A PIEZO-ELECTRIC METHOD FOR THE INSTANTANEOUS MEASUREMENT OF HIGH PRESSURES.

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ABSTRACT.

THE PRESSURE GAGE.—A number of quartz plates suitably cut are arranged condenser fashion in a stack and introduced in an electric circuit. The quartz plates are subjected to the gas pressure to be measured by means of a piston. The combination of these constitutes a gage which is screwed into the wall of the gas chamber and the leads are brought out through holes drilled through the wall.

THE RECORDING GALVANOMETER.—A ballistic galvanometer whose period is long compared to the duration of the pressure phenomenon is connected to the electrodes of the quartz plates. The galvanometer deflections are photographically recorded on a rapidly moving film. The pressure-time curve is the differential of the recorded deflection-time curve.

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1. INTRODUCTION.

The piezo-electric property of tourmaline has been used by J. J. Thomson¹ and D. A. Keys² for obtaining indicator diagrams of explosions and for recording the variation of pressure with time over small-time intervals. The proposal was made independently by Maj. G. F. Hull and Capt. H. O. Wood to utilize the piezo-electric property of quartz for obtaining indicator diagrams of gun discharges. In the following there are developed the principles of a method for reducing this proposal to practice, together with a description of appropriate experimental arrangements.

A piezo-electric pressure gage recommends itself for the recording of rapid pressure changes because of the absence of inertia effects. Unlike spring gages, the recorded effect depends only in a secondary manner upon displacement and the displacements involved are solely due to the elastic yielding of the materials.

¹ J. J. Thomson, *Engineering*, 107, 1919, pp. 543-544.

² D. A. Keys, A piezo-electric method of measuring explosion pressures, *Phil. Mag.*, October, 1921, pp. 473-488.

Sharp peaks in the pressure curves are, therefore, less likely to be rounded off and the difficulties arising from friction and inertia are more easily avoided.

2. PRINCIPLES OF THE METHOD.

A thin plate with its faces plane and parallel, cut from a quartz crystal so that the plane of the plate is perpendicular to an electric axis of a crystal, when subjected to a compressional force in the direction of the electric axis and normal to the faces, will liberate a quantity of electricity at these faces the magnitude of which is proportional to the force applied. The amount of charge liberated from a perfect crystal is given by P. Curie as 6.32×10^{-8} E. S. U. per dyne or approximately 10^{-11} coulombs per pound for small loads.

The published literature does not reveal any data on the relationship between charge and applied force for the large forces to be encountered in this work. Quartz plates were, therefore, subjected to pressures up to 50,000 pounds per square inch in a testing machine, and the relationship between charge and pressure was found to be linear over the full range of pressures used. The limit of error in these measurements was approximately 0.1 per cent. The manner of making them will be described in a later portion of this paper.

A gage may be constructed by using a stack of quartz plates with thin metal electrodes between adjacent plates. In this way more charge is obtained because of the greater plate area. The manner of construction is similar to that of a mica condenser with metal foil electrodes, except that the quartz plates possess polarity and must be so assembled that the faces of the two plates in contact with the same electrode develop charges of the same sign when force is applied. Moreover, quartz, unlike tourmaline, yields no charge when exposed uniformly to hydrostatic pressure so that the stack of plates must be inserted in a gas-tight housing and the pressures must be applied in the direction of the axis of the stack by means of a piston plunger, one end of which is in contact with the stack while the other is exposed to the gas pressures to be measured.

The charge Q developed on the electrodes of such a gage by a pressure P is given by the relation $Q = cP$ where c is a constant determined by the number and area of the quartz plates, the area of the piston, and some other features of construction. The constant c is readily evaluated experimentally. The validity of this

equation for even the most rapidly changing pressures to be encountered by the gage is assumed. Sufficient experimental evidence is already available to establish the correctness of this assumption to a high degree of approximation.

If the terminal leads of the gage are connected to a galvanometer through a suitable line, there will be a current.

$$i = \frac{dQ}{dt} = c \frac{dP}{dt} \quad (1)$$

This current in flowing through the galvanometer will subject the moving system to a torque, thereby causing an angular displacement given by the general relation

$$Gi = K \frac{d^2\theta}{dt^2} + D \frac{d\theta}{dt} + U\theta \quad (2)$$

where G is the dynamic constant of the moving system, K its moment of inertia, D the damping constant, and U the restoring constant of the galvanometer.

