MAGNETIC RELUCTIVITY RELATIONSHIP AS RELATED TO CERTAIN STRUCTURES OF A EUTECTOID-CARBON STEEL

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I. INTRODUCTION

It is to the close relationship existing between the heat treatment and the magnetic properties¹ of iron and steel as well as between their heat treatment and mechanical properties, that magnetic analysis owes its promise of increasing metallurgical and industrial importance. Moreover, the magnetic and many other physical properties of iron and steel are greatly affected by the thermal and mechanical treatments to which they are subjected during the process of manufacture. Changes in the physical properties resulting from thermal treatment are generally considered as indicative of certain transformations and changes in structure. Such changes² have been discussed for the case of a 1 per cent carbon steel and it has been shown that a very rapid

¹ Maurer, *Rev. de Metallurgie*, **6**, p. 721; 1908. Burrows and Fahy, *A. S. T. M. Proc.*, **19**, p. 5; 1919.

² C. Nusbaum, *A. S. T. M. Proc.*, **19**, p. 100; 1919.

rise of the maximum and residual induction and an even more pronounced decrease in the coercive force occur between the drawing temperatures of 150 and 250° C. Such changes very likely mark the transformation from martensite to troostite.

While in all magnetic measurements at ordinary magnetizing forces the properties of a given specimen are always determined in the aggregate, measurements made in intense magnetizing fields have the added advantage of permitting a study of the reluctivity relationship. This relationship is valuable because it is capable of indicating the presence of one or more constituents and, therefore, gives promise of differentiating between different structures of carbon steels by magnetic methods, which is the purpose of the present paper.

In this investigation a eutectoid-carbon steel (0.85 per cent carbon) has been chosen in order that the arrangement of the constituents may be as simple as possible. With suitable heat treatment there is no excess of ferrite or cementite to be rejected to the grain boundaries. Although this investigation has been limited to the one composition, the method is also applicable to the entire carbon series.

II. THE RELUCTIVITY RELATIONSHIP

1. DEFINITION OF THE TERM

The term "reluctivity" (ρ) is defined by the ratio

$$\rho = \frac{H}{B} \quad (1)$$

where H is the magnetizing force and B the corresponding magnetic induction or total flux density. For moderate values of the magnetizing force the relationship between the reluctivity (ρ) and the magnetizing force H is linear³ and can be expressed by the equation

$$\rho = \alpha + \beta H \quad (2)$$

where α and β are constants for a given material. For inductions resulting from very high magnetizing forces this linear equation no longer strictly holds, since the induction B does not approach a definite saturation value, but increases indefinitely; however,

$$B_0 = B - H$$

approaches a finite saturation value, so that Kennelly's law applies not to B , but to B_0 . The latter is usually called the metallic induction, and may be considered as the magnetic flux carried by the molecules of the magnetic material not including

³ Kennelly, *Am. Inst. Elect. Eng., Trans.*, 8, p. 485; 1891.

the flux H , carried by the space occupied by the material, but independent of it. The ratio

$$\frac{H}{B_0} = \rho_0 = \alpha + \beta H \quad (3)$$

is defined as the "metallic reluctivity" and this linear relation holds for high magnetizing forces as well as for moderate ones. From now on in this paper the "metallic reluctivity" will be referred to simply as the reluctivity and will be designated by ρ .

2. STAGES OF THE MAGNETIZATION AND RELUCTIVITY CURVES

In Fig. 1 are plotted the (1) magnetization and (2) reluctivity curves for an eutectoid-carbon steel quenched in water. The

FIG. 1.—Magnetization and reluctivity curves for eutectoid carbon steel quenched in water

general shape of the curves is that characteristic of a steel of high carbon content and in a hardened condition. The three stages of the curves are designated by the letters A, B, and C.

In stage A of the magnetization curve the relation between the induction B and the magnetizing force H is nearly a linear one and the slope of the curve depends on the state of the material, being generally steep for soft and more gradual for magnetically hard materials. The molecular magnets have been disturbed only slightly relative to their original condition of stable equilibrium. In stage B the groups of molecular magnets pass from their

previous condition of stability through an unstable state into a new position of stability. In magnetically soft substances this change in molecular stability within the various groups takes place almost simultaneously, and the slope of the magnetization curve is correspondingly steep. In magnetically hard substances this change occurs during a longer interval, and thus the slope of the curve is more gradual. In stage *C* the alignment of the molecular groups approaches more and more the direction of the magnetizing force as it increases in magnitude. The slope of the curve decreases in magnitude.

