THE VARIATION OF RESIDUAL INDUCTION AND COERCIVE FORCE WITH MAGNETIZING FORCE

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An analytical expression representing the relationship between magnetic quantities for ferromagnetic substances has been sought by many investigators. The law of Fröhlich, however, which was later modified by Kennelly¹ is the "only rational law of magnetic relation which has not been disproven."² This law states that the "permeability is proportional to the magnetizability." That is,

$$\mu = a (S - B) \quad (1)$$

where μ is the permeability, S is the saturation value of flux density, and B is the flux density.

If in equation (1) we substitute the relation

$$\mu = \frac{B}{H}, \quad (2)$$

we have, upon rearranging,

$$B = \frac{H}{a + \sigma H}, \quad (3)$$

where $\sigma = \frac{1}{S}$ = saturation coefficient, or reciprocal of the saturation density S , and $a = \frac{1}{\sigma S} = \frac{\sigma}{a}$. Kennelly's modification consisted of the introduction of the idea of reluctivity (ρ), which is the reciprocal of permeability. By substituting the relation

$$\rho = \frac{1}{\mu} = \frac{H}{B} \quad (4)$$

in equation (3) we can derive the equation

$$\rho = a + \sigma H. \quad (5)$$

¹ Kennelly, Trans. A. I. E. E., 8, p. 485; 1891.
181117°—20

² Steinmetz, G. E., Rev., 20, p. 135; 1917.
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Since B never reaches a saturation value or maximum, this equation was changed to the form

$$\rho_0 = a + \sigma H \quad (6)$$

where ρ_0 is the *metallic* reluctivity $\left(\rho_0 = \frac{H}{B-H}\right)$. This is the final expression of Kennelly's law and has been found to hold except for points near the origin for pure homogeneous materials.

If the values of residual induction (B_r) and coercive force (H_c) are plotted against maximum magnetizing force (H_m) as the independent variable, the resulting curves have the same characteristics as the curve of $B-H$ plotted against H . In view of this fact, it appears that there should hold for these quantities a relationship similar to that expressed by the reluctivity law of Kennelly.

In order to ascertain whether such a relationship does hold, hysteresis measurements have been made on a variety of materials ranging from Norway iron to an exceptionally hard steel, known as K S magnet steel, which was prepared by Prof. Honda. The magnetizing forces used ranged from small values up to about 2500 gauss, which in most cases was sufficient to produce magnetic saturation. The apparatus used in these experiments has already been fully described.³

Complete data for four typical specimens are given in Tables 1 to 4 and are plotted in the corresponding Figs. 1 to 4. The data include observed values of H_m , B_m , B_r , and H_c and calculated values of ρ_0 , H_m/B_r , and H_m/H_c . The actual observations are indicated in the graphs, and it is evident that the straight-line relationship holds within the limits of experimental error. Figs. 5 and 6 show curves for a number of other materials. In the case of Norway iron, the failure of the points for the H_m/H_c curve to fall on a straight line, and the consequent uncertainty in the calculation of the maximum value of H_c , are in all probability due to the fact that the sensitivity of the apparatus does not permit of determinations of very small values of H_c with as great an accuracy as can be obtained in other cases.

³ W. L. Cheney, B. S. Sci. Papers, No. 361, 1920; No. 383; 1920.

FIG. 1.—Illustrating the magnetic properties of annealed 0.85 per cent carbon steel

FIG. 2.—Illustrating the magnetic properties of 0.85 per cent carbon steel, quenched in oil from 800° C

FIG. 3.—Illustrating the magnetic properties of tool steel, as received

FIG. 4.—Illustrating the magnetic properties of hardened K S magnet steel

FIG. 5.—Showing the ratio H_m/H_c plotted against the magnetizing force, for a number of materials

FIG. 6.—Showing the ratio H_m/B_r plotted against the magnetizing force, for a number of materials

This linear relationship, which the experiments have confirmed, may be expressed analytically similarly to Kennelly's reluctivity relationship, as follows:

$$H_m/B_r = a_1 + b_1 H_m \quad (7)$$

$$H_m/H_c = a_2 + b_2 H_m \quad (8)$$

where

H_m = maximum magnetizing force,

B_r = residual induction,

H_c = coercive force,

a_1 and a_2 = intercepts on axis of ordinates,

b_1 and b_2 = slopes of the lines.

In each case the reciprocal of the slope of the line gives the maximum value of B_r or H_c , respectively, just as the reciprocal of the slope of the reluctivity line gives the maximum value of $B - H$.

Table 5 shows the agreement between the observed maximum values of B_r and H_c and the values calculated from the graphs. The highest magnetizing force employed was not sufficient to saturate the K S magnet steel so that the calculated value of the maximum H_c is higher than the highest observed value. In all other cases, the calculated values agree well within the limits of the probable experimental error.

