WEIGHING BY SUBSTITUTION

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ABSTRACT

This paper describes a plan for making substitution weighings, applicable either to equal-arm balances or compound-lever scales, that has been developed in connection with the standardization of large weights at the Bureau of Standards. It has been prepared to meet a demand for an explanation of substitution weighing which has come from practical scale men in the field who have seen the plan in use by the representatives of the Bureau and who desire to adopt it. The description, however, will also be of interest and value to many workers in engineering and other laboratories who have occasion to weigh large objects accurately. A record form and computation sheet is presented which it will be found advantageous to follow.

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1. INTRODUCTION

Substitution weighing is of great importance when it is desired to obtain very accurately the weight of some particular object and when calibrating or adjusting weights in the process of standardizing them. The purpose of the present paper is to set down for reference and application a procedure which can be employed for substitution weighing with great accuracy where either simple equal-arm balances or compound-lever scales are used. The method outlined gives a plan for using the apparatus in such a manner that the maximum precision in the results will be attained. The plan described is one that has been developed chiefly in con-

nection with the standardization of large weights at the Bureau of Standards on compound-lever scales. It is simple in its essential character and can be applied under almost any conditions where an accurate comparison of weights, large or small, has to be made. For convenience the description of the method has been developed in reference to compound-lever scales, but no difficulty should be experienced in applying it directly to equal-arm scales.

The virtue of the method lies in the fact that the multiplication ratio of the arms of the scale does not affect the accuracy obtained. The precision obtainable is determined by the constancy of the multiplication under given conditions of loading and by the sensibility of the device. It is especially efficient for the calibration or adjustment of a large number of weights of the same denomination.

This paper is prepared primarily for the practical scale man in charge of the construction and care of standards and their use in regular and precision work. The more progressive representatives of the workers in this field have shown a persistent desire to adopt for their use the best methods available, but the subject is not adequately covered in physics textbooks, or any other previous record. Secondly, it is expected that this paper will be of assistance to technical and laboratory workers who have occasion to use weighing apparatus for determining the weights of objects or to calibrate weights with special care.

The principles given herein apply in most cases with equal force to equal and unequal arm balances, such as are used in scale shops where it is the custom to make weighings by the method of substitution, though not with the precision here provided for. For instance, it is possible to take a scale of 10 000 pounds' capacity and by substitution accurately weigh and adjust 10 000-pound weights on the platform, and then using the same scale to standardize small weights from 1 to 10 pounds by weighing them on the counterpoise by the method of substitution.

In reference to the general value of the method of weighing by substitution it may be of interest to state that on a compound-lever scale of 10 000 pounds' capacity 10 000-pound weights were weighed and adjusted with an uncertainty of but 0.04 pound, including the errors in all of the steps. The error present in the scale was 3 or 4 pounds for a 10 000-pound load as weighed directly, but on account of the method used this did not enter into the results. It should also be understood that it is possible to use the substitution method with fine balances in precision weighing.

2. GENERAL SCHEME OF WEIGHING BY SUBSTITUTION

The general scheme of weighing by substitution is quite simple. For standardizing weights the standard first is placed on the scale pan or platform designed to receive it and counterpoised by any

FIG. 1.—*Photograph of scalebeam with graduated millimeter scale mounted upon it and pointer attached to trig loop as for weighing by substitution*

convenient material. When the balance of the beam is obtained, the standard is removed, the weight to be compared is substituted, and the correction is determined from the small weights that have to be added or subtracted to establish the same balance as before.

Where the weight of an object is desired and its weight is not already known approximately, it should be first placed upon the platform and balanced by a suitable counterpoise. It should then be removed and standard weights substituted until the same balance is established as before. The weight of the object will then be equal to the weights substituted for it.

The method of substitution weighing is rendered precise by observing certain details which are explained in the following portions of the paper, where they will be taken up in the order in which they present themselves for consideration in using this plan of weighing.

3. PREPARATION OF THE SCALE FOR WEIGHING

The scale should rest level on a solid support. The parts should be examined and aligned, all binds should be removed, and all anti-friction contacts should be cleared. Provisions should be made for applying and removing the loads without disturbance, so that the relative positions of the scale parts will not be altered, and the loads should be carried on the platform or pan in such a manner that their weight will always have the same distribution among the supporting main load knife-edges.