However, the last stage becomes more significant when use is made of the reluctivity relationship represented for the same substance by the curve (2). It is for this stage that equation (3) accurately holds. Stages *A* and *B* can not be represented generally by any such simple relationship. For magnetically homogeneous and soft substances these two stages practically disappear, while for magnetically hard substances they become increasingly pronounced.

Steinmetz,⁴ however, has suggested that both of these are stages of instability, and that, if the specimen whose magnetic properties are being determined is simultaneously subjected to vibration or to an alternating magnetic field applied at right angles to the unidirectional field *H*, these two characteristic stages practically vanish; that it is probable that the stable relation between the field intensity *H* and the flux density *B* is expressed over the entire range from zero to infinity by the above linear relationship.

3. CHARACTERISTICS OF THE RELUCTIVITY LINE

In equation (3) α is the intercept of the linear portion of the curve extended to intersect the axis of ordinates, as shown in Fig. 1, and is generally designated as the *coefficient of magnetic hardness*. If the intensity of magnetization and the susceptibility are represented by *I* and *K*, respectively, it can be shown easily that

$$\frac{H}{I} = \frac{I}{K} = 4\pi\rho = \alpha' + \beta'H, \quad (4)$$

so that the reciprocal of the susceptibility also bears a linear relationship to the magnetizing force. The ordinates of the reciprocal of the susceptibility are evidently 4π times those of the reluctivity curve. Also, if in equations (3) and (4) *H* is allowed

⁴ Steinmetz, Gen. Elect. Rev., 20, p. 135; 1917.

to increase indefinitely B_0 and I approach their saturation or maximum values B_s and I_m , respectively, so that

$$B_s = \frac{1}{\beta} \text{ and } I_m = \frac{1}{\beta'} \quad (5)$$

The constants β and β' are thus called the *coefficients of saturation* of the metallic induction and intensity of magnetization, respectively. Low values of these constants represent high-saturation values.

4. PURE MATERIALS

For pure and well-annealed materials, such as Norway, Swedish charcoal, and electrolytic iron, the $\rho - H$ relation is a straight line. This straight-line relationship is well illustrated by the admirable experiments of B. O. Peirce⁵ on various kinds of pure iron. Fig.

FIG. 2.—Reciprocal of susceptibility for "American ingot iron" (B. O. Peirce)

2, taken from Peirce's paper, shows the $1/K - H$ line for a specimen of "American ingot iron."

5. IMPURE MATERIALS AND COMPLEX STRUCTURES

In the more or less impure commercial materials the $\rho - H$ relation, although an approximately straight line, generally has one, and occasionally two, points where its slope changes. Since the change in slope of the line is in general greater with increase in impurity of the material, the cause evidently is lack of homogeneity; that is, the presence in the substance as aggregates or conglomerates of materials of different magnetic characteristics,

⁵ B. O. Peirce, Proc. Amer. Acad. Arts and Sci., 19, June, 1913.

such as cementite, silicide, etc. Such bodies generally have a much greater magnetic hardness and a lower saturation value, and give, therefore, the observed effect.

In carbon steel the arrangement of the constituents, such as pearlite and ferrite or pearlite and cementite, depends on the carbon content and also on its previous mechanical and thermal treatment, and is generally a random one. However, as magnetic circuits the arrangement of these constituents may be considered as combinations of any of three simple arrangements, viz, (a) the parallel, (b) the series, and (c) the spheroidal.

(a) *The Parallel Arrangement.*—This arrangement is the easiest for experimental verification and also for mathematical proof. An ideal parallel arrangement would be, if the alternate layers of cementite and ferrite in the pearlite grains of a eutectoid steel were all parallel to each other and to the direction of the effective magnetic field. If, then, ρ_1 and ρ_2 , and a_1 and a_2 are the reluctivities and cross-sectional areas of the two constituents, respectively, the resultant reluctivity ρ is given by the relation

$$\rho = \frac{\rho_1 \rho_2}{\rho_1 a_2 + \rho_2 a_1} \quad (6)$$

The reluctivity curve for two constituents in parallel approximates to two straight lines joined by a bend. This has been experimentally verified by Ball,⁶ using pure iron and cobalt in a closed magnetic circuit.