If the galvanometer used has a long period and small damping, that is, if U and D are small, it follows from equations (1) and (2) that

$$Gi = Gc \frac{dP}{dt} = K \frac{d^2\theta}{dt^2} \quad (3)$$

Integrating equation (3)

$$GcP = K \frac{d\theta}{dt} + c' \quad (4)$$

But $\frac{d\theta}{dt} = 0$ when $P = 0$. Hence, $c' = 0$ and

$$P = \frac{K}{Gc} \frac{d\theta}{dt} \quad (5)$$

Equation (5) shows that under the conditions assumed the angular velocity of the galvanometer mirror is proportional to the pressure. The assumed conditions are realized sufficiently well in a high-sensitivity, long-period galvanometer. A photographic record of the galvanometer deflections obtained on a rapidly moving film during the period of pressure change and a simultaneous record of a suitable time scale gives

$$\theta = f(t)$$

The differential of this curve multiplied by a suitable constant is the pressure-time curve.

The experimental arrangement, when the gage is to be used to obtain the indicator diagram of a gun discharge, is schematically illustrated by Figure 1. The quartz gage is set into the breech-block and extends slightly into the powder chamber. Two insulated leads are brought out and connected to an open overhead line which runs to the photographic laboratory which may be at some distance from the gun, or one terminal of the gage may be grounded on the gun and a single line to the laboratory used. Open wiring is used in order to keep the capacity at a minimum. The galvanometer is placed on a bracket attached to the brick wall of the photographic laboratory. A single-filament tungsten lamp

FIG. 1.—Assembly of apparatus.

is used as a source of light, and by means of a suitable lens system the image of the lamp reflected by the galvanometer mirror is focused on a moving film 4 inches wide placed at a meter's distance from the galvanometer. Time lines on the film $1/1000$ second apart in time and about 1 cm apart in distance are photographed simultaneously with the photographing of the galvanometer deflection.

The time lines are obtained in the following manner:

A Duddell oscillograph element is placed at a distance of about 60 cm from the camera lens and with the axis of vibration of the moving system horizontal. A parallel beam of light from a small arc lamp is thrown on the mirror of the oscillograph and is reflected into the cylindrical lens of the camera. This lens forms a fine-line image on the film. The oscillograph is tuned to a frequency of

500 cycles and is actuated by current supplied from the circuit of a 500-cycle vacuum tube driven tuning fork.³ When the oscillograph mirror is vibrating the reflected beam of light passes across the camera lens once every 1/1000 second. If the film has a speed of 10 m per second these lines will be 1 cm apart. The duration of a gun pressure record will be, roughly, from 0.01 to 0.05 second. It will, therefore, be from 10 to 50 cm long on the film.

One form of gage construction designed for use in the powder chamber of a gun involves the use of 6 quartz plates arranged condenser fashion. The gas pressure end and the end pressing on the plates are $\frac{5}{8}$ inch and $\frac{3}{4}$ inch in diameter, respectively. The electrodes consist of pieces of platinum foil 0.005 inch thick placed between adjacent plates. The leads were connected to a high sensitivity galvanometer which has a period of 12 seconds, a direct current resistance of 520 ohms, and gives a deflection of 1 mm at a meter's distance for a current of 10^{-10} amperes.

In order to prevent the galvanometer from being disturbed by steady current leaks along the line or by thermal e. m. f.'s., a 4-mf. condenser was connected in series with the circuit at one of the galvanometer terminals.

In order to show more clearly the sensitivity of the system, and the relation of the several factors which determine the performance of the apparatus, some quantitative data are here given.

Consider a galvanometer which has a period of 12.5 seconds and gives a deflection of 1 mm for a current of 10^{-10} amperes.

The expression for the period of the galvanometer in terms of K and U is

$$T = 2\pi\sqrt{\frac{K}{U}} = 12.5 \text{ sec.} \quad (6)$$

$$K = \frac{12.5^2}{4\pi^2} U = 4U \text{ (approximately)} \quad (7)$$

The continuous current sensitivity is given by the relation

$$Gi = U\theta \quad (8)$$

But $i = 2.10^{-7}$ amp. for $\theta = 1$ therefore

$$G = \frac{U}{2.10^{-7}} \quad (9)$$

³ E. A. Eckhardt, J. C. Karcher, and M. Keiser, Electron Tube Drive for Tuning Forks, *Phys. Rev.* 2d ser., 17, 1921, pp. 535, 563.