4. PROVISION FOR READING THE POSITION OF THE BEAM

A means must be supplied for reading the angular position of the beam. This is already provided for in the construction of the common even-arm laboratory balance by a pointer attached to the beam and arranged to read on a ruled scale. In many laboratories the image of a stationary ruled scale on a distant wall or support is reflected into an observing telescope by means of a mirror mounted on the beam. This is a very simple and effective method where the telescope and scale are available, and as this arrangement is perfectly practical for the shop the scale man should familiarize himself with it. For the ordinary case in substitution weighing on compound lever scales it is recommended as the simplest practical scheme to provide for reading the beam by mounting a graduated scale on the beam and a pointer on the trig stand, so that the angular position of the beam can be read in divisions on the graduated scale. Such an arrangement is shown in Fig. 1. Where a reverse arrangement is employed and the pointer is mounted on the beam and the reading scale is mounted on the trig loop, it is necessary to follow the motion of the beam by moving the eye up and down. This method is more trying to the observer and less

accurate on account of variations in illumination. Experience shows that it is distinctly better to use the moving scale and stationary pointer.

The reading scale and pointer should be so disposed that when the lever is balanced in a horizontal position the reading will be some simple value like 10 or 20, and the stops limiting the angular movement should be such that the play is equal on either side of this horizontal position.

5. CHOICE OF THE RULED SCALE

For making the readings a decimally graduated scale should be used. In this scale the length of the lines will have an important influence on the ease and accuracy with which they can be read. The particular form recommended is where the length of the ordinary graduations is about one and one-half times the distance between their centers, every fifth line about two times this distance, and every tenth line about three times this distance. With relatively longer lines for the graduations it is found that the eye tends to become confused and may not recognize the scale division until the time for making the reading has passed. An example of the scale recommended is shown in Fig. 2.

It has been found that a suitably graduated millimeter scale serves excellently this purpose. With proper lighting it is found that the readings can be estimated to 0.1 millimeter comfortably and consistently. The paper millimeter scales such as are provided at the Bureau of Standards are convenient; short sections can be cut out and attached to the beam. The weight is small, and the effect on the period and inertia of the beam of large scales is entirely negligible.

6. READINGS SHOULD INCREASE WITH WEIGHT

The values or markings selected for the graduation should be such that an increase in the weight being standardized will correspond to an increase in the reading. If the substitution weighing is being made for weights placed on the platform, then the scale

FIG.2.—*Proportion of scale graduations suitable for mounting on the beam for reading its position in weighing by substitution*

should be arranged to give increased readings as the beam moves up. If the substitution weighing is being carried out on the counterpoise, then the reading should be arranged to increase as the beam descends.

7. AVOIDANCE OF PARALLAX

If the pointer is well in front of the ruled scale, the exact reading will depend on the position of the eye, as indicated in Fig. 3. This effect is called parallax. The design and position of the pointer should be such that this is reduced to a negligible amount. This can

FIG. 3.—Diagram illustrating parallax

The pointer is well in front of the scale, and the reading depends upon the particular line of sight fixed by the eye. The proper reading is 12.0, as observed from position 1, where the line of sight is perpendicular to the scale. In position 2 the eye is too high and the pointer appears to read 9.0

be done by placing the pointer near to the scale; by having the pointer of such a design that it will help the observer to place his eye squarely in front of it; by the use of a mirror back of the pointer which will enable the eye to be placed properly (by moving the eye and lining up the pointer or shaft with its reflection); or by other methods which local circumstances will often suggest.

8. THE ESTIMATION OF TENTHS

The position of the beam is read as so many divisions and so many decimal fractions of a division. The fractions are estimated to the nearest tenth of a division by observation. This can be done by anyone with a little practice. In estimating tenths, however, it is important

that a decision be made at once and without hesitancy. It may be better that an error of a tenth or even two-tenths be made than to miss an observation and lose time in taking the observation over again. It is also found in practice that the most accurate and consistent estimations are made when it is done promptly, and that too much care, when it results in indecision and delay, actually causes a decrease in accuracy.

