A NEW SEISMOMETER EQUIPPED FOR ELECTROMAGNETIC DAMPING AND ELECTROMAGNETIC AND OPTICAL MAGNIFICATION (THEORY, GENERAL DESIGN, AND PRELIMINARY RESULTS)

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ABSTRACT

An elementary discussion is given of the principles involved in the functioning of seismometers equipped for electromagnetic damping and electromagnetic and optical magnification. These principles are then used in the development of an equation giving the relation between angular displacement of the winding of the galvanometer and the linear displacement of the ground. This equation, which is a linear differential equation of the fourth order, contains terms representing the reaction of the galvanometer upon the motion of the steady mass of the seismometer, a point which has previously been neglected and which is of importance unless the seismometer is made much larger than that for which there is any real need. A solution of this equation is given for the case of a sustained harmonic displacement of the ground. Also the corresponding equation and solution are given for the case of tilting of a horizontal-component instrument.

A procedure which may be followed in the development of a general design for a seismometer, to give approximately a specified performance, is illustrated by a concrete example. In this design the steady mass is about 500 g; the arrangement is such that the period may be determined and the damping adjusted from the recording station, which may be at a distance from the seismometer; and the magnification and its variation with the period of earth displacements, in the range from 2.5 to 60 seconds, is substantially the same as is given by a seismometer of the ordinary type having a magnification for short-period displacements of 1,250, a period of 12.5 seconds, and critical damping.

Preliminary results obtained with an experimental seismometer constructed substantially in accordance with this design are given, and a photograph of the instrument is shown.

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I. INTRODUCTION

This paper presents some results of work on the development of seismometers, which is a part of an investigation initiated by the Carnegie Institution of Washington, and in which among others the Bureau of Standards and the Coast and Geodetic Survey are cooperating. Other work on the development of seismometers, which constitutes a part of the same investigation, is described in a paper ¹ by Anderson and Wood. That paper described a new type of seismometer of high sensitivity employing only optical magnification and intended primarily for the registration of near-by earthquakes. This paper pertains to the development of seismometers of high sensitivity employing electromagnetic as well as optical magnification, sometimes referred to as galvanometer registration, and intended primarily for the registration of distant earthquakes.

The application of electromagnetic magnification to seismometry began to receive serious consideration about 20 years ago. In 1907 Galitzin published a paper ² on the electromagnetic registration method. However, he referred to this method in an earlier paper ³ in which he gave results obtained with an experimental equipment on a shaking table. In 1907 Goldschmidt ⁴ described a plan for a seismometer using electromagnetic magnification in conjunction with mechanical magnification by means of a pointer galvanometer for producing the record on a smoked paper. As no further reference to this plan has been

¹ Anderson and Wood, *Bull. Seis. Soc. Am.*, **15**, p. 1; 1925.

² Galitzin, *Comp. Rend. d. sea. d. l. Comm. Sism. Per.*, **3**, pt. 1; 1907.

³ Galitzin, *Comp. Rend. d. sea. d. l. Comm. Sism. Per.*, **1**, pt. 3; 1904.

⁴ *Bull. Mens. Soc. Bel. D'Elec.*, **24**, p. 605; 1907.

found, presumably no instrument was constructed in accordance with it, or, if so, its performance was found to be unsatisfactory.

In 1909 Grunmach⁵ described a seismometer, using electromagnetic magnification in conjunction with optical magnification and photographic registration, which was designed for and used in the study of vibration caused by operating machinery. This seismometer was used with a galvanometer of very short period, and therefore the characteristics of the system were such that it would give but a slight response to displacements having periods longer than one second. In other respects the equipment was not suitable for use in the usual teleseismic observations. Galitzin published a description of his vertical-component instrument⁶ in 1910 and of his horizontal-component instrument⁷ in 1911. He also published several other papers⁸ and gave a series of lectures.⁹ These papers and lectures are devoted mainly to the application of electromagnetic magnification to seismometry. He not only developed and used seismometers employing electromagnetic and optical magnification, electromagnetic damping, and photographic registration, but has given a fairly complete theory applicable to the instruments which he developed.

However, he did not take into consideration the reaction of the galvanometer upon the seismometer, except in an indirect and very incomplete way. He recommends that the period of the seismometer be adjusted to coincidence with that of the galvanometer and that the damping of the seismometer be adjusted with the galvanometer connected. With the rather large steady mass (5 kg or more) which he used, the galvanometer could affect the motion of the steady mass of the seismometer only to a nearly negligible extent. With the much smaller steady mass (1 kg or less) which I propose to use, it is necessary that the various reactions of the galvanometer upon the seismometer be taken definitely into consideration. Therefore, a theory more general than that given by Galitzin will be developed. In addition, the principles involved in electromagnetic damping and electromagnetic magnification will be discussed in an elementary way. A general design for a seismometer differing radically from any previously described will be developed and some idea given as to the performance to be expected from an instrument constructed in accordance with this design. Finally, a photograph of an experimental seismometer and parts of records obtained with it will be shown.

That this presentation of the subject may not be unnecessarily complicated no attempt will be made to state relations with extreme

⁵ Grunmach, *Phys. Zs.*, **10**, p. 853; 1909.

⁶ Galitzin, *Comp. Rend. d. sea. d. l. Comm. Sism. Per.*, **4**, pt. 2; 1910.

⁷ Galitzin, *Comp. Rend. d. sea. d. l. Comm. Sism. Per.*, **4**, pt. 1; 1911.

⁸ Galitzin, *Comp. Rend. d. sea. d. l. Comm. Sism. Per.*, **1** to **4**; 1902-1911. *Comp. Rend. d. sea. d. l. Acad. d. Sci.*, Paris, **150**, pp. 642, 816, 901, 1041, 1727; 1910. *Bull. d. l. Acad. Imp. d. Sci. d. St. Petersburg*; 1909 and 1910.

⁹ *Vorlesungen über Seismometrie*, published by B. A. Teubner; 1914.

exactness. For example, electric currents will be assumed to be in phase with the electromotive forces which cause them though the circuits are inductive, moments or torques caused by air friction will be assumed to be proportional to the relative velocity of parts of the apparatus, static friction and hysteresis will be assumed to be negligible, and the torque required to hold the steady mass in a displaced position will be assumed to be proportional to the displacement. Further, attention will be given mainly to the electrical, the electromagnetic, and electrodynamic relations, since seismologists generally may not be so familiar with these as with the mechanical and optical relations.

II. FUNDAMENTAL RELATIONS

Before proceeding to set up an equation giving the relation between the angular displacement of the coil of the galvanometer and the linear displacement of the ground, it will be to our advantage to refer to or establish a number of relations which, except for limitations of the type just referred to, may be considered as exact. It will also be to our advantage to refer to or establish other relations which must be considered as approximations or as applicable only in limiting or ideal cases. Both the more exact relations and the approximate relations will, in general, be stated in the form of equations or mathematical expressions, but to distinguish the one from the other the former will be designated by numbers and the latter by letters. It may be of assistance to the reader to know the need for both exact relations and approximate relations. The exact relations lead to expressions which are too complicated to be used in the development of a general design for a seismometer, and the approximate relations can not be relied upon to give definite information as to the performance to be expected from an equipment having known structural constants.

The symbols used will be explained as they are introduced and in connection with the more important equations. Most of them will also be explained in an appendix.

1. ELECTRODYNAMIC RELATIONS

The mechanical force acting between a conductor and a magnetic field when there is an electric current in the conductor is given by the equation

$$F = Hli \quad (1)$$

Here F is the mechanical force in a direction perpendicular to both the direction of the axis of the conductor and to the magnetic field, H is the component of the magnetic field perpendicular to the axis of the conductor, l is the length of that part of the conductor which is within the magnetic field, and i is the current in the conductor. (See

fig. 1) From equation (1) it follows that the moment or torque acting between a coil of insulated wire mounted, as shown in Figure 2, and the magnet is given by the equation

$$T = AHni \quad (2)$$

Here T is the torque, A is the area of the winding, H is the component of the magnetic field in the plane of the coil and perpendicular to the axis of rotation, and n is the number of turns of the coil. The product AHn will be referred to as the electrodynamic constant and designated as g . Therefore

$$T = gi \quad (3)$$

2. ELECTROMAGNETIC RELATIONS

If the conductor shown in Figure 1 is permitted to move perpendicular to its axis and to the magnetic field, either as the result of the

FIGURE 1.—Electrodynamic interaction between a straight conductor and a magnetic field

FIGURE 2.—Electrodynamic interaction between a coiled conductor and a magnetic field

force produced by the current or as a result of an applied mechanical force, there is developed in the conductor an electromotive force

$$e_i = -Hl \frac{\delta x}{\delta t} \quad (4)$$

Here $\delta x / \delta t$ is the rate of displacement of the conductor with respect to the magnet. If the conductor moves as a result of the current, the direction of this electromotive force is opposite to the electromotive force which causes the current, so it is sometimes referred to as a "back" electromotive force.

