

Non-fragile observer-based controller design for switched fuzzy systems

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Abstract

In this paper, the actuator dead zone problem is solved for switched fuzzy systems. Our attention is concentrated on the construction of nonlinear-solving method for switched fuzzy systems with time delay. Based on non-fragile observer, by the utilization of PDC technique and single Lyapunov function, a switched fuzzy controller is designed, which can guarantee the system to be asymptotical stable with performance. Finally, a simulation study is presented to show effectiveness of the proposed theory.

Keywords

Switched system; actuator dead zone; fuzzy controller; performance.

1. Introduction

Switching system is an important hybrid system consisting of a finite number of subsystems and a switching rule that represents the active subsystem at each moment. For the switching fuzzy system, the switching signal is divided into an arbitrary switching signal, a time related signal, and a state related switching signal. The average dwell time method was proposed in [1-3], and the state-dependent switching signal method was proposed in [4, 5]. However, in some special cases, it needs to switch arbitrarily. We cannot use this method. Then a single Lyapunov function was proposed in the literature [6,7].

Research on switching fuzzy systems has attracted widespread attention, especially in switching T-S fuzzy systems. In the past few decades, the T-S fuzzy model has been developed as an effective tool for approximating the most complex nonlinear systems [8-13]. Therefore, the analysis and synthesis of the problem of switching T-S fuzzy systems is still a hot spot for some researchers. For example, the authors in [14] proposed a class of switching control design methods for nonlinear systems based on the switch Lyapunov function. The authors in [15] studied the exponential and asynchronous stability of a class of nonlinear systems based on the switch Lyapunov function. problem. However, the above literature does not include the dead zone nonlinear phenomenon. What attracts us is that the nonlinear system contains dead zones. In fact, dead zone characteristics are often encountered in various engineering systems, and the presence of dead zones often leads to degradation and instability of system performance. In [16-18], adaptive dead zone inversion was established for linear and nonlinear systems with unmeasurable dead zones. [19] studied the robust stabilization problem of switched fuzzy systems with actuator dead zones. However, the above control method does not consider the conditions in which the time lag and the state are not available.

In practical applications, the observer gain may result in a change between the estimated state and the actual state. These undesired fluctuations can lead to degradation of the closed loop system. In [20], the stability effects of small fluctuations were discussed in depth, and non-fragile control methods were proposed to prevent this unfavorable behavior. In addition, in [21], the non-fragile control of a

class of uncertain systems was found. In [22], the non-fragile guaranteed cost fuzzy controller design of fuzzy systems was proposed. In [23], the uncertain Lur'e is proposed. etc. [24-29]. However, to the best of our knowledge, the design of non-fragile observers for switched fuzzy systems with time-delay and actuator dead zones has not been studied. Therefore, research in this field should have important theoretical and practical significance. It should be pointed out that so far, the work done in switching fuzzy systems is still limited, which prompted us to carry out this work.

This paper investigates the stability of switched fuzzy systems with time delay and actuator dead zone based on no-fragile observer, and considers the H_∞ performance. By using PDC design scheme and single Lyapunov function, a switched fuzzy control law is developed. The sufficient conditions of ensuring the switched fuzzy systems asymptotic stabilization are proposed in terms of LMIs, which can be solved very efficiently using the convex optimization techniques, it is proved that we can get the satisfying performance.

2. System Description

In this paper, we consider a class of fuzzy switched systems with time delay and actuator dead zone, which are described by the following model:

Rule \mathfrak{R}_σ^i : IF $z_1(t)$ is $F_{\sigma 1}^i$, $z_2(t)$ is $F_{\sigma 2}^i$, ..., $z_p(t)$ is $F_{\sigma p}^i$, Then

$$\begin{cases} \dot{x}(t) = A_{\sigma i}x(t) + A_{d\sigma i}x(t-d) + B_{\sigma i}\ell_\sigma(t) + \omega(t), \\ y(t) = C_{\sigma i}x(t) \end{cases} \quad (1)$$

where $i=1, 2, \dots, N_\sigma$, $z(t)$ are the premise variables, and $F_{\sigma p}^i$ are the fuzzy sets, $x(t) \in \mathbb{R}^n$ is the state vector; $\omega(t)$ is the bounded external disturbance, $y(t)$ is the output of the system, $\ell_\sigma(t)$ is the control input with actuator dead zone, which is defined as follows:

$$\ell_\sigma(t) = \begin{cases} u_\sigma(t) - a_l, & \text{if } u_\sigma(t) > a_l \\ 0, & \text{if } |u_\sigma(t)| > a_l \\ u_\sigma(t) + a_l, & \text{if } u_\sigma(t) < -a_l \end{cases} \quad (2)$$

where $u_\sigma(t)$ is the input to dead zone, and a_l is the breakpoint of the input nonlinearity.