(b) *The Series Arrangement.*—An ideal series arrangement would be if all of the lamellæ of ferrite and cementite in the pearlite grains of eutectoid steel were oriented parallel to each other and perpendicular to the direction of the effective magnetic field. If the reluctivities, lengths, and effective cross-sectional areas of the two constituents are, respectively, ρ_1 , ρ_2 , l_1 , l_2 , a_1 , and a_2 , the resultant reluctivity is expressed by the equation

$$\rho \frac{l_1 + l_2}{a} = \frac{\rho_1 l_1}{a_1} + \frac{\rho_2 l_2}{a_2} \quad (7)$$

where a is the equivalent cross-sectional area. The resultant reluctivity line consists of two or more approximately straight lines joined by more or less marked bends. The series arrangement can not be experimentally verified, since it is practically impossible to make satisfactory magnetic joints between the two materials. The solution can, however, be approximately obtained by a graph-

⁶ Ball, Jour. Frank. Inst., 181, p. 481; 1916.

ical method first outlined by J. and E. Hopkinson.⁷ The reluctivity is always greater in magnitude for the series than the parallel arrangement for a given ratio of the constituents.

(c) *Spheroidal Arrangement*.—The third arrangement may, for want of a better term, be called the spheroidal arrangement. It is well illustrated in a low-carbon (0.1 per cent) steel where the pearlite is distributed throughout the ferrite in irregularly shaped masses approximating in a general way to spheres or ellipsoids. The spherical masses (hard magnetically) carry practically no flux at low field intensities, all the flux being carried by the surrounding soft material. As the flux-carrying material occupies only part of the space, the flux density therefore rises rapidly, giving low α , and tends toward an apparent low-saturation value. At higher field intensities the spherical masses begin to carry flux. While in the softer material the flux increases at a lesser rate the increase of flux in the spherical masses gives a greater increase of the total flux density and a greater saturation value, but also a greater hardness, as the result of both materials. There is thus, in this arrangement, as well as in the others, a bend in the reluctivity line.

A special, but a very important, case is the distribution of very finely divided magnetic particles throughout a magnetically softer medium. If the particles are ultra microscopic in size, the medium is not only homogeneous but isotropic as well. It is also quite probable that the individual crystals would be composed of the two constituents rather than each crystal consisting of only one constituent. In the former arrangement the "intrinsic" magnetic field of the more magnetic of the constituents would entirely dominate the field of the weaker, so that the material would conduct itself as if magnetically homogeneous, and the reluctivity line would be a straight one.

(d) *General Arrangement*.—The arrangement of the constituents of a carbon steel as a function of the carbon content and the orientation of the respective grains is a random one. However complex the arrangement is, it may generally be considered as different combinations of any of the above simple arrangements.

⁷ J. and E. Hopkinson, Phil. Trans., 1886; p. 331.

III. APPARATUS AND HEAT TREATMENT

1. APPARATUS

In order to obtain values of the magnetic flux or intensity of magnetization which approach saturation, it is necessary to use high values of the magnetizing force. The apparatus suitable for this purpose and used in this investigation has already been described.⁸

2. HEAT TREATMENT

Two specimens (22 cm long) of eutectoid-carbon steel whose chemical analysis showed the following compositions were selected and turned down to a uniform diameter of approximately 7 mm:

	Per cent		Per cent
C.....	0.85	Cr.....	0.05
Mn.....	.23	P.....	.016
Si.....	.23	S.....	.014

Each of these specimens was quenched from a temperature of 800° C—one in a fairly light oil and the other in water at room temperature—and then carefully ground down to a uniform diameter of 6 mm. Induction tests showed that both specimens were magnetically uniform along their entire length. Each specimen was then cut into halves, designated as A and B, respectively, and the resulting ends ground, polished, and examined microscopically. The structure of both specimens was martensitic. The two halves of each rod were then drawn alternately to increasingly higher temperatures, as indicated in Table 1, and the normal induction curves determined after each operation. The specimens were kept at the designated drawing temperatures for a period of half an hour each, the water-quenched specimens being air cooled and the oil-quenched ones cooled in lime.