For the arrangement shown in Figure 2, it follows that

$$e_i = -g \frac{\delta \theta}{\delta t} \quad (5)$$

where $\delta\theta/\delta t$ is the rate of angular displacement of the coil. As the current, i , in the coil is equal to the total electromotive force in the circuit divided by the resistance,

$$i = (e + e_i)/r \quad (6)$$

Here e is the electromotive force of the battery or other source of electric power, e_i is the electromotive force developed within the coil, and r is the resistance of the circuit. From equations (3), (5), and (6) it follows that

$$T = \frac{ge}{r} - \frac{g^2\delta\theta}{r\delta t} \quad (7)$$

It should be understood that here T is only the torque resulting from the current in the winding and that an additional torque may be applied mechanically.

3. ELECTROMAGNETIC DAMPING

Equation (7) shows that the torque resulting from the current in the winding may be considered as composed of two parts, one of which is proportional to the rate of the angular displacement. Consequently, whether the coil rotates as a result of a current supplied by a battery or other source of electromotive force, or as a result of an applied mechanical torque or other cause, it experiences a torque as the result of the electromagnetic action caused by the motion equal to

$$-\frac{g^2\delta\theta}{r\delta t}$$

The negative sign here indicates that this torque is in opposition to the motion or tends to retard the motion. As the ratio of the retarding torque to the rate of angular displacement is the damping constant, it follows that that part of the damping constant resulting from electromagnetic action in the insulated electric circuit is equal to g^2/r . Other parts of the damping constant may be contributed by air friction, electric currents induced in the frame on which the coil is wound, as in some galvanometers, in a separate copper plate associated with another magnet such as is used in some seismometers, etc.

4. ELECTROMAGNETIC MAGNIFICATION

If the electric circuit includes two insulated windings each so arranged that it is free to move in the field of a magnet as shown in Figure 3, the motion of either develops an electromotive force in the circuit. The resulting current not only causes a torque tending to retard the motion of the winding which is caused to move as the result of an applied mechanical torque, but also a torque tending to displace the other winding. Therefore if a torque is applied to either

winding, both are rotated, and consequently both develop an electromotive force. However, these two electromotive forces are not in the same direction but tend to cause currents in opposite directions through the electric circuit.

It will be convenient to think of the winding on the left as attached to the steady mass of the seismometer in such a way as to rotate about the axis of suspension, and of the magnet as being attached to the support of the seismometer; and of the winding on the right as located in the galvanometer and as having a mirror attached to it for indicating or recording photographically its angular displacements. If a torque is applied to the winding on the left—that is, to the moving system of the seismometer—and the resulting angular displacement

FIGURE 3.—Two coiled conductors connected in series, each associated with a magnetic field

One of these coiled conductors may be thought of as attached to the steady mass of a seismometer and the other as being in the galvanometer and having a mirror attached for recording its motions photographically

of each winding is considered as positive, the total electromotive force, e_t , in the circuit is given by the equation

$$e_t = G \frac{\delta \phi}{\delta t} - g \frac{\delta \theta}{\delta t} \quad (8)$$

Here G and ϕ are the electrodynamic constant and the angular displacement of the winding of the seismometer with respect to its support, and g and θ are the electrodynamic constant and angular displacement of the winding of the galvanometer. For the arrangement shown in Figure 3,

G = the area times the number of turns of the winding, times the field strength of the magnet.

For the arrangement which is to be used in a seismometer, the winding will be circular and located at a distance from the axis of rotation,

and the magnetic field will be radial toward the axis of the winding so that

G = the circumference times the number of turns of the winding times the field strength of the magnet times the distance from the center of the winding to the axis of rotation.

The current i in the circuit is the total electromotive force $G \frac{\delta\phi}{\delta t} - g \frac{\delta\theta}{\delta t}$ divided by the resistance of the circuit $R + r$, or

$$i = \left(G \frac{\delta\phi}{\delta t} - g \frac{\delta\theta}{\delta t} \right) / (R + r) \quad (9)$$

Therefore the torque or moment tending to displace the winding of the galvanometer is

$$T = gi = g \left(G \frac{\delta\phi}{\delta t} - g \frac{\delta\theta}{\delta t} \right) / (R + r) \quad (10)$$

This torque is equal to the sum of the opposing torques brought into action by the displacement of the winding. These consist of inertia reaction, air friction, and other forms of damping exclusive of damping in the electrical circuit, and the elastic action of the suspension. If the winding is light (that is, has a small moment of inertia), if the motion is but slightly retarded by mechanical damping and there is no short-circuited turn to cause electromagnetic damping, and if there is but slight constraint by the suspensions, the torque required to produce the motion $\delta\theta/\delta t$ is small. If, in addition, both G and g are fairly large and $R + r$ is small, experience shows that, for conditions such as are met in teleseismic observations, $G \frac{\delta\phi}{\delta t} - g \frac{\delta\theta}{\delta t}$ is very small in comparison with either $G \frac{\delta\phi}{\delta t}$ or $g \frac{\delta\theta}{\delta t}$, or that

$$G \frac{\delta\phi}{\delta t} = g \frac{\delta\theta}{\delta t} \text{ approximately} \quad (A)$$

From this it follows by integration that

$$\frac{\theta}{\phi} = \frac{G}{g} \text{ approximately} \quad (B)$$

To the extent that these conditions are realized, the angular displacement of the winding of the galvanometer will be proportional to, and in phase with, the displacement of the steady mass with respect to its support, and the electromagnetic magnification will be equal to

$$\frac{G}{g} \quad (C)$$

A galvanometer giving an angular deflection at all times very nearly proportional to the angular displacement of the steady mass of the seismometer will be considered as ideal. The motion of the winding of such a galvanometer would be that necessary to develop a back electromotive force at every instant very nearly equal to the potential difference applied at its terminals or the electromotive force developed in the seismometer winding in case of a direct connection, as shown in Figure 3. Consequently, the current through the winding would be extremely small. For the galvanometer to give an exact duplication on a magnified scale of the motion of the seismometer winding with respect to its support, it would be necessary for the galvanometer to develop a back electromotive force exactly equal to the potential difference applied across its terminals. This in no case can be fully realized, and if it could would not be desirable, since the performance would then be the same as with mechanical and optical magnification. It is assumed that what may be considered ideal is that the galvanometer give a fairly accurate magnified reproduction of the displacement of the steady mass with respect to its support for frequencies of importance in teleseismic observations, but not follow the extremely short-period disturbances resulting from traffic, moving machinery, etc., nor the long-period disturbances resulting from temperature changes, etc.

The actions and reactions taking place in electromagnetic magnification are of such importance in connection with the theory and especially in connection with the design of seismometers of the type under consideration that it may be well to repeat a part of what has just been said in a somewhat different way. If each of the two windings is arranged to rotate about an axis parallel to its plane and passing through its geometric centers perpendicular to the field of its magnet, as shown in Figure 3, any rotation of either winding causes a change in the magnetic flux through it or a change in the magnetic flux linking the circuit, and the other winding tends to move in such a way as to keep the flux linking the circuit constant.

The change in the flux linking the circuit, assuming that only one of the windings is rotated, would be equal to its area, times the number of turns, times the field strength, times the angular displacement. Therefore the other winding tends to turn in the opposite direction by an amount such as to make its area times the number of its turns, times the field strength of its magnet, times its angular displacement, equal to that of the first. This is not only the action that tends to take place but actually would take place if the second winding had no moment of inertia, no damping, and no spring or other device tending to hold or bring it to a definite position. It is also the action which would take place if the electrical resistance of the circuit could be made zero or negligibly small. Further, if the

product of the area and number of turns of each of the windings times the field strength of each of the magnets is large, the inertia, damping, and restraint of each of the windings are small; and the resistance of the circuit is small, the relative motions of the two windings would be substantially the same as though they were mechanically connected in such a manner that any rotation of either would be duplicated by the other in reversed direction except that the magnitudes of the rotations would be in the inverse ratio of their electrodynamic constants. For example, if the two windings had equal areas and moved in equal magnetic fields but had turns in the ratio of 2 to 1, the one having the smaller number of turns would move through twice the angle moved through by the other.

In practice it is not possible to construct a galvanometer having a negligible moment of inertia, negligible damping and negligible restraint by the suspensions, a negligible resistance, or an extremely large electrodynamic constant. However, it is possible to construct a galvanometer having such relations between its structural constants that for the purpose of design it may be assumed that the angular displacement of its winding will be a magnified duplication of the angular displacement of the steady mass of the seismometer with respect to its support.

