FILTERS FOR PRODUCING THE COLOR OF THE EQUAL-ENERGY STIMULUS

By Raymond Davis and K. S. Gibson

ABSTRACT

Two filters have been prepared similar to other Davis-Gibson filters, which when combined with an illuminant at 2848 K give the color of the so-called equal-energy stimulus. One of these has been designed on the basis of the O.S.A. observer, the other on the basis of the I.C.I. observer. Spectrophotometric and colorimetric data are given.

So far as known, but one attempt has been made to produce accurately the color of the so-called equal-energy stimulus, i.e., the color of an hypothetical illuminant whose spectral energy per unit wave length is equal in amount at all wave lengths. A combination glass filter was designed by one of the authors in 1926¹ which, when used with Abbot-Priest sunlight² gave by computation a chromaticity match with the equal-energy stimulus. The resulting equal-energy color was used by Priest in "An Experiment Bearing on the Adoption of a Standard Neutral Stimulus in Colorimetry: The Choice as Between 'Sun' and 'Equal Energy'."³

This experiment illustrates one of the uses of an illuminant having the color of the equal-energy stimulus. Such an illuminant would also be useful as the heterogeneous stimulus in experimental determinations of dominant wave length and purity, the equal-energy stimulus having been adopted as the basic stimulus in the 1931 I.C.I. coordinate system.⁴

The present filters have been designed in order to make the color of the equal-energy stimulus readily available. Except for the necessary variations in composition they are identical with other Davis-Gibson filters.⁵ Readers are referred to B.S. Misc. Pub. No. 114 regarding selection of materials, preparation of solutions, and the filling of cells, and for general spectrophotometric and colorimetric data on the filter components.

Spectrophotometric and colorimetric data on these filters and on the resulting energy distributions are given in the accompanying figures and table. Figure 1 is a chart, similar to those of B.S. Misc. Pub. No. 114, which gives the spectral transmission⁶ of the filter, the resulting

¹ K. S. Gibson, The Production of Radiant Energy of Uniform Intensity Over the Visible Spectrum, Jour. Opt. Soc. Am. and Rev. Sci. Insts., vol. 13, p. 305, 1926.

² Irwin G. Priest, Standard Artificial Sunlight for Colorimetric Purposes, Jour. Opt. Soc. Am. and Rev. Sci. Insts., vol. 12, p. 479, 1926.

³ Jour. Opt. Soc. Am. and Rev. Sci. Insts., vol. 13, p. 306, 1926.

⁴ Deane B. Judd, The 1931 I.C.I. Standard Observer and Coordinate System, Jour. Opt. Soc. Am., vol. 23, pp. 359-374, 1933. See footnote 12 of Judd's paper.

⁵ Raymond Davis and K. S. Gibson, Filters for the Reproduction of Sunlight and Daylight and the Determination of Color Temperature, B.S. Misc. Pub. no. 114, 1931. Copy may be obtained from the Superintendent of Documents, Government Printing Office, Washington, D.C., at 45 cents.

⁶ Derived from the standard data on the filter components.

2848° K TO EQUAL ENERGY

CHART 39

λ ($m\mu$)	T	E''^*	E''/E'
350	.0259	7.0	.070
60	.351	12.3	.123
70	.449	20.0	.200
80	.543	30.3	.303
90	.626	43.2	.432
400	.684	57.4	.574
10	.721	72.7	.727
20	.736	88.2	.882
30	.716	100.7	1.007
40	.663	108.6	1.086
450	.585	110.6	1.106
60	.508	109.6	1.096
70	.451	110.3	1.103
80	.402	110.7	1.107
90	.353	108.5	1.085
500	.297	101.4	1.014
10	.248	93.5	.935
20	.217	89.9	.899
30	.205	92.6	.926
40	.201	98.5	.985
550	.196	103.8	1.038
60	.186	106.1	1.061
70	.172	105.2	1.052
80	.156	102.2	1.022
90	.1426	99.1	.991
600	.1316	96.8	.968
10	.1244	96.8	.968
20	.1195	97.9	.979
30	.1156	99.4	.994
40	.1121	101.1	1.011
650	.1103	103.8	1.038
60	.1080	106.0	1.060
70	.1051	107.2	1.072
80	.1016	107.5	1.075
90	.0969	106.2	1.062
700	.0926	104.8	1.048
10	.0881	102.8	1.028
20	.0834	100.2	1.002