9. PERIOD OF VIBRATION OF THE BEAM

With the use of the pointer and scale the position of the beam can be read to a high degree of relative refinement. In fact, the sensibility is often such that the action of the beam can be quickened and time can be saved by lowering or even in some cases removing the balance ball, and yet it will be possible to read to the

required accuracy. The beam, however, should not be caused to act too quickly. The observer should be allowed plenty of time to record the observations. If the readings are being made by one person and recorded by another, it is possible to have a much faster beam. However, there may be no particular gain in this, as it will be necessary to stop and make computations before proceeding with the test. A period of 15 seconds is recommended. By period is meant the time required for the beam to swing to and fro through a complete cycle.

10. THE REST POINT EXPLAINED

In rough forms of scales the position of balance is obtained by allowing the beam to come to rest. In the present instance, however, where precision is desired, and except in special forms of balances which it is not the purpose to discuss here, it is not practicable to determine the position of equilibrium by allowing the beam to come to rest. It is, however, possible to determine from the moving pointer where the beam would come to rest, and this point or position is known as the *rest point*.

If there is but very little friction present either in the knife-edges and connections or in the damping action of the air, the pointer will vibrate practically an equal distance on either side of the rest point. If the means for reading are sensitive enough, however, it will be found that the magnitude of the excursions of the pointer will gradually die down. The method for determining the rest point described in this paper takes into account the effect of this damping.

11. DETERMINATION OF THE REST POINT

The rest point is found from the moving beam by reading the extreme positions of the pointer as it swings and combining the readings in a certain simple manner. Thus, in Table I, we have a series of readings taken in order:

TABLE 1

Observation number	Readings of the pointer		Observation number	Readings of the pointer	
	Low position	High position		Low position	High position
1.....	2.0		4.....		27.0
2.....		29.0	5.....	6.0	
3.....	4.0		6.....		25.0

It will be noted that each downward swing is two divisions less in amplitude than the one before, as is indicated by an increase in

the successive figures, and that the successive upward readings also diminish by two divisions. This is on account of damping. The observations are used in such a manner as to eliminate the effects of this damping. For obtaining the rest point the following simple rules are given:

1. Use the observations in the order taken.
2. The first and last observations used in the computation should be at the same end of the swing—that is, in the same column.
3. Half the sum of the averages of both columns will be the rest point.

For example, we can use the observations just given in a number of different ways for obtaining the rest point. Using observations 1, 2, and 3 we have:

Low	High
2.0	
4.0	29.0
—	—
Average..... 3.0	

One-half the sum of 3.0 and 29.0 is 16.0, which is the rest point. Again, using observations 1, 2, 3, 4, and 5, we have:

Low	High
2.0	29.0
4.0	27.0
6.0	
—	—
Average..... 4.0	28.0

and $4.0 + 28.0 \div 2 = 16.0 = \text{rest point}$.

Or we could take observations 2, 3, 4, 5, and 6, in which case there results:

Low	High
...	29.0
4.0	27.0
6.0	25.0
—	—
Average..... 5.0	27.0

and $5.0 + 27.0 \div 2 = 16.0 = \text{rest point}$.

In the example just considered the damping of successive movements in each case was the same. In many balances, however, the amount of this damping falls off as the amplitude dies down, with the result that there is a small theoretical error in the determination of the rest point even when several observations are employed.

For most purposes the rest point can be determined with sufficient accuracy by using three consecutive observations—one on one

side and two on the other. The rule should be to start recording observations and continue until it is clear that the successive swings of the beam are uniform and consistent in character. Generally it is necessary to take only four readings. In this case the first reading serves as a check on the third, but it is not used in the computations, which fact is indicated by the use of brackets [], or, in practice, by striking a line through the figures.

12. USE OF EVEN VALUES IN ROUNDING OFF FIGURES

There is very frequent occasion to round off figures. For instance, in the following illustration the values are not recorded or computed, as a rule, to a greater refinement than 0.1 division. In obtaining the averages, however, it is a very common thing to get a value in the next place. The result is then usually rounded off to the nearest 0.1, but frequently the figure 5 appears in the decimal place to be dropped. Thus, in the series of readings:

Low	High
(1)....[16.3]	(2)....22.6
(3).... 16.5	(4)....22.5

The mean of 22.6 and 22.5 is 22.55. In rounding off, the value derived would be either 22.5 or 22.6. *The rule*, in this case where the last figure is 5 and where it is therefore equally accurate to round off to either the higher or lower figures, *is to round off to the nearest even number*, so the number selected would be 22.6. This would then be averaged with 16.5, as $(22.6 + 16.5) \div 2 = 19.55$, which, applying again the rule just given, would be recorded and used as 19.6.

