12):

$$P_{02}^{Max} \approx 30 \frac{A_{eff}}{g_r L_{eff}} + 30 \frac{A_{eff2}}{g_{r2} L_{eff2}}, \quad (14)$$

where P_{02}^{Max} is the pumping power required to store the maximum power in the second Stokes. If now we employ Eq. (9), a versatile equation can be obtained given by:

$$P_{02}^{Max} \approx 30 \frac{A_{eff}}{g_r L_{eff}} \left[1 + \frac{\lambda_p}{\lambda_s} \right]. \quad (15)$$

In general the pumping power for storing the maximum energy in N-order Stokes is given by:

$$P_{0N}^{Max} \approx 30 \frac{A_{eff}}{g_r L_{eff}} \left[1 + \frac{\lambda_p}{\lambda_s} + \frac{\lambda_s}{\lambda_{S2}} + \dots + \frac{\lambda_{N-1}}{\lambda_N} \right]. \quad (16)$$

With this equation, we obtain approximately the maximum power stored in the N-order Stokes before generating the $N + 1$ Stokes order. This equation is versatile because the parameters involved are simple to calculate. To date, Eq. (16), at least of our knowledge, has not been reported. And finally, the ratio between the

maximum power stored and coupled pump power [8], for the first Stokes is given by:

$$\frac{P_1^{Sto}}{P_{Max}^1} = \frac{\lambda_p}{\lambda_s} e^{-\alpha L}. \quad (17)$$

This equation depends on the attenuation and the fiber length, and describes the optical conversion efficiency for a particular fiber length (L) and fiber attenuation (α). If $L = 0$, Eq. (17) gives the maximum energy conversion efficiency or quantum efficiency [9].

3. Experimental configuration and results

The experimental setup consists of a 1064 nm CW fiber laser (FL) pumping 9.6 km LEAF fiber with cavity free of mirrors. At the end of the fiber, is installed a power meter (PM), for measuring the average power of the output signal, and an optical spectrum analyzer (OSA) for analyzing changes in the lines when the pumping power is released. With this experimental configuration the pump level was gradually increased to find the maximum power stored in Stokes lines. When the pumping power is released, instantaneously SRS occurs. The pump increases and the energy transfer to the first Stokes begins until it reaches its maximum value, and is ready to generate the following Stokes. Spectrum and the power output for each pumping serve to locate the value that corresponds to the residual power and Stokes signals. Figure 2 shows curves of the pump power and Stokes signals for various coupled pump powers.

Note that the maximum values of the first and second Stokes are located at positions 2.43 and 4.06 W, respectively. And the powers are ~ 0.45 and ~ 0.86 W, respectively.

4. Experimental analysis and theoretical simulations

Taking the characteristic data of LEAF fiber provided by the manufacturer, calculations were made, and resulted in the following values: $A_{eff} = 48 \mu m^2$, $A_{eff2} = 50 \mu m^2$, $\alpha = 0.75$ dB/km, $\alpha_1 = 0.6$ dB/km, $\lambda_p = 1064$ nm, $\lambda_1 = 1115$ nm,

Fig. 2: Experimental results, obtained for 9.6 km LEAF fiber.

Fig. 3: Two theoretical curves for 10 km of fiber, the circle represents the maximum power stored and the square represents the maximum pump power obtained experimentally for 9.6 km LEAF fiber. Note the overlap between the curves and points.

Fig. 4: Comparative figure for the second Stokes. Two theoretical curves for 10 km of fiber, the circle represents the maximum power stored and the square represents the maximum pump power obtained experimentally for 9.6 km LEAF fiber.

$\lambda_2 = 1175 \text{ nm}$, and $g_R = 1.27 \times 10^{-13} \text{ mW}^{-1}$. Numerical simulations for 10 km of LEAF fiber were obtained with these data. Figure 3 shows two curves, one showing the pump power necessary to obtain the maximum energy accumulated in the first Stokes, and a curve representing the energy stored in the first Stokes. For example, at 4 km fiber is necessary a pump power of 3.908 W to produce a stored maximum power of 1.876 W.

Note that at a distance of 9.6 km, the maximum pumping power is 2.4 W and the power stored in the first Stokes is 0.43 W with an optical conversion efficiency $\sim 18\%$, whereas experimentally found that the maximum pumping power is $\sim 2.43 \text{ W}$ and maximum power stored is $\sim 0.45 \text{ W}$ and the optical conversion efficiency $\sim 18.5\%$.

Figure 4 shows two theoretical curves, and two experimental points to the maximum pumping power and the maximum accumulated power that reaches the second Stokes. Note that at a distance of 9.6 km of fiber the maximum pumping power is 4.695 W, and the energy stored at the first Stokes is 0.832 W with an optical conversion efficiency of 17.7%; experimentally the maximum pumping power of $\sim 4.1 \text{ W}$ and maximum power is $\sim 0.85 \text{ W}$ stored and optical conversion efficiency $\sim 20\%$. For the

case of the maximum energy of the second Stokes, theoretical and experimental results are similar; however, that the error in the optical conversion efficiency is $\sim 2.3\%$. The great similarity between the theoretical and experimental results shows the consistency of the formulas shown in this work.

5. Conclusions

In this paper, we report a novel equation that predicts the maximum power stored in the first, second and subsequent generated Stokes Raman amplifier. Also, we report a simple equation that estimates the optical conversion efficiency, which shows that there is a strong correspondence between the simulated and experimental values for the first Stokes, however, that for the second Stokes the error in estimating the maximum power is $\sim 9\%$. The equation reported here is a powerful tool for designing efficient Raman lasers and amplifiers.

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