5. ELECTROMAGNETIC DAMPING WITH DIRECT CONNECTION BETWEEN THE SEISMOMETER AND GALVANOMETER

In the preceding section it was pointed out that the current in the circuit is equal to

$$\left(G \frac{\delta \phi}{\delta t} - g \frac{\delta \theta}{\delta t}\right) / (R + r)$$

If this current is brought about by the application of a mechanical torque tending to displace the winding of the seismometer with respect to its magnet, the resulting electrodynamic action is not only a torque

$$g \left(G \frac{\delta \phi}{\delta t} - g \frac{\delta \theta}{\delta t}\right) / (R + r)$$

tending to displace the winding of the galvanometer, but a torque

$$G \left(G \frac{\delta \phi}{\delta t} - g \frac{\delta \theta}{\delta t}\right) / (R + r) \quad (11)$$

tending to retard or damp the motion of the winding of the seismometer. However, it has just been assumed that the winding of the galvanometer should move in such a way as to develop a back electromotive force very nearly equal to the potential difference impressed across its terminals, or in this case that $G \frac{\delta \phi}{\delta t}$ and $g \frac{\delta \theta}{\delta t}$ should

be very nearly equal. Therefore the current must be very small, and consequently it can cause only a slight retardation of the motion of the winding of the seismometer. Thus, it will be seen that the desired performance of the galvanometer has the effect of practically destroying electromagnetic damping in the system. In fact, a satisfactory performance of both galvanometer and seismometer can not be obtained if the only provision for damping is in a simple series circuit, such as that showing in Figure 3.

The necessary damping may be obtained electromagnetically by the use of either of two fairly simple means. One of these means consists in the use of two circuits in the seismometer—one, which may consist simply of a copper plate, for producing the desired damping and the other for connection to the galvanometer to produce the desired electromagnetic magnification. The other means consists of the use of a bridge or shunt across the circuit as shown in Figure 4.

FIGURE 4.—*Bridge circuit used with the seismometer, the design of which is to be developed later*

Here R represents the resistance of the seismometer winding and the leads connecting it to the recording station; r represents the resistance of the galvanometer winding, suspensions, and leads; S represents the resistance of the bridge or shunt, the adjustment of which serves to bring about an optimum damping of the system

With this arrangement the electromotive forces induced in the windings tend, in general, to cause currents in opposite directions in them but in the same direction through the bridge. Therefore whenever there is a motion of the windings there will be a current in one or both, and consequently electromagnetic damping of the system. By a suitable choice of the resistance of the bridge, provided the apparatus is properly designed, this damping may be adjusted to an optimum value. The former of these means is used in the apparatus designed by Galitzin and similar apparatus designed by Wilip, but as compared with it the latter has a number of advantages, especially if the steady mass of the seismometer is made small. For this reason it will be presumed that the latter will be used though the analysis which will be made is applicable in either case.

6. ELECTROMAGNETIC DAMPING WITH BRIDGED CIRCUIT AND IDEAL GALVANOMETER

With an ideal galvanometer—that is, a galvanometer developing at every instant an electromotive force very nearly equal to but in opposition to the potential difference impressed across its terminals—there is but little current through its winding. Consequently the current through the seismometer winding is

$$G \frac{\delta\phi}{\delta t} / (R + S) \text{ approximately} \quad (D)$$

Here S is the resistance of the bridge. From this and equation (3) it follows that the retarding torque is

$$G^2 \frac{\delta\phi}{\delta t} / (R + S) \text{ approximately} \quad (E)$$

and that

$$G^2 / (R + S)$$

is approximately the contribution of the circuit to the damping constant of the seismometer. Since it is assumed that the motion of the galvanometer follows, at least approximately, that of the seismometer, $G^2 / (R + S)$ may be considered as the contribution of the circuit to the damping constant of the system.

7. ELECTROMAGNETIC MAGNIFICATION WITH BRIDGED CIRCUIT AND IDEAL GALVANOMETER

With an ideal galvanometer the potential difference across its terminals would be approximately equal to the current in the seismometer times the resistance of the bridge, or

$$G \frac{\delta\phi}{\delta t} \frac{S}{R + S}$$

and this is approximately equal to the electromotive force developed in the winding of the galvanometer or

$$g \frac{\delta\theta}{\delta t}$$

that is

$$G \frac{\delta\phi}{\delta t} \frac{S}{R + S} = g \frac{\delta\theta}{\delta t} \text{ approximately} \quad (F)$$

from which it follows that the electromagnetic magnification is

$$\frac{G}{g} \frac{S}{R + S} \text{ approximately} \quad (G)$$

8. DISTRIBUTION OF CURRENTS IN BRIDGED CIRCUIT

In a part of the above discussion it was assumed that the galvanometer would give an ideal performance. However, any galvanometer which might be used in connection with a seismometer must be expected to give a performance differing from this, at least slightly slightly under the most favorable conditions and greatly if the period of the disturbance is very short or very long. Consequently it is necessary that the relation between the currents and the electromotive forces be established without reference to the performance of the galvanometer.

The current in the seismometer winding which would result from an electromotive force developed only in its winding would be

$$G \frac{\delta \phi}{\delta t} / [R + rS / (r + S)] \quad (12)$$

and the fractional part of this passing through the galvanometer would be $S / (r + S)$; that is, if the winding of the galvanometer were clamped so as to prevent its motion while the winding of the seismometer were moved, the current in the seismometer winding would be

$$G(r + S) \frac{\delta \phi}{\delta t} / Q^2 \quad (13)$$

and the current in the galvanometer winding would be

$$GS \frac{\delta \phi}{\delta t} / Q^2 \quad (14)$$

where

$$Q^2 = rR + rS + RS \quad (15)$$

Likewise the current in the galvanometer winding resulting only from an electromotive force in its winding would be

$$-g \frac{\delta \theta}{\delta t} / [r + SR / (S + R)] \quad (16)$$

and the fractional part of this current passing through the seismometer winding would be $S / (R + S)$; that is, if the winding of the seismometer were clamped so as to prevent its motion while the winding of the galvanometer were moved in the direction in which it would move as a result of a positive displacement of the winding of the seismometer, the current in the galvanometer winding would be

$$-g(R + S) \frac{\delta \theta}{\delta t} / Q^2 \quad (17)$$

and the current in the seismometer winding would be

$$-gS \frac{\delta \theta}{\delta t} / Q^2 \quad (18)$$

However, as the apparatus is used both windings move and consequently both develop electromotive forces. Therefore if I is the current in the seismometer winding resulting from the electromotive forces developed in both windings, it follows from equations (13) and (18) that

$$I = G(r + S) \frac{\delta\phi}{\delta t} / Q^2 - gS \frac{\delta\theta}{\delta t} / Q^2 \quad (19)$$

and if i is the current in the galvanometer winding resulting from the electromotive forces developed in both windings, it follows from equations (14) and (17) that

$$i = GS \frac{\delta\phi}{\delta t} / Q^2 - g(R + S) \frac{\delta\theta}{\delta t} / Q^2 \quad (20)$$

Equations (19) and (20) will be used in the next section in establishing the relation between displacements of the support of a seismometer or the ground, and displacements of the winding of a galvanometer when a seismometer and galvanometer are connected as shown in Figure 4. Other parts of what has preceded will be used later in the development of a general design for a seismometer.

III. EQUATIONS OF MOTION

Having considered some of the electrical, electromagnetic, and electrodynamic relations, it is in order to set up the equation of motion of the steady mass of the seismometer, the equation of motion of the coil of the galvanometer, and the general seismometer equation; that is, the equation giving the relation between the motion of the coil of the galvanometer and the displacement of the support of the seismometer.

1. EQUATION OF MOTION OF STEADY MASS OF SEISMOMETER

With no externally applied moments or torques, no motion of the support of the seismometer, and with the electric circuit open, the equation of motion of the seismometer may be written as follows:

$$K \frac{\delta^2\phi}{\delta t^2} + D \frac{\delta\phi}{\delta t} + U\phi = 0 \quad (21)$$

Here K is the moment of inertia of the steady mass about the axis of rotation.

D is the damping constant or the ratio of the retarding torque to the angular velocity. It includes all forms of damping present when the electric circuit is open.

U is the restoring constant or ratio of the restoring torque to the angular displacement.

ϕ is the angular displacement; and

t is the time.

This is the equation of motion of a freely swinging pendulum for small amplitudes. It is developed in textbooks on physics, mechanics, and seismometry.

If the seismometer is moved linearly in a direction perpendicular both to the axis of rotation and to a line passing through the axis of rotation and the center of mass, and if the angular displacement ϕ resulting from this displacement is taken as positive, to prevent an angular displacement it would be necessary to apply a torque $-LM \frac{\delta^2 X}{\delta t^2}$. Here

L is the distance from the axis of rotation to the center of "steady" mass.

M is the mass of the moving system.

X is the displacement of the support.