The merit of this rule is that it is easy to remember, simple to apply and follow, it results just as often in increasing the value as in decreasing it, and it is possible by using this rule for different individuals working separately to take the same data and work it out in the same manner and arrive at the same conclusion. The odd values could be used instead of the even values, but custom has ordered otherwise. For cases where more than two numbers are averaged other decimal places may appear, but no difficulty should be encountered in determining what should be done in rounding off.

13. UNNECESSARY TO DIVIDE BY 2

In what has been given in the preceding discussion the rest point is obtained by dividing the sum of the average readings of the pointer on either side by 2. In practice it is not necessary to do this, but the undivided sum of the average readings of each column is used. This saves time by reducing the computations by

one step. In the illustration just given above, the sum $22.6 + 16.5 = 39.1$ would be taken instead of half the value, or 19.6. The number employed is therefore twice the actual rest point or, as it may be referred to, it is the number of half divisions of the actual rest-point reading. This introduces no error, as the subsequent computations are carried out consistently with regard to this practice.

14. THE SENSIBILITY RECIPROCAL

The sensibility reciprocal, or the *sr*, as it will be written, is defined as the change in weight required on the load-receiving part to change the reading of the rest point one division on the graduated scale. On account of not dividing by 2, the value actually employed in the computations is not the *sr*, but one-half the *sr*, or it is the weight required to change the actual rest point one-half of a division. The *sr* is obtained by changing the small weights, so that the rest point will be altered several divisions. Thus, in Table 2, there is obtained:

TABLE 2

Readings		Value taken for rest point	Load
Low	High		
[16.2]	24.7		10-pound standard in pan
16.2	24.6		
16.2	24.6	40.8	
[21.9]	28.4		10-pound standard in pan + 10.0 grains
22.0	28.2		
22.0	28.3	50.3	

$$50.3 - 40.8 = 9.5 \text{ half divisions} = 10 \text{ grains.}$$

$$1/2 \text{ } sr = \frac{10}{9.5} = 1.05 \text{ grains.}$$

For a given condition of loading and for a given load the value of the *sr* should remain substantially constant. It is recommended that the *sr* be taken for each load, and that in the computations each load be figured with the corresponding *sr*, and that average values be not used. The advantage of taking the *sr* each time is that if there is an error in reading the position of the pointer this fact will be disclosed by the obtaining of a value for the *sr* which is inconsistent with the values obtained at other times.

The *sr* should remain constant throughout the angle of the play of the beam. If this is not the case, there is some interference or imperfection which should be eliminated before the balance is used in actual weighing.

15. POSITION OF CENTER BALANCE DETERMINED BY COMPUTATION

In the method described here it is always determined what small weights would be required on the load-receiving part, platform, or counterpoise, in addition to the standard weight or object, to cause the pointer to reach a mid-balance position or to have a rest point in the middle reading of the scale. This could be accomplished by adding or removing small weights until this was established by trial; but in the plan given here time is saved and the action of the scale is made a matter of record by observing the rest point for two values of the small weights and then *computing* the amount of small weights required to bring the rest point to the mid-scale position.

It has been previously mentioned that under proper conditions the *sr* for a given load will be constant throughout the play of the beam. This is equivalent to assuming that the change in the rest point is proportional to the change in the small weights of the load. It should be easily understood that when this condition obtains it is a simple matter to compute the change in weights required to establish the center balance when the rest point for a given load is known together with the *sr*, or in this case one-half the *sr*. As the details of this are covered in the description of the record form and computation sheet, space will not be taken to illustrate the matter here.

16. RECORD FORM AND COMPUTATION SHEET

For recording data and making the computations the accompanying form, Table 3, can be used to advantage. This has been developed as the result of a careful study of the matter and from the desire to secure a record form and computation sheet uniform with that used in making precision tests on railroad master scales and grain scales. In securing this uniformity a few extra computations are needed compared with the method which has been used in the laboratories of the Bureau of Standards. This form was evolved, however, as a result of a belief very emphatically expressed by several practical railroad scale men that the form should be made as nearly uniform with the other one mentioned as possible, a point of view to which the writer has been converted after first holding a contrary belief, especially since the use of this form in the field by him recently has demonstrated its convenience.