TABLE 1.—Drawing Temperatures (Degrees Centigrade)

Oil-quenched specimen.....	A	100	200	250	300	410	475	600	
	B	170	230	270	370	450	500	700	
Water-quenched specimen.....	A	100	200	250	300	400	450	500	700
	B	170	230	270	350	430	470	600	

⁸ W. I. Cheney, B. S. Sci. Papers, No. 361; 1920.

IV. EXPERIMENTAL DATA

In Fig. 3 are represented several magnetization curves as illustrative of this particular type of steel on being quenched in water from 800° C and drawn in steps to increasingly higher temperatures. The data for curve 1 are displayed in Table 2, so that one may have a clearer idea of the actual observations from which the curve was plotted. Tables 3 and 4 show, in a condensed form, the data corresponding to all heat treatments. The values of

FIG. 3.—Magnetization curves for eutectoid carbon steel, quenched in water and drawn

the induction therein shown are not in all cases the actual observed values, but are interpolated from the curves for definitely chosen values of the magnetizing force (H). Curves for many of the intermediate drawing temperatures are omitted, and only those possessing distinctive features are given. Since the specimens are of small diameter and the rates of quenching are thus not markedly different, the curves for the oil-and-water-quenched specimens are similar though not identical.

TABLE 2.—Magnetic Properties of 0.85 per cent Carbon Steel Quenched in Water from 800° C

<i>H</i>	<i>B</i>	<i>I</i>	<i>z/K</i>	μ	<i>H</i>	<i>B</i>	<i>I</i>	<i>z/K</i>	μ
14	540	42	0.335	38.6	535	15 960	1225	0.437	29.8
30	1 320	102	.293	44.0	685	16 660	1268	.540	24.3
56	3 880	304	.184	69.3	860	17 300	1305	.658	20.1
88	8 040	631	.140	91.4	1090	17 900	1335	.817	16.4
118	10 220	800	.148	86.6	1380	18 570	1364	1.012	13.5
156	11 700	916	.170	75.0	1710	19 100	1380	1.240	11.2
232	13 250	1034	.224	57.1	2000	19 520	1390	1.440	9.8
318	14 400	1118	.284	45.3	2420	20 150	1407	1.720	8.3
430	15 320	1182	.364	35.6	2630	20 440	1414	1.860	7.8

TABLE 3.—Induction for Given Magnetizing Forces after Quenching Specimen of 0.85 per cent Carbon Steel in Water and after Drawing to Successively Higher Temperatures

<i>H</i>	Induction (<i>B</i>)							
	Water quenched	102° C	170° C	200° C	230° C	250° C	270° C	300° C
20.....	780	700	840	950	2 200	3 800	2 400	3 700
50.....	3 180	3 050	3 550	5 350	9 900	13 650	13 400	14 350
100.....	9 150	8 750	9 000	11 600	14 360	16 600	16 580	16 900
150.....	11 500	11 250	11 200	13 500	15 900	17 820	17 830	18 000
200.....	12 700	12 450	12 370	14 600	16 500	18 500	18 500	18 570
300.....	14 250	14 000	13 700	15 600	17 900	19 300	19 300	19 300
500.....	15 700	15 750	15 250	16 800	18 940	20 100	20 100	19 900
700.....	16 700	16 700	16 050	17 500	19 500	20 500	20 600	20 300
1000.....	17 650	17 700	16 850	18 250	20 100	21 000	21 100	20 800
1500.....	18 800	18 800	17 800	19 150	20 900	21 800	21 800	21 500
2000.....	19 600	19 650	18 600	19 800	21 500	22 400	22 300	22 100
2500.....	20 250	20 300	19 250	20 400	22 000	22 800	22 800	22 600

<i>H</i>	Induction (<i>B</i>)—Continued							
	370° C	410° C	450° C	475° C	500° C	600° C	700° C	
20.....	3 500	3 900	3 000	2 600	1 550	7 500	7 000	
50.....	14 350	15 000	14 250	13 450	12 650	14 100	13 350	
100.....	16 900	17 050	16 800	16 500	16 070	16 200	15 100	
150.....	17 700	18 000	17 600	17 500	17 080	17 170	15 900	
200.....	18 200	18 500	18 080	18 000	17 650	17 620	16 400	
300.....	19 000	19 100	18 800	18 800	18 450	18 250	17 100	
500.....	19 500	19 750	19 500	19 700	19 250	19 150	18 000	
700.....	19 950	20 200	19 900	20 100	19 720	19 700	18 600	
1000.....	20 380	20 700	20 400	20 570	20 300	20 300	19 200	
1500.....	21 050	21 400	21 100	21 200	21 000	21 000	20 000	
2000.....	21 650	21 950	21 400	21 700	21 600	21 650	20 700	
2500.....	22 220	22 400	21 900	22 130	22 120	22 250	21 200	