According to [19], we can represent the fuzzy switched fuzzy system as following:

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{N_\sigma} h_{\sigma i}(z(t)) [A_{\sigma i}x(t) + A_{d\sigma i}x(t-d) + B_{\sigma i}\ell_\sigma(t) + \omega(t)] \\ y(t) = \sum_{i=1}^{N_\sigma} h_{\sigma i}(z(t)) C_{\sigma i}x(t) \end{cases} \quad (3)$$

where

$$h_{\sigma i}(z(t)) = \frac{\prod_{j=1}^p F_{\sigma j}^i(z_j(t))}{\sum_{i=1}^{N_\sigma} \prod_{j=1}^p F_{\sigma j}^i(z_j(t))},$$

$0 < h_{\sigma i}(z(t)) < 1$, $\sum_{i=1}^{N_\sigma} h_{\sigma i}(z(t)) = 1$, $F_{\sigma j}^i(z_j(t))$ denotes the membership function where $z_j(t)$ belongs to the fuzzy sets $F_{\sigma j}^i$.

Lemma 1. Let $F \in \mathbb{R}^{m \times n}$ and $H \in \mathbb{R}^{m \times n}$ be given. If $x(t) \in \mathcal{V}(F)$, then $\text{sat}(Fx(t))$ can be represented as

$$\text{sat}(Fx(t)) = \sum_{k=1}^{2^m} \eta_k(t) (E_k F + E_k^- H) x(t)$$

$\eta_k(t)$ for $k=1, 2, \dots, 2^m$ are some scalars which satisfy $0 \leq \eta_k(t) \leq 1$ and $\sum_{k=1}^{2^m} \eta_k(t) = 1$.

Lemma 2. Given constant matrices X and Y , for arbitrary $\varepsilon > 0$, the following inequality holds:

$$X^T Y + Y^T X \leq \varepsilon X^T X + \frac{1}{\varepsilon} Y^T Y$$

3. Fuzzy Control Design

In this section, we will design an observer and develop the sufficient condition of asymptotical stability for the switched fuzzy system with time delay and actuator dead zone. If the state in (3) are unavailable for feedback control design, we can establish an observer to estimate the unmeasured state. The following fuzzy switched observer is presented:

Rule \mathfrak{R}_σ^i : IF $z_1(t)$ is $F_{\sigma 1}^i$, $z_2(t)$ is $F_{\sigma 2}^i$, ..., $z_p(t)$ is $F_{\sigma p}^i$, Then

$$\begin{cases} \dot{\hat{x}}(t) = A_{\sigma i} \hat{x}(t) + A_{d\sigma i} \hat{x}(t-d) + B_{\sigma i} \ell_\sigma(t) + L_{\Delta\sigma i} (y - \hat{y}) \\ \hat{y}(t) = C_{\sigma i} \hat{x}(t) \end{cases} \quad (4)$$

where $L_{\Delta\sigma i} = L_{\sigma i} + \Delta L_{\sigma i}$ is the i th observer gain for the σ -th switched fuzzy subsystem, and $\Delta L_{\sigma i}$ models the uncertain gain.

Assumption 1: The σ ith observer gain perturb $\Delta L_{\sigma i}$ is a structured uncertainty. This is, there exist known constant matrices $Q_{\sigma i} \in \mathbb{R}^{n_x \times n_y}$ and $R_{\sigma i} \in \mathbb{R}^{n_x \times n_y}$ and unknown time-varying matrices $F_{\sigma i}(t)$ such that

$$\Delta L_{\sigma i} = Q_{\sigma i} F_{\sigma i}(t) R_{\sigma i}$$

and $F_{\sigma i}(t)^T F_{\sigma i}(t) \leq I$ for all $t \in \mathbb{R}$ and for all $i=1, \dots, N_\sigma$.