TABLE 3.—Form for Recording Observations and Making Computations in Substitution Weighing

[This form has been arranged to facilitate its understanding and use by the ordinary scale man by making it as similar as possible to that used for the precision test of railroad master and grain scales.]

Place..... Date..... Observer..... Test No..... Page.....
 Identification of weights tested..... Standard wts..... Car No.....
 Balance used..... Temperature..... Computed by..... Checked by.....

Observation number	Time	Beam readings			20—sum	$\frac{1}{2}$ sr	Balance correction = 6×7	Load			Correction to standard weights	Small weights for center balance		Correction for unknown = 12—13	Computation for $\frac{1}{2}$ sr and notes
		Low	High	Sum = rest point $\times 2$				Identification	Additional small weights	Stand-ard, 8+ 10+11		Un-known, 8+10			
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	
371	4:47	{ [5.9] 6.0	{ 8.9 8.8	{ 14.8	+ 5.2	0.633	+ 3.29	10-pound standard	10 grains	+ 2.10 grains	15.39				
372		{ [12.1] 12.3	{ 18.3 18.3	{ 30.6	- 10.6			do.	{ do. + 10 grains					$\frac{10}{\frac{1}{2} \text{ sr} = \frac{15.8}{15.8} = 0.633}$	
373		{ [7.4] 7.4	{ 10.2 10.0	{ 17.5	+ 2.5	.625	+ 1.56	10-pound No. 4	10 grains			11.56	+ 3.83 grains		
374	4:57	{ [12.2] 12.3	{ 21.2 21.1	{ 33.5	- 13.5			do.	{ do. + 10 grains					$\frac{10}{\frac{1}{2} \text{ sr} = \frac{16.0}{16.0} = 0.625}$	

After the scale mechanism and all other details are ready for the test the standard weight should be placed upon the load-receiving platform or counterpoise, as the case may be, together with a small weight or two sufficient in amount to cover any probable variation between the standard and the other weights to be compared with it. The scale should then be balanced approximately by applying suitable material, and this counterpoising material should then remain undisturbed during the remainder of the test. The balance of the scale before the standard load is applied is of no importance in the method of test described here.

The record form and computation sheet will now be explained: The first column is for the computation number. Each series of weighings or swings taken together should be given a computation number. Numbers are found to be very convenient for reference and for identifying various data obtained. The second column is provided for recording time. It is not necessary that this be filled in for each observation. Columns 3 and 4 are used for recording the positions of the pointer at the extreme positions of its swing. The position for the lowest numerical readings are placed in 3, marked "low," and the positions of highest readings are recorded in 4, marked "high." The sum of the average low and high readings is placed in column 5. The resulting number is twice the reading for the rest point. In column 6 is placed the result of subtracting the value in column 5 from the number 20. Where the value in column 5 exceeds 20, the difference in column 6 should, of course, be given a negative sign. This assumes that the direct reading of the rest point is 10 in its mid-position, or that twice the reading of the pointer in this position, as would appear in column 5, is 20. If this is not the case, the number corresponding to twice the actual reading for the mid-position of the pointer on the reading scale should be used in place of 20.

The value in this column 6 represents the amount by which the rest point, as recorded in column 5, will have to be changed to bring the beam to balance in its mid-position, where the rest point will appear as 20. The value of one-half *sr* is recorded in column 7. The determination of this, of course, must await until two series of observations are taken corresponding to a known difference of weight on the load, pan, or platform. The computation for this one-half *sr* is usually made in column 15 at the extreme right. The theoretical weight change required to be added

to the platform to produce mid-balance position of the pointer is given in column 8. This is equal to the product of columns 6 and 7. It is the weight that should be added to or subtracted from the pan according to whether the sign is plus or minus, to change the rest point from the value observed, to a center position on the scale.

The character and identification of the weights are given in column 9. The small weights which are placed in the pan, in addition to the standard weight or weight being tested, are entered in column 10. In column 11 are recorded the corrections to the standard weights, which, of course, are known.