Next consider a quartz plate which has an area of 1 square inch and let this be subjected by means of a piston of equal area to the pressure in the gun.

Then

$$Q = CP \quad (10)$$

where $C = 10^{-11}$ coulombs per pound. Using this value and substituting the values obtained for K in equation 7 and for G in equation 9 for K and G , respectively, in equation 5, we have

$$P = 8.10^4 \frac{d\theta}{dt} \quad (11)$$

If the galvanometer deflections are photographed on a film placed at a meter's distance and moved at the rate of 100 cm per second, let the unit of deflection x be 1 mm and let the unit of time t' be $1/1000$ second = 1 mm on the record, then

$$x = 2000 \theta \quad (12)$$

and

$$\frac{d\theta}{dt} = \frac{1}{2} \frac{dx}{dt'}$$

Substituting this value in (1)

$$P_t = 40,000 \frac{dx}{dt'} \text{ pounds per square inch or} \quad (13)$$

$$P_t = 40,000 \tan \phi \text{ pounds per square inch} \quad (14)$$

where $\tan \phi$ is the slope of the time-deflection curve at the time t . Since the record is 10 cm wide, and from 10 to 50 cm long, it is feasible to determine the slopes graphically with a precision greater than 4 per cent for the maximum pressure. By the use of a comparator even greater precision is obtainable. When a comparator is used it is well to run the film at the rate of about 1,000 cm per second and the deflections may be measured by means of the comparator at $1/1000$ second intervals or less. By using these measured values in conjunction with a suitable analytical method of differentiation the desired curve is obtained.

3. CALIBRATION.

If a given load on the piezo gage is suddenly released, a charge is developed by the gage which is proportional to the load released. The ballistic throw of the galvanometer is a measure of this total

charge, if the load release is effected quickly enough. This forms the basis of the calibration which is performed readily in the following manner: The gage is placed between the blocks of a testing machine, preferably a sufficiently large one, so that high-speed screw feeds may be used. The galvanometer is connected across the gage terminals and provided with a short-circuiting key to be used while the load is being applied. A known load is applied to the gage and the screw feed gears are shifted so that a high-speed screw feed may be used to release the load. The short circuit is removed and the load is suddenly released. The galvanometer deflection is noted. Let this observed deflection $= x'$. If the logarithmic decrement λ is given as the logarithm of the decrement observed in successive swings of the galvanometer, then the undamped deflection

$$= x_o = \left(1 + \frac{\lambda}{2}\right) x' \quad (15)$$

The equation of motion of the undamped galvanometer is

$$x = x_o \sin \frac{2\pi}{T} t \quad (16)$$

$$\frac{dx}{dt} = \frac{2\pi}{T} x_o \cos \frac{2\pi}{T} t \quad (17)$$

If the galvanometer is given a ballistic throw by a sudden release of pressure from the quartz plate

$$P = C'' \frac{dx}{dt} (t \doteq 0) \quad (18)$$

But from (17)

$$\frac{dx}{dt} (t \doteq 0) = \frac{2\pi}{T} x_o \quad (19)$$

Therefore,

$$P = C'' \frac{2\pi}{T} x_o \quad (20)$$

and

$$C'' = \frac{PT}{2\pi x_o} = \frac{PT}{2\pi \left(1 + \frac{\lambda}{2}\right) x'} \quad (21)$$

C'' is the desired calibration constant. By repeating this calibration for a series of loads up to 50,000 pounds per square inch the linearity of the pressure-charge relation was established for the full pressure range.

Also, the gage may be calibrated as used, that is, a photographic record may be made when a known load is suddenly removed from the gage, and the calibration determined from the slope of the photographed curve in a manner heretofore described.

Because of the constancy of performance of both the quartz gage and the galvanometer the arrangement will retain its calibration indefinitely, and since the performance of the galvanometer is dependent only on the moment of inertia and electrodynamic constants of the coil the replacement of a broken suspension does not alter the calibration.

4. RESULTS.

Actual pressure measurements in guns have been made with several gages of the type described. A comparison of the time-pressure records obtained, with other data obtained at the same time, indicates that the performance of the gage is in entire accord with the theory here developed. The results are at present not available for publication.

In passing, it may be noted that the method of calibrating the gage indicates the feasibility of its use in the calibration of other testing machines.

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