The same as (3), the fuzzy switched observer are inferred as follow:

$$\begin{cases} \dot{\hat{x}}(t) = \sum_{i=1}^{N_\sigma} h_{\sigma i}(z(t)) \\ \quad \times [A_{\sigma i} \hat{x}(t) + A_{d\sigma i} \hat{x}(t-d) + B_{\sigma i} \ell_\sigma(t) + L_{\Delta\sigma i} (y - \hat{y})] \\ \hat{y}(t) = \sum_{i=1}^{N_\sigma} h_{\sigma i}(z(t)) C_{\sigma i} \hat{x}(t) \end{cases} \quad (5)$$

In this paper, we consider the switching signal is arbitrary, the switching signal $\sigma(\hat{x}(t))$ subjects to

$$\nu_r(t) = \begin{cases} 1 & \text{signal}_r = 1 \\ 0 & \text{signal}_r = 0 \end{cases}$$

The overall fuzzy switched observer is inferred as follows:

$$\begin{cases} \dot{\hat{x}}(t) = \sum_{r=1}^l \sum_{i=1}^{N_r} \nu_r(t) h_{ri}(z(t)) \\ \quad \times [A_{ri} \hat{x}(t) + A_{dri} \hat{x}(t-d) + B_{ri} \ell_r(t) + L_{\Delta ri} (y - \hat{y})] \\ \hat{y}(t) = \sum_{r=1}^l \sum_{i=1}^{N_r} \nu_r(t) h_{ri}(z(t)) C_{ri} \hat{x}(t) \end{cases} \quad (6)$$

According to [19], the dead zone is related to the saturation nonlinearity by:

$$\ell_r(t) = u_r(t) - \text{sat}(u_r(t)) \quad (7)$$

To realize the control objective, we will use the single Lyapunov function method here. Based on PDC scheme, we consider the switched fuzzy controller for the switched system

$$u_r(t) = \sum_{i=1}^{N_r} h_{ri}(z(t)) K_{ri} \hat{x}(t) \quad (8)$$

Then, the closed-loop fuzzy switched system is represented as follows:

$$\begin{cases} \dot{x}(t) = \sum_{r=1}^l \sum_{i=1}^{N_r} \sum_{j=1}^{N_r} \nu_r(t) h_{ri}(z(t)) h_{rj}(z(t)) [A_{ri} x(t) + A_{dri} x(t-d) \\ \quad + B_{ri} [(K_{rj} \hat{x}(t)) - \text{sat}(K_{rj} \hat{x}(t))] + \omega(t)] \\ y(t) = \sum_{r=1}^l \sum_{i=1}^{N_r} \sum_{j=1}^{N_r} \nu_r(t) h_{ri}(z(t)) h_{rj}(z(t)) C_{ri} x(t) \end{cases} \quad (9)$$

and the switched fuzzy observer is written as follows:

$$\begin{cases} \dot{\hat{x}}(t) = \sum_{r=1}^l \sum_{i=1}^{N_r} \sum_{j=1}^{N_r} v_r(t) h_{ri}(z(t)) h_{rj}(z(t)) \{ A_{ri} \hat{x}(t) + A_{dri} \hat{x}(t-d) \\ + B_{ri} [(K_{rj} \hat{x}(t)) - \text{sat}(K_{rj} \hat{x}(t))] + L_{dri} C_{rj} \tilde{x}(t) \} \\ \hat{y}(t) = \sum_{r=1}^l \sum_{i=1}^{N_r} \sum_{j=1}^{N_r} v_r(t) h_{ri}(z(t)) h_{rj}(z(t)) C_{ri} \hat{x}(t) \end{cases} \quad (10)$$

The sufficient conditions on the asymptotical stability for system (9) are provided in the following theorem.