TABLE 4.—Induction for Given Magnetizing Forces After Quenching Specimen of 0.85 per cent Carbon Steel in Oil and After Drawing to Successively Higher Temperatures

<i>H</i>	Induction (<i>B</i>)							
	Oil quenched	100° C	170° C	200° C	230° C	250° C	270° C	300° C
20.....	700	900	1 300	1 200	1 550	2 350	2 350	5 650
50.....	3 250	3 300	4 850	5 850	9 600	12 750	12 920	14 000
100.....	8 950	8 900	10 450	11 400	14 370	16 300	14 950	16 630
150.....	11 380	11 450	12 520	13 230	15 900	17 450	17 120	17 700
200.....	12 750	12 700	13 630	14 220	16 850	18 150	17 700	18 250
300.....	14 050	13 950	14 900	15 320	17 950	19 100	18 350	18 850
500.....	15 550	15 550	16 050	16 660	19 460	20 200	19 380	19 650
700.....	16 500	16 550	16 850	17 420	20 220	20 700	19 750	20 150
1000.....	17 450	17 500	17 650	18 250	20 850	21 250	20 200	20 700
1500.....	18 580	18 550	18 620	14 150	21 630	22 000	20 900	21 400
2000.....	19 350	19 380	19 400	19 900	22 280	22 600	21 420	21 950
2500.....	20 000	20 100	20 100	20 600	22 800	23 150	21 920	22 400

<i>H</i>	Induction (<i>B</i>)—Continued.							
	350° C	400° C	430° C	450° C	470° C	500° C	600° C	700° C
20.....	3 550	3 550	3 550	2 850	2 750	3 350	4 000	7 100
50.....	14 350	14 450	14 650	14 250	13 850	13 950	13 650	14 050
100.....	16 560	16 580	16 800	16 600	16 450	16 250	15 870	15 700
150.....	17 400	17 550	17 650	17 400	17 420	17 120	16 800	16 450
200.....	18 000	18 050	18 150	18 100	18 000	17 700	17 300	16 950
300.....	18 900	18 650	18 800	19 100	18 750	18 550	18 000	17 600
500.....	19 400	19 400	19 500	19 250	19 180	19 300	18 900	18 700
700.....	19 900	19 880	20 000	19 900	19 650	19 720	19 400	19 250
1000.....	20 360	20 400	20 500	20 400	20 120	20 250	20 020	19 850
1500.....	21 050	21 100	21 200	21 100	20 850	21 000	20 700	20 680
2000.....	21 660	21 650	21 750	21 650	21 500	21 620	21 450	21 350
2500.....	22 250	22 120	22 200	22 150	22 050	22 200	22 050	21 950

The successive drawing operations produce an increasing steepness in the second stage of the curve, indicating that the material is becoming increasingly softer magnetically. The third stage of the curve is increasingly greater in magnitude for lower but not for intermediate or high drawing temperatures, as is illustrated by the crossing of the curves 3, 4, and 5 of Fig. 3. This decreasing in magnitude of the saturation intensity is evidently due to a rearrangement of the constituents. While such curves are valuable as criteria of the heat treatment, yet they often are difficult to interpret.

In Fig. 4 the values of the induction for constant values of the magnetizing force are plotted as ordinates against the drawing temperatures as abscissæ. The agreement between the respective values of the induction for the two specimens is generally quite

good except for low and high drawing temperatures. Special attention is called to the very rapid rise of the induction for the drawing-temperature intervals 200 to 250° C, the significance of which will be discussed later.

The reciprocal of the susceptibility ($\frac{I}{K}$) is plotted against the magnetizing force in Fig. 5 for the water-quenched specimens in

FIG. 4.—Variation of induction for a given magnetizing force with the drawing temperatures

four conditions of treatment, viz, (1) quenched, (2) drawn to 250°, (3) to 370°, and (4) to 700° C. Curves 1 and 4 consist of two straight lines joined by a bend *a*. The lower part of each curve is extended so as to show more clearly the indicated bend. The curves 2 and 3 are straight lines throughout their entire third stage.