Consequently the torque resulting from the displacement or rather the acceleration of the support of the seismometer is

$$LM \frac{\delta^2 X}{\delta t^2} \quad (22)$$

If, in addition, the circuit is closed so that a current may flow as a result of the electromotive forces induced in the winding of the seismometer and the connected galvanometer, there is an additional torque supplied through the winding of the seismometer. This torque is equal to

$$-GI \quad (23)$$

where G is the electrodynamic constant of the seismometer and I is the current in the seismometer winding.

If, therefore, there is a motion of the support of the seismometer and the resulting motion of the steady mass is permitted to cause a current in the winding, the equation of motion of the steady mass with respect to its support is

$$K \frac{\delta^2 \phi}{\delta t^2} + D \frac{\delta \phi}{\delta t} + U\phi = LM \frac{\delta^2 X}{\delta t^2} - GI \quad (24)$$

2. EQUATION OF MOTION OF THE COIL OF THE GALVANOMETER

The equation of motion of the coil of the galvanometer may be written as follows:

$$k \frac{\delta^2 \theta}{\delta t^2} + d \frac{\delta \theta}{\delta t} + u\theta = gi \quad (25)$$

Here

k is the moment of inertia of the moving system,

d is the damping constant with the circuit open,

u is the restoring constant,

g is the electrodynamic constant,

θ is the angular displacement,

t is the time, and

i is the current through the winding.

It may be noted that in equations (24) and (25) the terms GI and gi have opposite signs. The difference in sign results from the assumption that the relative motion of the winding and support of the seismometer is the primary cause for the current in the seismometer winding and that the current in the galvanometer winding is the primary cause for its motion with respect to its support. The other difference between these two equations results from the assumption that the axis of rotation of the moving system of the galvanometer passes through its center of mass so that a linear acceleration of the support of the galvanometer has no tendency to cause an angular displacement.

3. GENERAL SEISMOMETER EQUATION

With a bridged circuit such as is shown in Figure 4, expressions for I and i are given by equations (19) and (20). Substituting these in equations (24) and (25) gives

$$K \frac{\delta^2 \phi}{\delta t^2} + [D + G^2(r + S)/Q^2] \frac{\delta \phi}{\delta t} + U\phi = LM \frac{\delta^2 X}{\delta t^2} + \frac{GgS}{Q^2} \frac{\delta \theta}{\delta t} \quad (26)$$

and

$$k \frac{\delta^2 \theta}{\delta t^2} + [d + g^2(R + S)/Q^2] \frac{\delta \theta}{\delta t} + u\theta = \frac{GgS}{Q^2} \frac{\delta \phi}{\delta t} \quad (27)$$

It may be noted that each of these equations contains a term representing electromagnetic damping in the circuit, that the former contains a term representing the effect of the motion of the galvanometer winding upon the relative motion of the steady mass and support of the seismometer, and that the latter contains a term representing the effect of the relative motion of the steady mass and its support upon the motion of the galvanometer winding.

With seismometers of the type under consideration the user will wish to know the ratio of the displacements as shown on the record, which will be proportional to the angular displacements of the winding of the galvanometer, to displacements of the ground, but need not be concerned with the relative motion of the steady mass with respect to its support. Consequently equations (26) and (27) may be combined in such a way as to eliminate ϕ , $\frac{\delta \phi}{\delta t}$ and $\frac{\delta^2 \phi}{\delta t^2}$. To do this it is only necessary to differentiate the equations and make the proper substitutions.

Differentiating equation (26) gives

$$K \frac{\delta^3 \phi}{\delta t^3} + [D + G^2(r + S)/Q^2] \frac{\delta^2 \phi}{\delta t^2} + U \frac{\delta \phi}{\delta t} = LM \frac{\delta^3 X}{\delta t^3} + \frac{GgS}{Q^2} \frac{\delta^2 \theta}{\delta t^2} \quad (28)$$

Differentiating equation (27) gives

$$k \frac{\delta^3 \theta}{\delta t^3} + [d + g^2(r + S)/Q^2] \frac{\delta^2 \theta}{\delta t^2} + u \frac{\delta \theta}{\delta t} = \frac{GgS}{Q^2} \frac{\delta^2 \phi}{\delta t^2} \quad (29)$$

Differentiating equation (29) or equation (27) a second time gives

$$k \frac{\delta^4 \theta}{\delta t^4} + [d + g^2(R + S)/Q^2] \frac{\delta^3 \theta}{\delta t^3} + u \frac{\delta^2 \theta}{\delta t^2} - \frac{GgS}{Q^2} \frac{\delta^3 \phi}{\delta t^3} \quad (30)$$

Equations (27), (29), and (30) contain expressions for $\frac{\delta \phi}{\delta t}$, $\frac{\delta^2 \phi}{\delta t^2}$, $\frac{\delta^3 \phi}{\delta t^3}$, which when substituted into equation (28) give

$$\frac{\delta^4 \theta}{\delta t^4} + m \frac{\delta^3 \theta}{\delta t^3} + n \frac{\delta^2 \theta}{\delta t^2} + o \frac{\delta \theta}{\delta t} + p\theta = q \frac{\delta^3 X}{\delta t^3} \quad (31)$$

and this will be referred to as the general seismometer equation. Here

$$\begin{aligned} m &= \frac{D}{K} + \frac{G^2(r+S)}{KQ^2} + \frac{d}{k} + \frac{g^2(R+S)}{kQ^2} \\ n &= \frac{Dd}{Kk} + \frac{u}{k} + \frac{U}{K} + \frac{dG^2(r+S) + Dg^2(R+S)}{kKQ^2} \\ o &= \frac{dU + Du}{Kk} + \frac{Ug^2(R+S) + uG^2(r+S)}{kKQ^2} \\ p &= \frac{Uu}{Kk} \\ q &= \frac{LMgGS}{KkQ^2} \end{aligned} \quad (32)$$

The general seismometer equation (31) is a fourth-order linear differential equation with constant coefficients. It is much more complicated than the second-order linear differential equation with constant coefficients, which is usually given for the relation between the relative displacement of the steady mass and the support of the seismometer when the latter is subjected to accelerations. Therefore, no attempt will be made to get a solution of it except for the case of a sustained harmonic displacement of the support; that is, of the ground. This, however, will suffice to show how the magnification and phase angle depends upon the constants of the seismometer, the constants of the galvanometer, the resistance of the bridge, and the frequency or period of the disturbance.

IV. A PARTICULAR SOLUTION OF THE GENERAL SEISMOMETER EQUATION

In deriving a particular solution of the general seismometer equation the complex number or vector notation will be used; that is, it will be assumed that the displacement X is the real part of the expression

$$\mathfrak{X} \cos(\omega t + \alpha) - j\mathfrak{X} \sin(\omega t + \alpha) \quad (33)$$

This may be written

$$\begin{aligned} X &= \Re[(\epsilon^{j(\omega t + \alpha)} + \epsilon^{-j(\omega t + \alpha)})/2 + (\epsilon^{j(\omega t + \alpha)} - \epsilon^{-j(\omega t + \alpha)})/2j] \\ &= \Re \epsilon^{j\alpha} \epsilon^{j\omega t} \\ &= X \epsilon^{j\omega t} \text{ (real part only).} \end{aligned} \quad (34)$$

Here $\omega = 2\pi$ times the frequency or 2π divided by the period of the earth displacement.

t = the time from any arbitrary starting point.

α = the phase angle and depends upon the starting point of t .

$j = \sqrt{-1}$.

ϵ = the base of the Napierian logarithm.

X = a complex quantity equal in magnitude to the amplitude of the earth displacement and containing the phase angle α .

Since X is a harmonic function of time, derivatives of X , and the angular displacement of the galvanometer winding and its derivatives, are harmonic functions of time and all have the same frequency or period. Consequently the angular deflection of the galvanometer winding is given by the equation

$$\theta = \Theta \epsilon^{j\omega t} \text{ (real part only)} \quad (35)$$

where Θ is a complex quantity equal in magnitude to the amplitude of the deflection of the galvanometer winding and containing a phase angle β , differing in general from the phase angle α . Successive differentiation of equation (35) gives

$$\begin{aligned} \frac{\delta \theta}{\delta t} &= j\omega \Theta \epsilon^{j\omega t} \\ \frac{\delta^2 \theta}{\delta t^2} &= -\omega^2 \Theta \epsilon^{j\omega t} \\ \frac{\delta^3 \theta}{\delta t^3} &= -j\omega^3 \Theta \epsilon^{j\omega t} \\ \frac{\delta^4 \theta}{\delta t^4} &= \omega^4 \Theta \epsilon^{j\omega t} \end{aligned} \quad (36)$$

Differentiation of equation (34) three times gives

$$\frac{\delta^3 X}{\delta t^3} = -j\omega^3 X \epsilon^{j\omega t} \quad (37)$$

Substituting these expressions for Θ and its derivatives and the derivative of X in equation (31) gives

$$[\omega^4 - j\omega^3 m - \omega^2 n + j\omega O + p] \Theta \epsilon^{j\omega t} = -j\omega^3 q X \epsilon^{j\omega t}$$

or

$$\Theta/X = -j\omega^3 q/[\omega^4 - j\omega^3 m - \omega^2 n + j\omega o + p] \quad (38)$$