Theorem 1. For the switched fuzzy system (9), if there exist symmetric positive matrices P_e, Q_e, P, Q with appropriate dimensions and $\{\alpha_{rij}, \beta_{rij}, \xi_{rij}, \varepsilon_{ri}, \varsigma_{ri}, \delta_{ri}\} > 0, (i, j=1, 2, \dots, N_r)$ such that

$$\begin{pmatrix} \Xi_{rij} & PA_{dri} & 0 & 0 & 0 \\ * & -Q & 0 & 0 & 0 \\ * & * & \Theta_{rij} + I & P_e A_{dri} & P_e \\ * & * & * & -Q_e & 0 \\ * & * & * & * & -\gamma^2 I \end{pmatrix} < 0 \quad (11)$$

where

$$\begin{aligned} \Xi_{rij} = & A_{ri}^T P + P A_{ri} + \varepsilon_{rj} P B_{rj} B_{ri}^T P + \varepsilon_{rj} P B_{rj} B_{rj}^T P - \varepsilon_{rj} P B_{rj} E_k B_{ri}^T P \\ & - \varepsilon_{rj} P B_{ri} E_k B_{rj}^T P - \varsigma_{rj} P B_{rj} E_k^{-T} B_{ri}^T P - \varsigma_{rj} P B_{ri} E_k^{-T} B_{rj}^T P + Q \\ & + \beta_{rij}^{-1} \delta_{ri}^2 P P_e^{-1} C_{ri}^T C_{rj} C_{rj}^T C_{ri} P_e^{-1} P + \xi_{rij}^{-1} P Q_{ri} Q_{ri}^T P \\ \Theta_{rij} = & A_{ri} P_e + P_e A_{ri} - \alpha_{rij} P_e Q_{ri} Q_{ri}^T P_e - \alpha_{rij}^{-1} C_{rj}^T R_{ri}^T R_{ri} C_{rj} + Q_e + \beta_{rij} I \\ & + \xi_{rij} C_{rj}^T R_{ri}^T R_{ri} C_{rj} - \delta_{ri} C_{rj}^T C_{ri} - \delta_{ri} C_{ri}^T C_{rj} \end{aligned}$$

Then for any switching signals, the switching controller (8) could guarantee the switched fuzzy system (9) is asymptotical stable with H_∞ performance.

Proof. Choose a Lyapunov function as follows

$$V(t) = V_1(\tilde{x}(t)) + V_2(\hat{x}(t)) \quad (12)$$

where

$$\begin{aligned} V_1(\tilde{x}(t)) &= \tilde{x}^T(t) P_e \tilde{x}(t) + \int_{t-d}^t \tilde{x}^T(t) Q_e \tilde{x}(t) d(t) \\ V_2(\hat{x}(t)) &= \hat{x}^T(t) P \hat{x}(t) + \int_{t-d}^t \hat{x}^T(t) Q \hat{x}(t) d(t) \end{aligned}$$

Then, we have

$$\begin{aligned} \dot{V}_1(\tilde{x}(t)) &\leq \sum_{k=1}^{2^m} \eta_k(t) \sum_{r=1}^l \sum_{i=1}^{N_r} \sum_{j=1}^{N_r} v_r(t) h_{ri}(z(t)) h_{rj}(z(t)) \{ \tilde{x}^T(t) \\ &\times [A_{ri}^T P_e + P_e A_{ri} - \alpha_{rij} P_e Q_{ri} (P_e Q_{ri})^T - \alpha_{rij}^{-1} (R_{ri} C_{rj})^T R_{ri} C_{rj} \\ &- C_{rj}^T L_{ri}^T P - P_e L_{ri} C_{rj} + Q_e] \tilde{x}(t) + \omega^T(t) P_e \tilde{x}(t) + \tilde{x}^T(t) P_e \omega(t) \\ &+ \tilde{x}^T(t-d) A_{dri}^T P_e \tilde{x}(t) + \tilde{x}^T(t) P_e A_{dri} \tilde{x}(t-d) - \tilde{x}^T(t-d) Q_e \tilde{x}(t-d) \} \end{aligned} \quad (13)$$