Bureau of Standards, Davis-Gibson Filters

T-- Spectral Transmission of Filter
 V-- Relative Visibility Function
 E-- Relative Energy of 2848° K
 E'-- Equal Energy

$E'' (=T \times E)^*$ --- o o o o --- Relative Energy of
 2848° K and Filter Combination

Light Transmission of Filter**
 for 2848° K = 0.174

FILTER FORMULA

A

Copper Sulphate ($\text{CuSO}_4 \cdot 5\text{H}_2\text{O}$) 3.134 grams
 Mannite ($\text{C}_6\text{H}_8(\text{OH})_6$) 3.134 grams
 Pyridine ($\text{C}_5\text{H}_5\text{N}$) 30.0 cc
 Water (distilled) to make 1000. cc

B

Cobalt Ammonium Sulphate
 ($\text{CoSO}_4 \cdot (\text{NH}_4)_2\text{SO}_4 \cdot 6\text{H}_2\text{O}$) 31.180 grams
 Copper Sulphate ($\text{CuSO}_4 \cdot 5\text{H}_2\text{O}$) 17.154 grams
 Sulphuric Acid (sp. gr. 1.835) 10.0 cc
 Water (distilled) to make 1000. cc

These data are for a one centimeter layer
 each of solutions A and B in a double cell
 with three plates of borosilicate crown glass
 (refractive index, D line, $n_D = 1.51$), each 2.5 mm thick.

* Adjusted to make sum of $E'' - E'$ from 400
 to 720 $m\mu$ equal practically to zero

** Factor to be used to multiply the candle-power
 of the light source to obtain the candle-power
 of the source-and-filter combination

FIGURE 1.—Equal-energy filter derived on the basis of the O.S.A. standard observer.
 This chart is numbered 39 to follow in sequence chart no. 38 of B.S. Misc. Pub. No.
 114.

2848°K TO EQUAL ENERGY

I.C.I. OBSERVER

λ ($\mu\mu$)	T	E''^*	E''/E'
350	0.270	6.9	.069
60	.363	11.9	.119
70	.460	19.2	.192
80	.553	29.0	.290
90	.636	41.1	.411
400	.693	54.5	.545
10	.730	69.0	.690
20	.747	83.8	.838
30	.728	96.0	.960
40	.679	104.2	1.042
450	.607	107.3	1.073
60	.533	107.7	1.077
70	.478	109.4	1.094
80	.430	110.8	1.108
90	.380	109.5	1.095
500	.324	103.6	1.036
10	.274	96.7	.967
20	.242	93.6	.936
30	.228	96.3	.963
40	.222	101.9	1.019
550	.215	106.9	1.069
60	.204	108.9	1.089
70	.189	108.0	1.080
80	.172	105.1	1.051
90	.157	102.1	1.021
600	.1449	99.9	.999
10	.1370	99.8	.998
20	.1312	100.7	1.007
30	.1264	101.8	1.018
40	.1220	102.9	1.029
650	.1190	104.9	1.049
60	.1157	106.3	1.063
70	.1114	106.4	1.064
80	.1067	105.7	1.057
90	.1007	103.3	1.033
700	.0952	100.9	1.009
10	.0897	98.0	.980
20	.0841	94.5	.945

Bureau of Standards. Davis-Gibson Filters

T--Spectral Transmission of Filter
V--Relative Visibility Function
E--Relative Energy of 2848°K
E'--Equal Energy

$E'' (=T \times E)^* \sim \circ \circ \circ \circ \circ$ --Relative Energy of
2848°K and Filter Combination

Light Transmission of Filter **
for 2848°K = 0.191

FILTER FORMULA

A

Copper Sulphate ($\text{CuSO}_4 \cdot 5\text{H}_2\text{O}$) 2.954 grams
Mannite ($\text{C}_6\text{H}_8(\text{OH})_6$) 2.954 grams
Pyridine ($\text{C}_5\text{H}_5\text{N}$) 30.0 cc
Water (distilled) to make 1000. cc

B

Cobalt Ammonium Sulphate
($\text{CoSO}_4 \cdot (\text{NH}_4)_2 \cdot \text{SO}_4 \cdot 6\text{H}_2\text{O}$) 28.440 grams
Copper Sulphate ($\text{CuSO}_4 \cdot 5\text{H}_2\text{O}$) 17.840 grams
Sulphuric Acid (sp.gr. 1.835) 10.0 cc
Water (distilled) to make 1000. cc

These data are for a one centimeter layer
each of solutions A and B in a double cell
with three plates of borosilicate crown glass
(refractive index, D line, = 1.51), each 2.5 mm thick.