Table 2 shows the values from which curve 1 was plotted, while Tables 5 and 6 show the reciprocal of the susceptibility for the other heat treatments. The statements made with reference to Tables 3 and 4 also apply to this case.

FIG. 5.—Reciprocal of susceptibility for eutectoid carbon steel, quenched in water and drawn

TABLE 5.—Reciprocal of Susceptibility for Given Magnetizing Forces After Quenching Specimen of 0.85 Per Cent Carbon Steel in Water and After Drawing to Successively Higher Temperatures

<i>H</i>	Reciprocal of susceptibility (<i>t</i> / <i>K</i>)							
	Quenched	102° C	170° C	200° C	230° C	250° C	270° C	300° C
20.....	0.320	0.340	0.270	0.240	0.120	0.070	0.100	0.065
50.....	.200	.220	.200	.120	.070	.050	.048	.045
100.....	.145	.150	.138	.110	.090	.078	.078	.075
150.....	.165	.180	.175	.142	.120	.108	.110	.110
200.....	.200	.212	.210	.180	.155	.138	.140	.140
300.....	.270	.277	.275	.250	.220	.200	.200	.200
500.....	.413	.413	.435	.390	.345	.320	.322	.325
700.....	.550	.548	.580	.530	.475	.442	.445	.450
1000.....	.760	.750	.800	.735	.665	.630	.630	.640
1500.....	1.098	1.085	1.163	1.078	.980	.940	.935	.955
2000.....	1.435	1.425	1.525	1.420	1.300	1.245	1.240	1.270
2500.....	1.785	1.770	1.890	1.770	1.620	1.555	1.550	1.585

<i>H</i>	Reciprocal of susceptibility (<i>t</i> / <i>K</i>)—Continued							
	370° C	410° C	450° C	475° C	500° C	600° C	700° C	
20.....	0.055	0.065	0.085	0.100	0.160	0.045	0.050	
50.....	.045	.046	.040	.050	.055	.046	.050	
100.....	.080	.080	.080	.078	.080	.080	.083	
150.....	.110	.110	.110	.110	.112	.110	.120	
200.....	.140	.140	.140	.140	.145	.145	.155	
300.....	.205	.205	.208	.205	.210	.210	.225	
500.....	.332	.330	.335	.332	.340	.340	.370	
700.....	.460	.455	.460	.460	.465	.465	.500	
1000.....	.650	.642	.660	.650	.655	.655	.700	
1500.....	.968	.968	.980	.970	.970	.970	1.025	
2000.....	1.283	1.270	1.300	1.288	1.290	1.282	1.355	
2500.....	1.605	1.590	1.625	1.610	1.608	1.595	1.680	

TABLE 6.—Reciprocal of Susceptibility for Given Magnetizing Forces After Quenching Specimen of 0.85 per cent Carbon Steel in Oil and After Drawing to Successively Higher Temperatures

<i>H</i>	Reciprocal of susceptibility (<i>I/K</i>)							
	Quenched	100° C	170° C	200° C	230° C	250° C	270° C	300° C
20.....	0.332	0.300	0.188	0.225	0.150	0.105	0.105	0.060
50.....	.215	.200	.145	.115	.070	.050	.045	.042
100.....	.145	.145	.115	.108	.090	.070	.078	.075
150.....	.165	.170	.152	.142	.122	.110	.108	.107
200.....	.200	.208	.188	.180	.150	.140	.140	.140
300.....	.275	.278	.260	.250	.212	.200	.205	.200
500.....	.425	.420	.405	.392	.333	.320	.335	.325
700.....	.560	.560	.546	.525	.455	.440	.465	.450
1000.....	.770	.766	.755	.730	.628	.623	.655	.640
1500.....	1.115	1.110	1.105	1.068	.940	.925	.978	.955
2000.....	1.460	1.450	1.450	1.405	1.245	1.228	1.300	1.265
2500.....	1.805	1.795	1.800	1.742	1.555	1.530	1.620	1.585