The amplitude of displacements as shown on the record is equal to Θl where l is the effective length of the galvanometer pointer. Since the record is to be produced photographically by means of a light beam reflected from a mirror attached to the galvanometer winding, usually l is equal, at least approximately, to twice the distance between the mirror and the recording paper. Consequently the magnification M_x —that is, the ratio of the displacements shown on the record to the displacements of the ground—is equal to $\Theta l/X$. Therefore

$$M_x = -j\omega^3 ql/[\omega^4 - j\omega^3 m - \omega^2 n + j\omega o + p]$$

Replacing m , n , O , p , and q by their equivalents, as given by equation (32), gives

$$M_x = \frac{MLGgSl}{(Dk + dK)Q^2 + G^2k(r+S) + g^2K(R+S) - [(Du + dU)Q^2 + G^2u(r+S) + g^2U(R+S)]/\omega^2 + j\omega kKQ^2 - j[(Dd + uK + Uk)Q^2 + G^2g^2 + G^2d(r+S) + g^2D(R+S)]/\omega + juUQ^2/\omega^3} \quad (39)$$

This equation will be used in determining the performance characteristics of a seismometer equipment having structural constants corresponding to those of the general design to be developed later and may be used for this purpose in cases in which the structural constants are determined experimentally. This equation may also be used in determining the performance characteristics of a seismometer equipment of the Galitzin type, providing the structural constants are known or determined experimentally. However, in such cases there may or may not be a bridge or shunt across the terminals of the galvanometer. If no shunt is used, S is infinite, so terms not containing S may be neglected and the equation simplified by dividing both numerator and denominator by S . It should be noted that Q^2 contains S and that in this case

$$Q^2/S = R + r$$

Equation (39) applies only after the displacement has continued for a sufficiently long time for a steady state to be reached and does not give any indication as to how long that is. Therefore the equa-

tion gives directly but little information concerning the relation between the displacement shown on the record and the displacement of the earth just after the beginning or end of a disturbance or the beginning or end of a phase. However, if the apparatus is so designed that the performance of the galvanometer approximates what has been referred to as ideal and the system is properly damped, the effect of transients will be substantially the same as for properly damped seismometers having mechanical or mechanical and optical magnification.

Equation (39) might be rationalized, but to do so would make it more rather than less complicated. Calculations may be made by first substituting numerical values, then summing the real and imaginary (those containing j) terms separately, squaring each, adding and extracting the square root. The ratio of the sum of the imaginary terms to the sum of the real terms multiplied by -1 is the tangent of the angle by which the displacements shown on the record lag in phase behind the displacements of the ground.

V. TILTING OF HORIZONTAL-COMPONENT SEISMOMETER

The discussion given above is applicable to both vertical and horizontal component instruments. With either, the tilting or rotation accompanying seismic disturbance having a distant origin have but slight or negligible effect. However, with horizontal-component instruments the long-period tiltings of local origin, such as changes in the level of the pier on which the instrument is mounted, either as the result of temperature changes, gradual settling, or change in position of large masses near it, have a marked effect. Likewise the change in the stiffness of the spring of a vertical-component instrument as a result of gradual weakening or as a result of temperature changes has a marked effect. It will, therefore, be of interest to see how apparatus of the type under consideration behaves for such disturbances.

If a horizontal-component seismometer is subjected to tilt about a horizontal axis through the center of its steady mass and normal axis of rotation instead of a linear acceleration, the equation of motion of the steady mass with respect to its support has the term $ML \frac{\delta^2 X}{\delta t^2}$ replaced by the term $ML\mathfrak{g}\psi$ where $\mathfrak{g}$ is the acceleration of gravity and ψ is the angle of tilt. Therefore, $LM\mathfrak{g} \frac{\delta\psi}{\delta t}$ may be substituted for $LM \frac{\delta^3 X}{\delta t^3}$ in the general seismometer equation (31). Since for a harmonic disturbance

$$\frac{\delta^3 X}{\delta t^3} = -j\omega^3 X e^{j\omega t} \quad (40)$$

and

$$\frac{\delta\psi}{\delta t} = j\omega \Psi e^{j\omega t}$$

the magnification of tilts, M_ψ , is the same as the magnification of displacements, M_x , except for the factor $-\mathfrak{g}/\omega^2$. The magnification of tilts, therefore, may be obtained by multiplying the right-hand member of equation (39) by $-\mathfrak{g}/\omega^2$. This gives

$$M_\psi = \frac{-ML\mathfrak{g}GS}{\omega^2[(Dk+dK)Q^2+G^2k(r+S)+g^2K(R+S)]} \quad (41)$$

$$- (Du+dU)Q^2+G^2u(r+S)+g^2U(R+S)$$

$$+j\omega^3kKQ^2$$

$$-j\omega[(Dd+uK+Uk)Q^2+G^2g^2+G^2d(r+S)+g^2D(R+S)]$$

$$+juUQ^2/\omega$$

Here the minus sign in the numerator of the right-hand member signifies that 180° is to be subtracted from the phase angle when determined as explained above.

In cases in which the magnification and phase angle of displacements have already been calculated, the magnification and phase angle of tilts may be obtained by multiplying the calculated values by $\mathfrak{g}/\omega^2$ and adding 180° .

As used here, M_ψ is not strictly a magnification, since it is equal to the amplitude of the linear displacement shown on the record divided by the amplitude of the angle of tilt instead of a ratio of two similar quantities.

VI. GENERAL DESIGN

1. STATEMENT OF THE PROBLEM

Equations (39) and (41) show how the magnification and phase angle of earth displacements and earth tilts depend upon the various structural constants of the seismometer and galvanometer, the resistance of the bridge, the distance between the galvanometer mirror and the recording paper, and the period of the disturbance. It might appear, therefore, that simply by a suitable choice of values for these almost any performance desired might be obtained. However, in such apparatus values which may be taken for certain of the constants are more or less definitely limited by values taken for other constants. A case in point is the ratio of the moment of inertia to the restoring constant. If this is made greater than 4, the period is longer than 12.5 seconds, and, in general, it can not be made more than two or three times this without the instrument becoming definitely unstable. Likewise, with the galvanometer, the electrodynamic constant can not be increased indefinitely while both the electrical resistance and moment of inertia of the winding are kept below definite limits. There are many such limitations, so while it would be possible to select a set of values for the various constants to give, for example, the same magnification for displacements of all periods, difficulties would be encountered in either or both the con-

struction and use of the apparatus. Therefore, in deciding what not only should be considered as a good performance but one that may be within the possibility of attainment, we shall be guided mainly by the experience of others.

It seems that except for the shifting of the rest point or general lack of stability, and in some cases the response to short-period disturbances of local origin, the usual type of seismometer having a high static magnification, a very long period, and critical or near-critical damping would give the type of record desired for teleseismic observations. Difficulties in maintaining a suitable stability increase both as the magnification is increased and as the period is increased, so that usually with a magnification of as much as 400 it is not practicable to have the period longer than about 10 seconds. With electromagnetic magnification there can be a gradual shift in the position of the steady mass without a corresponding shift in the position of the galvanometer winding. The difficulties arising from a moderate lack of stability are therefore much less, and consequently a somewhat longer period and a considerably higher magnification may be used. In fact, this constitutes one of the main advantages to be gained by the use of electromagnetic magnification. It will, therefore, be assumed that the free period is to be 12.5 seconds and the combined optical and electromagnetic magnification is to be such that the displacements shown on the record will be approximately the same as would be given an ordinary type of seismometer having the same period, critical damping, and a static magnification of 1,000.

Galvanometers known as the fluxmeter type, which have been on the market for a number of years, have such relations between their constants that for conditions such as are met in teleseismic observations they give approximately the ideal performance referred to above; that is, their characteristics are such that when connected to a suitably designed winding on a seismometer they should give a reasonably accurate record of the displacements of the steady mass with respect to its support. In fact, except for minor defects, which need not be considered here, they seem to be nearly as good as any which might be made, especially for the purpose. It will, therefore, be assumed that a particular one of these galvanometers is to be used without change and that the problem is to so design the seismometer that when used with this galvanometer the performance will approximate that just stated, except that the record should have the appearance of much greater stability of the rest or zero position, and the magnification of very short period displacements should be relatively small. Further, it will be assumed that the seismometer should be small in size, reasonably inexpensive to construct, be fairly rugged, and require but few adjustments and these of a type easily made.

2. CONSTANTS OF GALVANOMETER AND ELECTROMAGNETIC MAGNIFICATION REQUIRED

It now becomes necessary to consider the magnitude of the various constants affecting the performance of the apparatus and, therefore, to decide upon a system of units. While the equations given above may be used with any consistent system of units, that which will be used is based on the centimeter as the unit of length, the gram as the unit of mass, the second as the unit of time, and the magnetic permeability of air (or vacuum) as the unit of magnetic permeability.