* Adjusted to make sum of $E'' - E'$ from 400
to 720 $\mu\mu$ equal practically to zero

** Factor to be used to multiply the candle-power
of the light source to obtain the candle-power
of the source-and-filter combination

FIGURE 2.—Equal-energy filter derived on the basis of the 1931 I.C.I. standard observer.

energy distribution when in combination with a 2848 K illuminant, the light transmission of the filter for 2848 K, and the chemical composition of the solutions.

As in the case of the other Davis-Gibson filters, the concentrations of the components have been so chosen that the trilinear coordinates of the illuminant-filter combination agree by computation precisely with those for the ideal energy distribution, differences appearing only in the fifth decimal place; at the same time the unavoidable deviations between the actual and ideal energy distributions are held to a minimum. Because of these deviations in energy distribution, however, the particular distribution required to give the same trilinear coordinates as those of the ideal equal-energy distribution will depend upon the particular "excitation" curves used to define the standard observer. Similarly, any two actual observers would probably not agree exactly

FIGURE 3.—Comparison of the spectral energy distributions obtained with the two equal-energy filters illustrated in figures 1 and 2.

as to the degree of chromaticity match given by the ideal and actual stimuli, if both were available for observation.

The filter described in figure 1 was designed on the basis of the O.S.A. standard observer, in terms of which all the filters of B.S. Misc. Pub. No. 114 were designed. A similar filter has also been prepared on the basis of the 1931 I.C.I. standard observer; a chart giving the composition and characteristics of this filter is given in figure 2. The two energy distributions are shown together for comparison in figure 3.

The colorimetric differences resulting from these two distributions are shown in table 1. It is apparent from these data that the filter designed on the I.C.I. basis is slightly yellowish green relative to the filter designed on the O.S.A. basis when both are combined with 2848 K. The chromaticity difference is readily perceptible in a good photometric field but is hardly perceptible when the two filters are merely viewed side by side. The difference is equivalent to approximately two thirds of a Jones hue step, to approximately 10 microreciprocal-degrees, or to a Lovibond yellow glass of value N-0.16. It

is thus very small, but might not be negligible for all purposes. Differences of the same order of magnitude would be expected between the filters of series II of B.S. Misc. Pub. No. 114 and similar filters designed on the basis of the I.C.I. observer. Much smaller differences would be found in a similar comparison for the filters of series I of that publication because of the notably better agreement between the ideal and actual spectral energy distributions.

TABLE 1.—*Colorimetric characteristics of equal-energy filters*

Colorimetric quantity	Ideal equal-energy stimulus	Illuminant 2848 K combined with—		Differences O.S.A. minus I.C.I.
		O.S.A. filter	I.C.I. filter	
O.S.A. coordinates: ¹				
<i>r</i> -----	0.3142	0.3142	0.3206	-0.0064
<i>g</i> -----	.3062	.3062	.3125	-.0063
<i>b</i> -----	.3796	.3796	.3669	+.0127
I.C.I. coordinates: ²				
<i>x</i> -----	.3333	.3301	.3333	-.0032
<i>y</i> -----	.3333	.3264	.3333	-.0069
<i>z</i> -----	.3333	.3435	.3333	+.0102
Correlated color temperature ³ , θ_i -----		5356 K	5271 K	+85 K
Color departure, ³ σ/δ -----		10.1	5.8	
Conjunctive wavelength ³ , λ_e , equal energy to black body-----		565.7 m μ	566.1 m μ	
Conjunctive wavelength ³ , O.S.A. to I.C.I.-----		572.0 m μ		

¹ Values of *r*, *g*, *b* computed on basis of O.S.A. excitation curves with Davis-Gibson mean sun as basic stimulus; see reference 5, figure 12.

² Values of *x*, *y*, *z* computed on basis of 1931 I.C.I. standard observer and coordinate system with equal energy as basic stimulus; see reference 4, table 3.

³ Definitions of these terms and description of the methods of computation are given in a paper by Raymond Davis, A Correlated Color Temperature for Illuminants, B.S. Jour. Research, vol. 7 (RP365), pp. 659-681, 1931. It should be mentioned that the I.C.I. data on the *r*, *y* and *z* coordinate system cannot be used to compute correlated color temperature, color departure and conjunctive wave length by the method described in Research Paper No. 365.

WASHINGTON, December 11, 1933.

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