<i>H</i>	Reciprocal of susceptibility (<i>I/K</i>)—Continued							
	350° C	400° C	430° C	450° C	470° C	500° C	600° C	700° C
20.....	0.060	0.060	0.070	0.075	0.115	0.075	0.085	0.040
50.....	.048	.045	.048	.040	.048	.048	.045	.045
100.....	.080	.078	.078	.080	.080	.080	.080	.078
150.....	.110	.110	.110	.112	.110	.110	.110	.115
200.....	.142	.142	.140	.145	.145	.145	.145	.147
300.....	.205	.207	.205	.210	.210	.210	.210	.215
500.....	.332	.335	.330	.335	.340	.340	.340	.350
700.....	.460	.460	.457	.460	.468	.468	.468	.475
1000.....	.650	.650	.648	.655	.658	.655	.665	.668
1500.....	.968	.968	.965	.970	.975	.970	.980	.980
2000.....	1.284	1.285	1.278	1.285	1.290	1.285	1.295	1.300
2500.....	1.605	1.605	1.595	1.605	1.610	1.600	1.615	1.620

As has been previously suggested, the saturation value of the intensity of magnetization can be calculated from the relation

$$I_m = \frac{I}{\beta}$$

where β is the slope of the $\frac{I}{K}-H$ line. If the line consists of two lines joined by a bend there are two values of β . The greater of these slopes is the "apparent" saturation value while the lesser is the "true" saturation value for the material.

In Fig. 6 the saturation values of the intensity of magnetization are plotted against the drawing temperature as the independent variable. The real saturation values are indicated in full lines, while the apparent values are in broken lines. In Fig. 7 the intercepts (α) are plotted and represented in a similar manner.

V. DISCUSSION OF THE RESULTS

1. PRECIPITATION OF THE CEMENTITE

The significance of the very rapid rise of the induction within the temperature limits of 150 and 250° C has already been discussed, as relating to a 1 per cent carbon steel,⁹ in a previous paper. However, for the sake of completeness the essential points will be given briefly.

According to Tammann's¹⁰ law for solid solutions, the intensity of magnetization of a solid solution is less than that of the more magnetic of the two constituents and also less than would be calculated from a knowledge of the proportion of the constituents. This is well illustrated by the difference in induction (Fig. 4) for the material when in the quenched condition and drawn to 250° C. The experiments of Honda¹¹ on the magnetic transformation of cementite in quenched carbon steels are additional evidence. Cementite is approximately one-tenth as magnetic as pure iron and becomes nonmagnetic at 215° C. As a quenched high-carbon steel is repeatedly drawn to increasingly higher temperatures, the magnitude of the magnetic transformation of cementite increases and reaches its maximum in the neighborhood of 300° C.

These experimental facts indicate that in a martensitic structure the cementite is in solid solution and is precipitated within the drawing-temperature interval of 150 to 250° C. The magnetic changes on tempering accompanied by heat evolution¹² at least represent the completion of a suppressed transformation, which completion may be considered as the transition from martensite to troostite. Whether such a criterion is acceptable remains for the future to decide.

2. SATURATION AND INTERCEPT VALUES

The regions *A*, *B*, and *C* of Figs. 6 and 7 show the existence of three magnetically differentiable arrangements or structures very probably of martensite, troostite, and sorbite, respectively.

(a) *Region A*.—The fact that there is a break in the reluctivity line throughout this entire region indicates that two magnetically different constituents are present. This may be due to one of two factors, or both, namely, the stresses in the steel or the existence of two physically different constituents.

⁹ Nusbaum, A. S. T. M. Proc., 19, p. 100; 1919.

¹⁰ Tammann, Zeit. für Phys. Chem., 65, p. 79; 1908.

¹¹ Honda, Tohoku University Repts., 6, p. 149; 1917.

¹² Scott and Movius, B. S. Sci. Papers, No. 396; 1920.

Owing to the stresses set up during the quenching process, a quenched material is always in a state of strain. The outside portion of the material is in a state of tension, while the inside portion is in a state of compression. The effect of tension¹³ is in general to increase the induction at low field intensities and decrease it at high intensities relative to that of the material in the unstressed state. The stressed material thus acts as a mag-

FIG. 6.—Variation of maximum intensity of magnetization with drawing temperature

netically softer substance of lower maximum intensity of magnetization. Its reluctivity line would have a lower intercept and a higher slope when in the stressed (tension) state than when in the normal state. Under compression¹⁴ the induction is lower in value at low field intensities than for the material in the normal state. Thus, under compression, the material conducts itself as a magnetically harder substance, and probably approaches a

¹³ Villari, Pogg. Ann.; 1868. Honda and Shimizu, Phil. Mag., 4, p. 388; 1903.