and

$$\begin{aligned} \dot{V}_2(\hat{x}(t)) &\leq \sum_{k=1}^{2^m} \eta_k(t) \sum_{r=1}^l \sum_{i=1}^{N_r} \sum_{j=1}^{N_r} v_r(t) h_{ri}(z(t)) h_{rj}(z(t)) \{ \hat{x}^T(t) \\ &\times [A_{ri}^T P + P A_{ri} + (K_{rj} - E_k K_{rj} - E_k^{-T} H_{rj})^T B_{ri}^T P + P B_{ri} \\ &\times (K_{rj} - E_k K_{rj} - E_k^{-T} H_{rj}) + Q + \beta_{rij}^{-1} P L_{ri} C_{rj} C_{rj}^T L_{ri}^T P + \xi_{rij}^{-1} P Q_{ri} Q_{ri}^T P] \\ &\times \hat{x}(t) + \beta_{rij} \tilde{x}^T(t) \tilde{x}(t) + \xi_{rij} \tilde{x}^T(t) C_{rj}^T R_{ri}^T R_{ri} C_{rj} \tilde{x}(t) + \hat{x}^T(t-d) \\ &\times A_{dri}^T P \hat{x}(t) + \hat{x}^T(t) P A_{dri} \hat{x}(t-d) - \hat{x}^T(t-d) Q \hat{x}(t-d) \} \end{aligned} \quad (14)$$

Combining (13) and (14), one has

$$\begin{aligned} & \dot{V}(t) + \tilde{x}^T(t) \tilde{x}(t) - \omega^T(t) \gamma^2 \omega(t) = \\ & \sum_{k=1}^{2^m} \eta_k(t) \sum_{r=1}^l \sum_{i=1}^{N_r} \sum_{j=1}^{N_r} \nu_r(t) h_{ri}(z(t)) h_{rj}(z(t)) \times \\ & \begin{pmatrix} \hat{x}(t) \\ \hat{x}(t-d) \\ \tilde{x}(t) \\ \tilde{x}(t-d) \\ \omega^T(t) \end{pmatrix}^T \begin{pmatrix} \Xi_{rij} & PA_{dri} & 0 & 0 & 0 \\ * & -Q & 0 & 0 & 0 \\ * & * & \Theta_{rij} + I & P_e A_{dri} & P_e \\ * & * & * & -Q_e & 0 \\ * & * & * & * & -\gamma^2 I \end{pmatrix} \begin{pmatrix} \hat{x}(t) \\ \hat{x}(t-d) \\ \tilde{x}(t) \\ \tilde{x}(t-d) \\ \omega(t) \end{pmatrix} \end{aligned} \quad (15)$$

Then we obtain the system (9) is stable with H_∞ performance index for the given γ^2 , this completes the proof.

From (11) in Theorem 1, the stability conditions for the switching fuzzy system are transformed into the following matrix inequality

$$\begin{pmatrix} \Xi_{rij} & PA_{dri} & 0 & 0 & 0 \\ * & -Q & 0 & 0 & 0 \\ * & * & \Theta_{rij} + I & P_e A_{dri} & P_e \\ * & * & * & -Q_e & 0 \\ * & * & * & * & -\gamma^2 I \end{pmatrix} < 0 \quad (16)$$

Note that matrix inequalities (16) are not LMI, To obtain positive definite matrix P_e, Q_e, P, Q , and control gain matrix K_{ri} , the observer gain matrix L_{ri} and matrix H_{ri} .

For the convenience of design, define (16) as

$$\begin{pmatrix} \Xi_{rij} & PA_{dri} & 0 & 0 & 0 \\ * & -Q & 0 & 0 & 0 \\ * & * & \Theta_{rij} + I & P_e A_{dri} & P_e \\ * & * & * & -Q_e & 0 \\ * & * & * & 0 & -\gamma^2 I \end{pmatrix} = \begin{pmatrix} M & 0 \\ 0 & N \end{pmatrix} < 0 \quad (17)$$

with

$$M = \begin{pmatrix} \Xi_{rij} & PA_{dri} \\ * & -Q \end{pmatrix}, \quad N = \begin{pmatrix} \Theta_{rij} + I & P_e A_{dri} & P_e \\ * & -Q_e & 0 \\ * & * & -\gamma^2 I \end{pmatrix}$$

Let $M < 0$ and $N < 0$. By the Schur complements, $N < 0$ is equivalent to the following LMI

$$\begin{pmatrix} \Lambda_{rij} + I & P_e A_{dri} & P_e \\ * & -Q_e & 0 \\ * & * & -(\gamma^2 I - \alpha_{rij} Q_{ri} Q_{ri}^T)^{-1} \end{pmatrix} < 0 \quad (18)$$

where

$$\begin{aligned} \Lambda_{rij} = & A_{ri} P_e + P_e A_{ri} - \alpha_{rij}^{-1} C_{rj}^T R_{ri}^T R_{ri} C_{rj} + Q_e + \beta_{rij} I \\ & + \xi_{rij} C_{rj}^T R_{ri}^T R_{ri} C_{rj} - \delta_{ri} C_{rj}^T C_{ri} - \delta_{ri} C_{ri}^T C_{rj} \end{aligned}$$