In this system of units the structural constants of the galvanometer as determined from five independent measurements of its performance are as follows:

$$k = 1.62$$

$$\bar{d} = .05$$

$$u = .318$$

$$g = 623,000$$

$$r = 10^{10}$$

Also the focal length of the mirror is such that the distance between the mirror and the recording paper can conveniently be made 1 m. Consequently it may be presumed that

$$l = 200$$

If the steady mass remained strictly at rest, the magnification M_x would be equal to

$$\frac{l}{L} \times \frac{\Theta}{\Phi}$$

where l/L may be referred to as the optical magnification and Θ/Φ is the electromagnetic magnification, which will be designated as h .

l is the effective length of the galvanometer pointer which, with the optical system to be used, is equal to twice the distance between the galvanometer mirror and the recording paper.

L is the distance from the center of the steady mass to the axis of rotation.

Θ is equal in magnitude to the amplitude of the angular deflection of the galvanometer winding.

Φ is equal in magnitude to the amplitude of the angular deflection of the moving system of the seismometer with respect to its support.

Since l equals 200 cm for M_x , to equal 1,000 it would be necessary for the electromagnetic magnification h to equal 1,000 $L/200$ cm or 5 L/cm . However, the steady mass is displaced when the support is displaced both because all of the steady mass can not be concentrated at the same distance from the axis of rotation and because of the moment of inertia of the galvanometer, and the resulting loss in

magnification, in proportional parts, is equal to the displacement of the steady mass divided by the displacement of the support.

An assumption as to how much should be allowed as the loss in magnification resulting from these two causes constitutes a starting point in the development of a general design for a seismometer. For the present, it will be assumed that this loss will amount to 50 per cent. As the design progresses, this assumption may be revised and the design started anew should it be found that it is unreasonable. This loss in magnification can be compensated for by doubling the electromagnetic magnification; that is, by making it $10 L/\text{cm}$ instead of $5 L/\text{cm}$.

3. MOMENT OF INERTIA

With an ideal galvanometer, the reactions between the galvanometer and the seismometer are substantially the same as though there were a direct mechanical connection between them. Consequently the loss in magnification¹⁰ because of the moment of inertia of the winding of the galvanometer is the same as would result if h^2k/L^2 grams could be added to the steady mass at a distance L from the axis of rotation without in any way affecting the displacing constant, ML . Since $h=10L/\text{cm}$ and $k=1.62$, h^2k/L^2 is equivalent to 162 g. Therefore, if the loss in magnification because of the moment of inertia of the galvanometer winding is not to be excessive, the steady mass must be large in comparison with 162 g. However, there is not much to be gained by making it larger by a factor of more than 5, and in order that the cost may not be unnecessarily large it is desirable to keep the steady mass small. Therefore 500 g will be taken as representing what seems to be a fair compromise between somewhat conflicting requirements.

If the steady mass is taken as 500 g and placed with its center at a distance of 20 cm from the axis of rotation, which seems might be convenient, the moment of inertia will be somewhat greater than 500×20^2 . It must be somewhat greater than this, since the steady mass must include the winding and various supports, as well as the steady mass proper, all of which can not be placed at the same distance from the axis of rotation. It will, therefore, be assumed that the moment of inertia K of the moving system of the seismometer is to be $2.5 \times 10^5 \text{ g cm}^2$. With L equal to 20 cm, the optical magnification as defined above becomes 10 and the electromagnetic magnification h becomes 200.

4. DISPLACING CONSTANT

With a steady mass of 500 g and the distance from the center of mass to the axis of rotation equal to 20 cm the displacing constant ML equals 10,000 g cm.

¹⁰ Reid, The California Earthquake of Apr. 18, 1906, 2, p. 155; 1910.

Since $K=2.5 \times 10^5$ and this, in effect, is increased by h^2k or 0.648×10^5 , and $ML^2=2 \times 10^5$, the loss in magnification on account of a lack of concentration of the steady mass and on account of the moment of inertia of the galvanometer winding is $(3.148-2)/3.148$, or 37 per cent. Therefore the assumption made above that the loss in magnification from these two causes would amount to 50 per cent was somewhat over liberal, but scarcely so much as to justify a revision of the values selected for the steady mass and moment of inertia.

5. RESTORING CONSTANT

According to the pendulum formula, the free or undamped period, T_0 , is given by the equation

$$T_0 = 2\pi\sqrt{K/U} \quad (42)$$

Since it has been assumed that the free period would be adjusted to 12.5 seconds and the value 2.5×10^5 has been selected for the moment of inertia K , it is necessary that the restoring constant

$$\begin{aligned} U &= 4\pi^2 \times 2.5 \times 10^5 / 12.5^2 \\ &= 6.25 \times 10^4 \text{ g cm}^2/\text{sec}^2 \end{aligned}$$

6. DAMPING CONSTANT WITH THE CIRCUIT OPEN

No effort will be made to damp the motion of the steady mass with the circuit open. There will, however, be some damping because of air friction, etc., but this probably will amount to not more than one-tenth that which would be required to produce critical damping.

The equation for the motion of the seismometer with the circuit open and no driving torque is

$$K \frac{\delta^2 \phi}{\delta t^2} + D \frac{\delta \phi}{\delta t} + U \phi = 0$$

as given above, equation (21), and for the motion to be critically damped it is necessary that¹¹

$$D = 2\sqrt{KU} \quad (43)$$

Since

$$K = 2.5 \times 10^5 \text{ and } U = 6.25 \times 10^4$$

for critical damping it would be necessary that $D = 25 \times 10^4$. It will, therefore, be considered that $D = 25 \times 10^3 \text{ g cm}^2/\text{sec}$. This value for D will have but a slight effect upon the performance of the apparatus, so it would make but little difference if by change it should be larger or smaller by as much as 100 per cent.

7. ELECTRODYNAMIC CONSTANT

What has preceded fixes the electromagnetic magnification at 200. With an ideal galvanometer the electromagnetic magnification is equal to $\frac{G}{g} \frac{S}{R+S}$, so if $S=2R$, which seems reasonable, it is necessary since $g=623,000$, that $G=1.5 \times 623,000 \times 200 = 1.87 \times 10^8$ dyne cm/E. M. U. current.

8. RESISTANCES OF SEISMOMETER WINDING AND BRIDGE

For the system to be critically damped, assuming the galvanometer to give an ideal performance, it is necessary that¹¹

$$D' + G^2/(R+S) = 2\sqrt{K'U'} \quad (H)$$

or

$$R+S = G^2/(2\sqrt{K'U'} - D')$$

where

$$D' = D + h^2d$$

$$K' = K + h^2k$$

$$U' = U + h^2u$$

Here D' , K' , and U' represent the combined effects of the damping constants with the circuits open, the moments of inertia, and the restoring constants of the galvanometer and seismometer.

Substituting for D , d , G , K , k , U , u , and h the values given above gives for $R+S$

$$12.3 \times 10^{10} \text{ E. M. U. resistance (or 123 ohms).}$$

Since it is assumed that S is to be equal to $2R$, it follows that R may be taken as 4.1×10^{10} and S as 8.2×10^{10} .

The selection of a proper value for S is of considerable importance, since it determines the damping and the damping to a considerable extent determined the performance. In the use of the apparatus, the proper resistance for the bridge will be determined experimentally and so should present no difficulty, but the problem here is to determine in advance what the resistance of the bridge should be to give critical damping. The procedure followed above is based on the assumption that the galvanometer would develop a back electromotive force equal to the potential difference applied across its terminals, while it is known that this will be only approximately true. Therefore a second approximate calculation will be made based on the assumption that the galvanometer and the seismometer have the same period. This permits the consideration of the bridge as made up of two

¹¹ See general design of critically damped galvanometers. B. S. Sci. Papers, 19 (No. 273), pp. 216-219; 1916, for discussion of conditions giving critical damping.

conductors in parallel, one connected to the galvanometer and of the proper resistance to critically damp the galvanometer, and the other connected to the seismometer and of the proper resistance to critically damp the seismometer. Assuming the resistance of the seismometer winding and connecting line to be 4.1×10^{10} , this leads to a value for the resistance of the bridge differing by but a few per cent from that stated above. As the galvanometer has a period of about 14 seconds, and it is assumed the seismometer will be adjusted to a period of 12.5 seconds, 8.2×10^{10} may be considered a proper value to use for the bridge in making an advance determination of the performance characteristics of the apparatus.

In the construction of a seismometer in accordance with this design, the value taken here for R should be looked upon as an upper limit for the resistance of the seismometer winding. If it is made less than 4.1×10^{10} in adjusting the bridge to given critical damping, it may be expected that it will become somewhat more than 8.2×10^{10} , and this will give some increase in the magnification and make the performance characteristics approach more nearly to those of the usual type of seismometer.