¹⁴ Smith and Sherman, Phys. Rev., 4, p. 267; 1914.

higher saturation value. Its reluctivity line would have a higher intercept and also, probably, a lower slope under compression than in the normal state. In a material in a quenched state the exterior portions are thus magnetically softer and have a relatively lower saturation value than the interior portions, and there is a gradual transition from the one to the other. Such an arrangement corresponds to a complex parallel-series arrangement of

FIG. 7.—Variation of the coefficient of magnetic hardness (intercept) with drawing temperature

materials of uniformly varying magnetic properties. From earlier arguments in this paper it is thus evident that there should be a break in the reluctivity line for a quenched material. Drawing the material so as to relieve the strains should thus decrease the change in slope and also decrease in magnitude the difference between the apparent and the true saturation values of the intensity of magnetization. While the stresses in the material

undoubtedly contribute to the effects observed, they very likely are not the only cause, or perhaps not even the predominating one. It is not probable that the strains are entirely removed at a temperature of 250° C. The questions here raised can be answered only by further experimentation.

(b) *Region B*.—Throughout this entire drawing-temperature interval the steel behaves like a magnetically pure material. However, the experiments of Honda¹⁵ show quite clearly that the carbon present in this region is in the form of cementite. Since there are thus two magnetic constituents present, in order that the material in its entirety may be magnetically homogeneous according to the arguments in the earlier part of the paper, the cementite is evidently distributed very uniformly in very finely divided particles (ultra microscopic in size) throughout the entire mass of the ferrite, and there are no separate cementite and ferrite crystals. This agrees with the conception of troostite generally accepted by metallographists and first presented by Benedicks. Throughout this entire region the maximum intensity of magnetization generally gradually decreases in magnitude, while the intercept values remain constant.

(c) *Region C*.—With increasing drawing temperatures the ultra microscopic particles of cementite of region *B* form aggregates of continually increasing size, which finally attain such dimensions as to produce a magnetically nonuniform or inhomogeneous material, as is indicated by a more or less marked break in the reluctivity line. The line of demarcation between regions *B* and *C* then marks the transition between the regions of magnetic homogeneity and nonhomogeneity. Since this transition is so sharply defined, it may be chosen as the beginning of the metallographic constituent sorbite. Beyond this transition point as the aggregate particles or conglomerates either increase in size with increasing drawing temperatures or as the medium approaches more and more toward stratification the bend in the reluctivity line becomes more pronounced, as is indicated by the increasing magnitude of the difference between the apparent and real saturation values of the intensity of magnetization and also the intercept values.

¹⁵ Honda, Tohoku, Univ. Reports, 6, p. 149; 1917.

VI. SUMMARY

The essential and important points of this paper may be summarized as follows:

1. The very rapid rise of the maximum induction in the temperature interval of 150 to 250° C, accompanied by heat evolution, viewed in the light of Tammann's law and Honda's experiments, indicates that the cementite in this temperature interval is thrown out of solid solution, and thus that in martensite the cementite is in solid solution. It also indicates the completion of a previously suppressed transformation, which completion may be considered as the beginning of the metallographic constituent troostite.

2. The magnetic inhomogeneity within the region *A* may be due to one of two causes or both, viz, (1) the presence of another constituent, (2) the stresses set up in the material during the quenching operation. The stresses undoubtedly play an important part, but are not necessarily the predominating factor.

3. In region *B* the material is magnetically homogeneous. This fact is not an experimental verification, but is evidence in favor of Benedicks's theory of troostite.

4. In region *C* the material is magnetically nonhomogeneous. This inhomogeneity is due to the increasing size of the aggregates and to their approaching stratification.

5. The transition point between regions *B* and *C* may be considered as the beginning of the metallographic constituent sorbite.

6. The magnetic reluctivity relationship provides a method for magnetically differentiating the metallographic constituents of the carbon-iron series.

Credit is due to H. A. Wadsworth for most of the heat treating.

WASHINGTON, August 17, 1920.