Pre- and post-multiplying both side of M by the matrix $\text{diag}\{X \ X\}$, let $X = P^{-1}$, $Y = X Q X$, then we have

$$\begin{pmatrix} \Psi_{rij} & A_{dri} X \\ * & -Y \end{pmatrix} < 0 \quad (19)$$

where

$$\begin{aligned} \Psi_{rij} = & X A_{ri}^T + A_{ri} X + \varepsilon_{rj} B_{rj} B_{ri}^T + \varepsilon_{rj} B_{ri} B_{rj}^T - \varepsilon_{rj} B_{rj} E_k B_{ri}^T \\ & - \varepsilon_{rj} B_{ri} E_k B_{rj}^T - \varsigma_{rj} B_{rj} E_k^{-1} B_{ri}^T - \varsigma_{rj} B_{ri} E_k^{-1} B_{rj}^T + Y \\ & + \beta_{rij}^{-1} \delta_{ri}^2 P_e^{-1} C_{ri}^T C_{rj} C_{rj}^T C_{ri} P_e^{-1} + \xi_{rij}^{-1} Q_{ri} Q_{ri}^T \end{aligned}$$

The matrices X and Y can be obtained by solving the LMIs, by substituting P and P_e into

$$K_{ri} = \varepsilon_{ri} B_{ri}^T P, H_{ri} = \varsigma_{ri} B_{ri}^T P, L_{ri} = \delta_{ri} P_e^{-1} C_{ri}^T \quad (20)$$

we can easily obtain K_{ri} , H_{ri} and L_{ri} .

4. Simulation Study

In this section, an example is presented to show the effectiveness of the proposed technique.

Consider a stirred tank reactor, whose reaction form takes place from a to b (a is a reaction species and b is a product species). The real application requires switching between two inlet streams consisting of the species a at flow rate F_1, F_2 , concentrations C_{a1}, C_{a2} , and temperatures T_{a1}, T_{a2} , respectively. Therefore, the process can be modeled by

$$\begin{aligned} \dot{C}_a(t) &= \frac{F_{\sigma(t)}}{V} (C_{a\sigma(t)}(t) - C_a(t)) + \frac{3F_{\sigma(t)}}{V} C_a(t-d) - k e^{-E/RT(t)} C_a(t) \\ \dot{T}(t) &= \frac{F_{\sigma(t)}}{V} (T_{a\sigma(t)}(t) - T(t)) - \frac{\Delta H}{\rho c} k e^{-E/RT(t)} C_a(t) + \frac{o_{\sigma(t)}(t)}{\rho c V} \end{aligned}$$

where $C_a(t)$ denotes the concentration; $F_{\sigma(t)}$ denotes the flow rate a , $T(t)$ denotes the temperature.

Take $V = 0.1$, $k = 1.2 \times 10^{10} s^{-1}$, $E = 8.314 \times 10^4 kJ/mol$, $\Delta H = -4.78 \times 10^4 kJ/mol$, $c = 0.0239 kJ/kgK$, $\rho = 1000.0 kg/m^3$. In addition, $C_a - C_{a\sigma(t)}$ is $0.06 kmol/m^3$, when $C_a - C_{a\sigma(t)} < 0.006 kmol/m^3$, system 1 is activated, when $0.006 < C_a - C_{a\sigma(t)} kmol/m^3$, system 2 is activated. Then, the system described as follows:

Subsystem 1:

$$\begin{aligned} \dot{C}_a(t) &= -0.334 C_a(t) - 1.2 \times 10^{10} e^{-10000T(t)} C_a(t) \\ &\quad + 1.002 C_a(t-d) + 0.26386 \\ \dot{T}(t) &= -0.334 T(t) + 2.4 \times 10^{12} e^{-10000T(t)} C_a(t) + 117.7684 + \frac{o_{\sigma(t)}(t)}{2.39} \end{aligned}$$

where $F_1 = 0.0334 m^3/s$, $T_1 = 352.6 K$, $C_{a1} = 0.79 kmol/m^3$.