The procedure followed here has led to the following set of values for the structural constants of the seismometer and resistance of the bridge:

$$\begin{aligned} LM &= 10^4 \\ K &= 2.5 \times 10^5 \\ D &= 2.5 \times 10^4 \\ U &= 6.25 \times 10^4 \\ G &= 1.87 \times 10^8 \\ R &= 4.1 \times 10^{10} \\ S &= 8.2 \times 10^{10} \end{aligned}$$

Here all values are in c g s mechanical or electromagnetic units.

This design is based on the supposition that the galvanometer gives an ideal performance, though it is known that it gives only approximately such a performance. Therefore it should be expected that a seismometer constructed in accordance with it will give only approximately the performance specified above.

It should be noted that in the development of this design no reference has been made to the general seismometer equation (31) or to its solution for the case of a sustained harmonic displacement, equation (39). Equation (39) and equation (41), which gives the magnification of tilts, will serve to give more definite ideas concerning the performance to be expected and may be used to show the effects of various modifications in the design, especially if modifications may be made in the galvanometer as well as the seismometer.

VII. PERFORMANCE TO BE EXPECTED

Having thus arrived by a rather indirect procedure and to some extent by guessing at a set of values for the intrinsic or structural constants of the seismometer; it is in order to see more definitely what should be expected of a seismometer constructed in accordance with this design, when used with a galvanometer having the constants stated above. Substituting the values given for the structural constants of the seismometer and the galvanometer, and the resistance of the bridge, in equations (39) and (41), and dividing numerator and denominator by 10^{27} , gives for the magnification of displacements

$$M_x = \frac{19,200}{17.4 - 4.07/\omega^2 + j1.86\omega - j15.76/\omega + j0.091/\omega^3} \quad (44)$$

and for the magnification of tilts

$$M_\psi = \frac{81.8 \times 10^6}{17.4\omega^2 - 4.07 + j1.86\omega^3 - j15.76\omega + j0.091/\omega} \quad (45)$$

The substitution in these equations of different values of ω in the range from 62.8 to 0.001, corresponding to periods from 0.1 to 6,280 seconds, gives the data for the magnification of displacements and tilts tabulated in Table 1. This table also contains data for the phase angle between the deflection as shown on the record and the displacement and tilt of the earth. A part of these data are shown graphically in Figures 5, 7, and 9.

TABLE 1.—Data showing performance to be expected of seismometer constructed in accordance with general design developed above and used with galvanometer having constants stated above

ω	T_e	M_x	$M_\psi \times 10^{-6}$	γ_x	γ_ψ
62.8	0.100	163	0.00004	-81.5	-261.5
15	.419	600	.00261	-57.1	-237.1
10	.628	790	.00775	-44.4	-224.4
5	1.26	1,050	.0457	-19.9	-199.9
4	1.57	1,097	.0675	-11.8	-191.8
3	2.09	1,132	.123	-1.1	-181.1
2.5	2.51	1,140	.178	5.6	-174.4
2	3.14	1,135	.2781	15.3	-164.7
1.67	3.80	1,120	.3935	21.7	-158.3
1.50	4.19	1,104	.4775	26.3	-153.7
1.00	6.28	1,000	.980	46.0	-134.0
.628	10.00	781	1.94	73.3	-106.7
.500	12.57	643	2.52	87.9	-92.1
.333	18.85	397	3.50	113.6	-66.4
.250	25.1	259	4.013	130.0	-50.0
.200	31.4	178	4.360	141.6	-38.4
.100	62.8	48.5	4.75	170.2	-9.8
.060	104.5	16.8	4.575	188.0	8.0
.030	209.5	3.60	3.96	212.3	32.3
.010	628	.193	1.89	247.0	67.0
.002	3,140	.00169	.413	264.8	84.8
.001	6,280	.00021	.207	267.4	87.4

Here

$\omega = 2\pi/T_e$.

T_e = the period of earth displacement.

M_x = the magnification of earth displacements.

M_ψ = the magnification of earth tilts.

γ_x = the phase displacement between the record and earth displacement, in degrees.

γ_ψ = the phase displacement between the record and earth tilt, in degrees.

Figure 5 shows the magnification of earth displacements having periods in the range from 0 to 30 seconds. The reduced magnification of short-period displacements, especially those below 2.5 seconds,

FIGURE 5.—Magnification of earth displacements of different periods

is one of the important advantages to be gained by electromagnetic magnification. In the range of periods from about 2.5 to 30 seconds, the magnification of earth displacements varies with the period much as it does with the usual type of seismometer having the same

FIGURE 6.—Magnification of earth displacements of different periods as given by a seismometer of the usual type having a static magnification of 1,250, a period of 12.5 seconds, and critical damping

free period and critical damping, as may be seen by a comparison of Figures 5 and 6.

Figure 6 shows the magnification of earth displacements in the range of periods from 0 to 30 seconds as given by the ordinary type of seismometer having a free period of 12.5 seconds, critical damping, and a magnification of short-period displacements of 1,250. Here a

magnification of 1,250 is used instead of 1,000, that assumed in the development of the design, since it is found that this gives a closer correspondence between the two characteristic curves and so furnishes a better basis for a comparison of the performances of the two types of apparatus.

Figure 7 shows the phase angle between displacements as shown on the record and ground displacements having periods from 0 to 30 seconds. It may be seen that for very short periods the displacements shown on the record lags 90° , or a quarter period, behind the earth displacement. For periods of about two seconds the displacement as shown on the record and the displacement of the ground are approximately in phase, and for longer periods the record leads

FIGURE 7.—Phase displacements between the record and earth displacements of different periods

The ordinates are in degrees and a negative sign indicates that the record lags behind the earth movement

the ground displacement much as with the usual type of seismometer, as may be seen by a comparison of Figures 7 and 8.

Figure 8 shows the phase angle between the displacement shown on the record and the ground displacement in the range of periods from 0 to 30 seconds as given by the ordinary type of seismometer having critical damping and a period of 12.5 seconds.

Figure 9 shows the magnification of tilts of the ground having periods from 6 to 6,000 seconds.

Figure 10 shows the magnification of tilts of the ground in the same range of periods as given by the ordinary type of seismometer having a free period of 12.5 seconds, critical damping, and a magnification of short-period displacements of 1,250.

In Figures 9 and 10 the periods are plotted on a geometric scale, the better to bring out the characteristics of the apparatus.

It will be seen that for the design under consideration the magnification of very long period earth tilts is much less than that given by the ordinary type of seismometer. This is an important advantage

FIGURE 8.—Phase displacements between the record and earth displacements of different periods as given by the usual type of seismometer having a period of 12.5 seconds and critical damping

to be gained by electromagnetic magnification, since in many cases it is tilting, having in effect a period of several hours, which limits the magnification which can be used with a seismometer having a long period. It may be of interest, therefore, to note that for a

FIGURE 9.—Magnification of earth tilts of different periods

period of 6,000 seconds the magnification is only about 4 per cent of that of the ordinary type of seismometer having similar characteristics in the teleseismic range of periods. For disturbances of longer periods the magnification is inversely proportional to the period.

As to the performance which should be expected under conditions to be met in teleseismic observations—that is, where the displacement of the ground is more or less irregular instead of a sustained harmonic function of time—no very definite conclusions can be reached from the analysis which has been given. This phase of the subject does not lend itself readily to a theoretical analysis, since the general seismometer equation is of the fourth order and, therefore, rather complicated. However, the design is such that in this respect the performance should be substantially the same as that given by the usual type of seismometer having the same period and critical damping.

As shown by the tabulated data and characteristic curve (fig. 5), the magnification is about 25 per cent higher than that specified in

FIGURE 10.—Magnification of earth tilts of different periods as given by the usual type of seismometer having a static magnification of earth displacements of 1,250, a period of 12.5 seconds, and critical damping

the statement of the problem. This difference is largely accounted for by the assumption made tentatively as a starting point in the design, namely, that the loss in magnification on account of the moment of inertia of the winding of the galvanometer and lack of concentration of the steady mass would amount to 50 per cent, whereas a calculation of this loss showed it to be about 37 per cent. Had the design been modified so as to obviate this discrepancy, presumably the magnifications would have been made smaller by the factor 50/63 and, if so, would have been very nearly that specified. However, it is possible there may be no serious objection to the use of the higher magnification.

The reduced magnification of short-period displacements results mainly from the moment of inertia of the winding of the galvanometer and the resistance of the electrical circuit. By changing the design of the galvanometer, a reduction of the resistance of the seismometer

winding, or both, it would be possible to materially increase the magnification of short-period displacements without making much change in the magnification of displacements having periods of 2.5 seconds and more. By making the bridge across the line connecting the galvanometer and seismometer highly inductive¹² and of low resistance, and the seismometer considerably larger, or the coil of the galvanometer considerably lighter, it would be possible to make the performance characteristics both for short and intermediate period displacements correspond very closely to those of the usual type of seismometer having the same period and same damping. However, this would make the construction slightly more expensive, the apparatus somewhat more complicated and much more subject to very short period disturbances of local origin. Further, it would result in only a slight improvement in records of distant earthquakes.