TABLE 1.—Magnetic Properties of Annealed 0.85 per cent Carbon Steel

H_m	B_m	B_r	H_c	ρ_o	H_m/B_r	H_m/H_c
32	12 080	7 500	5.0	0.0027	0.0043	6.40
92	15 020	10 980	6.5	.0062	.0084	14.15
120	15 570	11 650	9.0	.0078	.0130	13.35
183	16 280	12 100	10.0	.0114	.0151	18.30
246	16 990	12 550	10.5	.0148	.0196	23.40
325	17 690	13 000	11.0	.0187	.0250	29.55
418	18 140	12 620	11.5	.0236	.0332	36.34
500	18 380	13 100	12.0	.0280	.0382	41.65
680	18 970	13 240	12.0	.0370	.0514	56.60
850	19 380	13 480	12.5	.0459	.0630	68.0
1080	19 820	13 600	14.0	.0577	.0794	77.0
1260	20 350	13 650	12.5	.0660	.0924	100.7
1520	20 890	14 050	13.0	.0785	.1082	116.8
1840	21 240	14 000	13.0	.0948	.1314	141.5
2090	21 930	14 310	13.0	.1053	.1460	160.8
2300	21 940	14 300	13.0	.1172	.1610	177.0
2430	21 860	14 200	13.0	.1250	.1710	187.0

TABLE 2.—Magnetic Properties of 0.85 per cent Carbon Steel, Quenched in Oil from 800° C

H_m	B_m	B_r	H_c	ρ_o	H_m/B_r	H_m/H_c
42	2 720	950	9	0.0157	0.044	4.67
86	8 050	4540	22	.0108	.0190	3.91
137	11 210	6900	34	.0124	.0199	4.03
190	12 680	7600	44.5	.0152	.0250	4.27
220	13 160	7700	46.0	.0170	.0286	4.78
304	14 360	8150	49.5	.0216	.0373	6.14
410	15 240	8300	50.0	.0276	.0494	8.20
485	15 690	8440	50.5	.0318	.0575	9.60
570	16 290	8550	52.0	.0363	.0667	10.95
670	16 580	8600	52.0	.0421	.0780	12.88
748	16 660	8600	52.0	.0470	.0870	14.38
842	17 220	8750	52.0	.0514	.0963	16.20
920	17 420	8800	53.0	.0558	.1045	17.35
1050	18 040	8900	52.5	.0618	.1180	20.00
1310	18 570	9000	52.5	.0760	.1455	25.00
1500	18 750	9100	53.0	.0870	.1650	28.30
1710	19 000	9120	52.5	.0990	.1875	32.60
2000	19 540	9300	52.5	.1140	.2150	38.10
2290	19 930	9380	52.0	.1300	.2440	44.00
2420	19 970	9300	52.5	.1378	.2600	46.10

TABLE 3.—Magnetic Properties of Tool Steel, as Received

H_m	B_m	B_r	H_c	ρ_o	H_m/B_r	H_m/H_c
34	10 680	6 500	8.0	0.0032	0.0052	4.25
90	14 740	9 840	10.0	.0061	.0092	9.00
144	16 010	11 100	13.5	.0091	.0130	10.65
202	17 170	12 100	14.0	.0119	.0167	14.43
260	17 340	12 300	14.5	.0152	.0211	17.90
380	18 160	12 600	15.0	.0214	.0302	25.35
510	18 680	13 000	15.5	.0280	.0392	32.90
610	19 210	13 200	15.5	.0328	.0462	39.40
710	19 200	13 090	15.5	.0384	.0543	45.8
870	19 750	13 300	16.0	.0461	.0654	54.4
1110	20 300	13 600	16.0	.0579	.0816	69.4
1370	20 630	13 800	16.0	.0711	.0994	85.7
1800	21 060	13 800	16.0	.0935	.1310	112.5
2260	21 770	14 100	16.0	.1175	.1600	141.0

TABLE 4.—Magnetic Properties of Hardened K S Magnet Steel

H_m	B_m	B_r	H_c	ρ_c	H_m/B_r	H_m/H_c
26	210	0		0.1470		
48	450	80	5.5	.1195	0.6000	8.73
84			9			9.34
100	1 140	200		.0962	.5000	
149	2 280	580	16	.0694	.2569	9.32
224	4 660	2100	48	.0505	.1074	4.67
296	9 080	5610	116	.0337	.0528	2.55
410	11 340	7200	160	.0375	.0570	2.56
490	12 290	7800	178	.0415	.0628	2.75
665	13 780	8360	194	.0506	.0795	3.43
850	14 710	8650	201	.0613	.0982	4.23
1060	15 530	8800	205	.0733	.1205	5.17
1380	16 510	8780	210	.0912	.1572	6.57
1675	17 190	8850	210	.1080	.1893	7.97
1975	17 750	8840	211	.1250	.2240	9.36
2260	18 370	8900	212	.1400	.2540	10.70
2500	18 720	8900	215	.1540	.2820	11.62

TABLE 5.—Showing the Agreement between Calculated and Observed Values of Maximum Residual Induction and Maximum Coercive Force for a Number of Specimens

Material	Maximum B_r		Maximum H_c	
	Calculated	Observed	Calculated	Observed
Norway iron.....	16 400	15 900	7.4	6.0
Bessemer steel.....	16 000	15 900	9.4	9.0
0.85 per cent carbon steel (annealed).....	14 500	14 310	13.5	13.0
Tool steel.....	14 200	14 100	16.2	16.0
0.85 per cent carbon steel (oil quenched).....	9 550	9 380	52.7	53.0
Chromium steel (oil quenched).....	11 300	11 200	53.8	53.5
0.85 per cent carbon steel (water quenched).....	9 750	9 800	57.0	56.5
Tungsten steel (oil quenched).....	11 300	11 200	69.0	68.0
K S magnet steel.....	9 000	8 900	220	^a 215.

^a Saturation not reached.

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