Subsystem 2:

$$\begin{aligned} \dot{C}_a(t) &= -0.167 C_a(t) - 1.2 \times 10^{10} e^{-10000T(t)} C_a(t) \\ &\quad + 0.501 C_a(t-d) + 0.167 \\ \dot{T}(t) &= -0.167 T(t) + 2.4 \times 10^{12} e^{-10000T(t)} C_a(t) + 51.77 + \frac{o_{\sigma(t)}(t)}{2.39} \end{aligned}$$

where $F_2 = 0.0167 m^3/s$, $T_2 = 310.0 K$, $C_{a2} = 1.0 kmol/m^3$.

The control objective is to stabilize the reactor at reactor at the unstable equilibrium point $(C_a, T_s)_1 = (0.57, 395.3)$ and $(C_a, T_s)_2 = (0.738, 509.12)$ using the rate of heat input $o_{\sigma(t)}(t)$ and change in inlet concentration of species a .

Define fuzzy system of state vector is

$$[x_1(t) \ x_2(t)]^T = [C_a(t) \ -C_{a\sigma(t)}(t)T(t) - T_{\sigma(t)}(t)]^T,$$

and the system of control input is $u_{\sigma(t)}(t) = o_{\sigma(t)}(t) - o_{s\sigma(t)}(t)$, where $\Delta C_{a\sigma(t)}(t) = C_{a\sigma(t)}(t) - C_{a\sigma(t)s}(t) \leq 1 kmol/m^3$, $|u_{\sigma(t)}(t)| \leq 1 kJ/h$, and $o_{s\sigma(t)}(t) = 0 kJ/h$.

We can obtain

$$\begin{aligned} L_{11} &= \begin{bmatrix} 551.7944 & 361.0006 \\ 43.2492 & 26.3885 \end{bmatrix}, \quad L_{12} = \begin{bmatrix} 656.2956 & 429.3685 \\ 51.4399 & 31.3861 \end{bmatrix} \\ L_{21} &= \begin{bmatrix} 441.6490 & 49.6059 \\ 34.6161 & 3.7307 \end{bmatrix}, \quad L_{22} = \begin{bmatrix} 686.3645 & 77.0923 \\ 53.7967 & 5.7979 \end{bmatrix} \end{aligned}$$

and

$$K_{11} = \begin{bmatrix} 0 & 0 \\ -0.1755 & 0.4518 \end{bmatrix}, K_{12} = \begin{bmatrix} 0 & 0 \\ -0.1755 & 0.4518 \end{bmatrix}$$

$$K_{21} = \begin{bmatrix} 0 & 0 \\ -0.1755 & 0.4518 \end{bmatrix}, K_{22} = \begin{bmatrix} 0 & 0 \\ -0.1755 & 0.4518 \end{bmatrix}$$

The initial condition is chosen as $[0.12 \ 0.15 \ 0 \ 0]^T$, $d = 0.4$. Then the simulation results are shown in Figs. 1-6, where Figs. 1 and 2 show the trajectories of $x_i (i=1,2)$ and their estimates $\hat{x}_i (i=1,2)$, Fig. 3 presents the trajectories of $\tilde{x}_i (i=1,2)$, Fig. 4 and 5 show the trajectories of $u_{i2} (i=1,2)$, Fig. 6 shows the trajectory of switching signal. From the simulation results, it is clear that the fuzzy feedback control law can guarantee the stability of the fuzzy switched system with time delay and actuator dead zone.

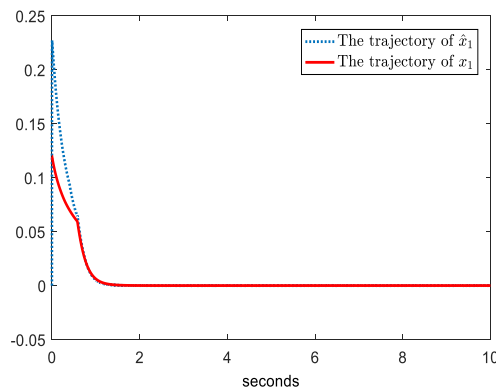


Fig. 1 The trajectories of x_1 (solid line) and \hat{x}_1 (dotted line)