It would be an advantage if the magnification of tilts having periods of one minute and longer were materially reduced, and this could be brought about by stiffening the suspensions of the galvanometer winding, an increase of the resistance of the electrical circuit, or both. However, a stiffening of the suspensions or an increase of the resistance of the electrical circuit would have a detrimental effect upon the magnification of displacement of periods in the teleseismic range. It seems, therefore, that there are no obvious changes which should be made in this general design so it will be considered as completed.

VIII. DETAILED DESIGN AND CONSTRUCTION

The general design outlined above differs radically from any which, in so far as I know, has been used, and the analysis just made shows that an instrument constructed in accordance with it should give at least a fairly satisfactory performance, assuming, of course, proper attention to the detailed design and first-class workmanship throughout. It seemed, therefore, that it might be well to carry this project further to see what if any difficulties might be encountered in the detailed design and construction, to see more definitely what sort of performance such an instrument might give, and to see what difficulties might be encountered in its adjustments and use. Accordingly, a detailed design was made for a horizontal-component seismometer to have the values for the structural constants approximately the same as those given in the general design and an experimental instrument constructed approximately in accordance with this detailed design.

In the detailed design and construction no special effort was made to follow any particular general design strictly, since it was felt that considerable liberty might be taken without materially affecting the performance. However, care was taken to see that the structural

¹² Wenner, *Phys. Rev.*, **19**, p. 410; 1922.

constants should have such relative values that there would be no question but that critical damping and high magnification could be realized even should it be found necessary to adjust the apparatus so as to have a considerably shorter free period than that assumed in the statement of the problem. Further, where it was found convenient to make departures in a direction thought to be beneficial, this was done. As constructed, most of the constants have values smaller than those given by the general design. The largest departure is in the resistance of the seismometer winding, which is only about half as large. This lower resistance should result in making the performance characteristics approach more closely to those of the usual type of seismometer, having the same period and critical damping.

The experimental seismometer was shown at the meeting of the eastern section of the Seismological Society of America in May, 1927, and a photograph of it is reproduced in Figure 11. It has been installed on a pier located on undisturbed ground beneath the floor of a basement room of one of the buildings of the Bureau of Standards where conditions are not especially favorable for obtaining records of teleseismic disturbances. The galvanometer, recording camera, and bridge are located on the first floor of the same building, which permits of changing records without disturbing the seismometer. This also permits of a test to see that the motion of the steady mass is not obstructed, a determination of the period, and an adjustment of the damping without entering the room in which the seismometer is located. Parts of a few records obtained with this equipment are shown in Figures 12 to 16, inclusive. In these records the recording drum made one revolution per hour and the minute marks were made by momentarily increasing the intensity of the light source. A shutter was operated in the light beam as suggested by Anderson and Wood.¹³ This gave exposures of about 0.07 second at intervals of about 0.6 second. Because of the limited space available for the recording apparatus, a lens was used to reduce the distance between the galvanometer mirror and the recording paper to about 80 cm. Therefore in the records the magnification is only 0.8 that which would have been obtained had the normal distance of 100 cm been used. The motion recorded is the N-S component. All save one of these records were made with a rather poor mirror on the galvanometer. The replacement of this mirror with one of good quality is giving much clearer and sharper records.

It is expected that a more complete description of the apparatus, a procedure for determining its structural constants, a procedure for making the adjustments, more definite information concerning its

¹³ Anderson and Wood, *Bull. Seis. Soc. Am.*, **15**, p. 36; 1925.

FIGURE 11.—*Experimental seismometer constructed substantially in accord with the general design*

FIGURE 12.—Parts of record of earthquake of March 16, 1928, near New Hebrides
a shows L phase; b shows M phase

FIGURE 13.—Part of record of earthquake of March 22, 1928, off the coast of southern Mexico, showing P, S, and other earlier phases

FIGURE 14.—Parts of record of earthquake of April 18, 1928, in Bulgaria
a shows P, PR₁, PR₂, and possibly other phases; *b* shows S, SR₁, and possibly other phases

FIGURE 15.—Parts of record of earthquake of June 17, 1928

a, About 1 hour and 30 minutes after time of origin; *b*, about 1 hour and 45 minutes after time of origin

FIGURE 16.—Part of record showing microseisms of considerably more than average intensity

performance, and possibly suggestions for improving either or both the general design and the detailed design will be given in another paper.

IX. SUMMARY

1. An elementary discussion is given of the principles involved in electromagnetic damping and electromagnetic magnification.

2. It is shown that proper electromagnetic damping may be secured by the use of a bridge or shunt across the circuit connecting the seismometer and the galvanometer.

3. Expressions are derived for the current in the galvanometer winding and in the seismometer winding when both windings are developing an electromotive force and the circuit is bridged.

4. The equation is established for the motion of the steady mass relative to its support for the case in which the displacing torque arises both from an acceleration of the support and a current in the winding. From this and the corresponding equation for the motion of the winding of the galvanometer there is obtained a general seismometer equation; that is, an equation stating the relation between the angular displacement of the winding of the galvanometer and the linear displacement of the ground on which the seismometer is located.

5. The general seismometer equation which is a linear differential equation of the fourth order is solved for the case of a sustained harmonic displacement of the ground.

6. It is shown that by a slight modification of this solution it gives the relation between the deflection of the galvanometer winding or the displacements shown on the record and the angle of tilt of a horizontal-component seismometer.

7. A general design is developed for a seismometer to give at least approximately a specified performance. This general design permits of making the seismometer of small size, simple construction, and requires no special magnets for producing proper damping.

8. The damping of both the galvanometer and the seismometer is controlled by a single bridge or shunt across the line connecting the galvanometer and the seismometer. As this bridge may be located adjacent to the galvanometer, the period may be determined and the damping adjusted from the position of the galvanometer, which normally would be in a room other than that in which the seismometer might be located.

9. Characteristic curves for a seismometer of this general design and, for comparison, corresponding curves for the ordinary type of seismometer are shown.

10. Finally, a photograph of an experimental seismometer constructed substantially in accordance with this general design and parts of a few records obtained with it are shown.

X. APPENDIX—SYMBOLS

The symbols used in more or less separated parts of this paper and the quantities which they represent are as follows:

- d , damping constant of galvanometer with circuit open.
- D , damping constant of seismometer with circuit open.
- g , electrodynamic constant of galvanometer.
- g , acceleration of gravity.
- G , electrodynamic constant of seismometer. The factors on which it depends are explained on page 970.
- H , magnetic field strength.
- h , the electromagnetic magnification or the ratio of Θ to Φ .
- i , current usually in galvanometer winding.
- I , current in seismometer winding.
- j , square root of minus one.
- k , moment of inertia of winding and mirror of galvanometer.
- K , moment of inertia of moving system of the seismometer.
- l , equivalent length of galvanometer pointer or twice the distance between the optical center of the galvanometer mirror or lens and the recording paper.
- L , distance from center of mass of moving system of seismometer to the axis of rotation.
- M , mass of the moving system of seismometer.
- M_x , magnification of linear displacements of the support of the seismometer; that is, the ratio of displacement as shown on the record to displacements of ground.
- M_ψ , magnification of angular displacements or tilts.
- Q^2 , defined by equation (15).
- r , resistance, usually of the galvanometer winding and suspensions.
- R , resistance of seismometer winding and connecting leads.
- S , resistance of shunt or bridge across circuit connecting galvanometer and seismometer.
- t , time.
- T_0 , the undamped period of the seismometer.
- u , restoring constant of galvanometer.
- U , restoring constant of seismometer.
- x , linear displacement.
- X , linear displacement of support of seismometer.
- $\mathfrak{X}$, amplitude of linear displacement of support of seismometer.
- X , complex quantity equal in magnitude to amplitude of linear displacement of support of the seismometer and containing a phase angle depending upon the assumed origin of time.
- e , base of Napierian logarithm.
- ϕ , angular deflection of moving system of seismometer.

Φ , complex quantity, equal in magnitude to amplitude of angular deflection of steady mass of seismometer, and containing a phase angle.

ψ , angle of tilt.

Ψ , complex quantity, equal in magnitude to the amplitude of angle of tilt and containing a phase angle depending upon the assumed origin of time.

θ , angular deflection, usually of moving system of galvanometer.

Θ , complex quantity, equal in magnitude to amplitude of angular deflection of moving system of galvanometer and containing a phase angle.

ω , 2π divided by period of earth displacement or tilt.

WASHINGTON, December 7, 1928.