In column 12 is recorded the amount of small weights in addition to a perfect standard weight which would be required to bring the rest point to its mid-position. It relates to the standard weight only. It is equal to the sum of the quantities in columns 8, 10, and 11, with due regard for sign. This weight required may, of course, be negative. Column 13 is used only for data for the unknown weight. It represents the sum of the values given in columns 8 and 10 for the unknown weight. It represents the small weights required on the pan, in addition to the unknown weight, to bring the position of equilibrium of the pointer to the mid-position of the scale.

Column 14 is for the correction of the previously "unknown" weight. The value in column 13 subtracted from the value of the standard weight in column 12 is the correction for the weight which is being standardized.

On the form are given the results of a test made to standardize a 10-pound weight, here designated as 10-pound weight No. 4. With the standard and 10 grains placed on the pan observation 371 was made.

The time is recorded in column 2 and the readings of the pointer are entered, in accordance with the principles previously given, in columns 3 and 4. Four observations are taken, and it appears that the change in the successive readings for the lower observations is consistent with the difference in the two readings for the high positions. This indicates that no mistake has been made, and that there is no excessive or irregular damping. The rest point is obtained by adding the value of 6.0 in column 3 to the average of the two readings in column 4. The first reading of 5.9, indicated in brackets, is not used in the computation. The result

is 14.8, which is put in column 5. Subtracting 14.8 from 20.0 the value of +5.2 results, which is placed in column 6.

Meanwhile, in column 9 has been entered the identification of the weight, and in column 10 the additional small weights used with it—in this case 10 grains—are recorded. The correction to the 10-pound standard weight is +2.10 grains, which is placed in column 11.

Following the first observation 10 grains are added to the weight pan, as indicated in column 10 of observation 372, and the rest point is determined, as before. It appears that the rest point is changed from the previous value of 14.8 to 30.6, or 10 grains has caused it to change 15.8 half-divisions. The value of one-half *sr* is then computed, as indicated in column 15, and the value obtained for it of 0.633 is entered in column 7. The product of the values in columns 6 and 7 is placed in column 8. In this case it is $+5.2 \times 0.633 = +3.29$. This signifies that 3.29 grains added to the pan for the conditions of observation 371 should bring the rest point to the center of the scale, so that it would appear in column 5 as 20.0.

In column 12 is placed the sum of the values in columns 8, 10, and 11, which is 15.39. It represents the weights in addition to a perfect 10-pound standard on the pan which would be necessary to bring the rest point $\times 2$ to the value 20.

The unknown weight is then substituted and observations are made as before, which are recorded in observations 373 and 374. By this means it is determined that the unknown weight would require in addition to itself 11.56 grains on the pan to establish for the double rest point. This is obtained by taking the sum of the values in columns 8 and 10 and entering it in column 13.

The unknown weight is heavy. In the previous observation it was determined that 15.39 grains would be required on the pan for center balance in addition to a perfect 10-pound standard. The unknown weight only requires 11.56 grains to establish this condition. It is therefore heavy, or has a plus correction. The value of this correction, +3.83 grains, is recorded in column 14 and is obtained by subtracting the value in column 13 for observation 373 from that given in column 12 for observation 371.

In observation 373 a weighing was carried out similarly to that previously done in observation 371. It would be possible at this point to determine the value for the unknown weight without

requiring additional data to be taken. However, in compliance with the practice recommended here, that the *sr* be obtained each time, an additional observation is made, observation 374, by which the *sr* is again determined. This reduces very largely the chance for an observational error remaining undetected. Here 10 grains produce a change in the rest point of 16.0 divisions, where in the previous case it was 15.8 divisions. The difference is but 0.2 division, and the agreement is quite satisfactory, which indicates both that the apparatus appears to work properly and that no mistake has been made in the readings.

This same form and the same method of procedure is adaptable to practically all needs for substitution weighing that will ordinarily arise. The modifications or the special considerations required in weighing with other forms of apparatus are of such a simple character and of so obvious a nature as to make it unnecessary to refer further to them here.

17. SUMMARY

In conclusion it may be stated that the plan of substitution weighing presented here not only can be used with laboratory precision apparatus, but, what is of more importance here, it offers a means for obtaining weighings of considerable refinement with ordinary forms of compound-lever scales that are often available. Large weights or objects can be weighed, or fine laboratory weights can be tested and calibrated by following the outline of substitution weighing given herein. The form will be found simple to follow and use, particularly by those who are familiar with the form used by the Bureau of Standards in the test of railroad master and grain-hopper scales.

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