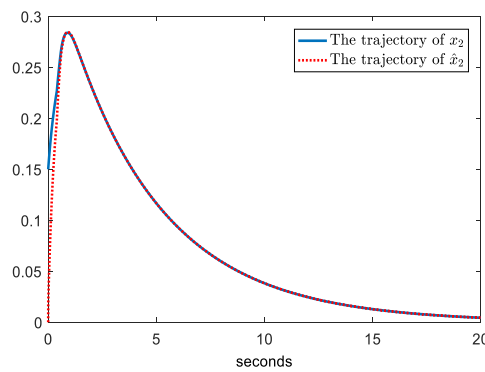


Fig. 2 The trajectories of x_2 (solid line) and \hat{x}_2 (dotted line)

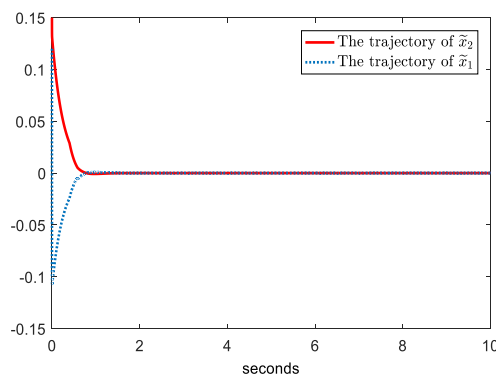


Fig. 3 The trajectories of \tilde{x}_2 (solid line) and \tilde{x}_1 (dotted line)

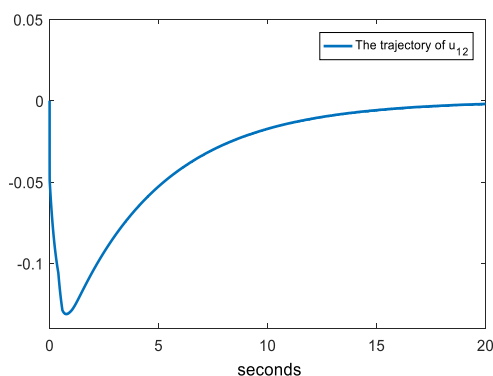


Fig. 4 The trajectory of u_{12}

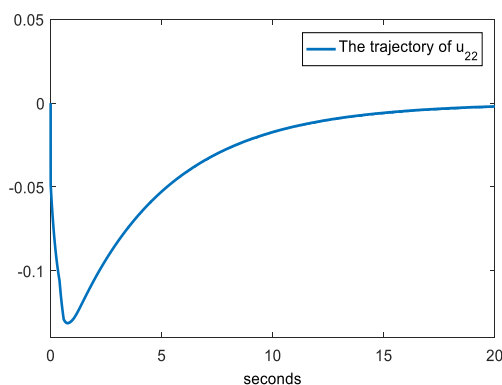


Fig. 5. The trajectory of u_{22}

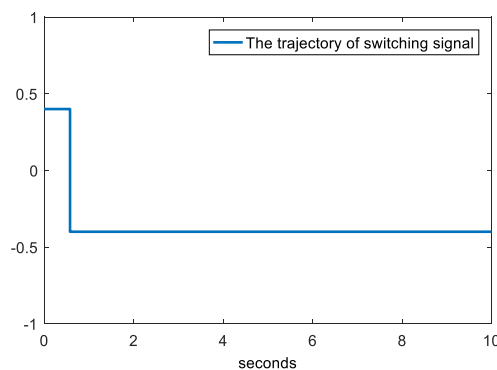


Fig. 6 The trajectory of switching signal

5. Conclusion

In this paper, the switched fuzzy control design problem has been investigated for a class of switched fuzzy systems with time delay and actuator dead zone. By using PDC design method, a non-fragile switched state observer is designed to obtain the estimations of the unmeasured states. And we have considered H_∞ performance, which improves anti-interference ability of the system. Moreover, the sufficient conditions for ensuring the asymptotic stability of switched fuzzy system have been derived and formulated in the form of LMIs. It has been proved that the proposed control approach can guarantee that the whole closed-loop system is asymptotically stable by using the single Lyapunov function method. A real application example was given to illustrate the effectiveness of the proposed control method. In fact, the main approaches utilized in this work can be extended to switched stochastic systems with time-varying delay, which could be our future work.

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