CONSTANTS OF SPECTRAL RADIATION OF A UNIFORMLY HEATED INCLOSURE OR SO-CALLED BLACK BODY, II

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CONTENTS

	Page
I. Introduction.....	459
II. Recomputation of data.....	461
III. Discussion of other data.....	468
IV. Summary.....	470
Appendix—Note on methods of calculating the spectral radiation constants...	471

I. INTRODUCTION

In a previous communication¹ the results of an investigation of the constant of spectral radiation of a black body were presented. The spectral energy curves were obtained by means of a vacuum bolometer, a mirror spectrometer, and a perfectly clear fluorite prism.

In the isothermal spectral energy curves obtained with these instruments the position λ_m of the maximum emission E_m , was computed by taking the wave lengths, λ_1 and λ_2 , corresponding to equal intensities, $E_{\lambda_1} = E_{\lambda_2}$, on the assumption that the observed energy curve fits the Planck equation. The equation which is used in computing λ_m is

$$\lambda_m = \frac{\alpha (\log \lambda_2 - \log \lambda_1) \lambda_1 \lambda_2}{\alpha' (\lambda_2 - \lambda_1) \log e} - \frac{\lambda_1 \lambda_2 [\log(1 - e^{-c/\lambda_1 T}) - \log(1 - e^{-c/\lambda_2 T})]}{\alpha' (\lambda_2 - \lambda_1) \log e}$$

The second term in this equation is rather complicated. It requires an approximate value of the spectral radiation constant, C , which was assumed, for the purpose of calculating this correction (for the series of 1909 to 1911), to be $C = 14\,500$. A decrease of 100 units in C (say $C = 14\,400$) decreases the mean value² of λ_m by 0.0012μ , which at that time was considered negligible.

¹ This Bulletin, 10, p. 2; 1913.

² In the previous paper this value was stated to be 0.005μ , which is erroneous.

The second term in the above equation, which is usually mentioned as a "correction factor" has always been a stumbling block in the computations. After the completion of the calculations of λ_m it was found that the computer had omitted a factor from this "second term" which introduced an error of about 0.5 per cent, giving a mean value of $C = 14\ 280$ (0.5 per cent too low).

Some doubts arose also as to the accuracy of part of the calibration curve of the prism, known as the "calibration curve of 1912," in view of the general belief, held at that time, that the value can not be lower than $C = 14\ 500$. To save work in correcting this error in the "second term," fewer points were taken, and the values of λ_2 , at 4 to 5μ , were increased by a slight amount to correspond with the "calibration curve of 1910." The data thus obtained gave a mean value of $C = 14\ 456$, which is about 0.2 per cent higher than would have been obtained by adhering to the "calibration curve of 1912," published in the previous paper.

These data were published in 1913, the intention being to revise them, if the calibration curve of 1910 should be found wrong. In the meantime Paschen³ has published further data on the dispersion of fluorite which shows that the part of the calibration curve of 1910, used by the writer in the above computations, is less reliable than the calibration curve of 1912.

Recently, on examining the previous computations, it was found that the above-mentioned "second term" still contains a small error, giving a mean value of λ_m , which is 0.2 to 0.3 per cent too high. As a result of these errors, which happen to be of the same sign, the value of $C = 14\ 456$, previously published, is 0.5 to 0.6 per cent too large, owing to faults in the calibration curve and to a small error in the reduction of the data.

The computations have therefore been made anew, using the values of λ_1 and λ_2 which had been read from the spectral energy curves obtained with the prism calibration curve of 1912. The recomputation of these data turned out to be a simple matter, in view of the fact that it merely required dividing the above-mentioned "second term" factors by 2.3 and subtracting 0.0038μ from the mean value of λ_m . The subtraction of 0.0038μ is necessary because $C = 14\ 600$ instead of $C = 14\ 300$ had been used in computing the second-term factors of the "data of 1912."

³ Paschen., *Ann. der Phys.*, 41, p. 670; 1913.

The difficulty encountered in this investigation was an overabundance of spectral energy curves, the viewing of which caused such fatigue to the eyes that, after having drawn the curves and indicated the best positions for obtaining λ_1 and λ_2 , it was necessary to assign to another the task of reading off the numerical values of λ_1 and λ_2 from the curves and computing the results. After a lapse of more than three years the writer finds no corrections to the values of λ_1 and λ_2 as then read from the energy curves by his assistant, showing that the work had been faithfully done. As for the errors in the computations, owing to eye fatigue, it was impossible for months thereafter to give attention to the reduction of the data. Furthermore, at that time a correction of 0.3 to 0.5 per cent was considered of minor importance in view of the great difficulties involved in the work. It is of interest to note that the discrepancies in the previous paper are due to faults in reduction of the data and not in the experimental work. The present disagreement between theory and experiment is very small, and may be further reduced when we know more about the optical constants of the prism used in producing the spectrum.

II. RECOMPUTATION OF DATA

As already mentioned, the recomputation of the data consisted simply in eliminating the errors in the "second-term correction" which is used in obtaining λ_m . The total number of points on the energy curve was usually greatly increased. In this manner a mean value of λ_m was obtained which is more representative of the whole energy curve. The values of λ_2 were taken in the regions where there is no atmospheric absorption, viz, 2.3 to 2.5 μ , 3.3 to 4.1 μ , 4.4 to 4.6 μ , and 4.8 to 5.6 μ . In the previous paper the value of C was based upon data obtained in 1911 and the "series of 1912." The "series of 1911" contained many curves taken at extreme temperatures. For the low temperatures the curves were greatly affected by atmospheric absorption, and at high temperatures the scale of temperatures was questioned. Correcting these data a mean value of $C = 14\,390$ was obtained for the "series of 1911." However, considering the quality of the spectral energy curves, it does not seem fair to give them any weight in deducing a new mean value, although it would increase the corrected mean value but little.

As mentioned in the previous paper the "series of 1912" observations are considered the most accurate (1) as regards adjustments of apparatus, (2) temperature scale, (3) bolometric control, (4) elimination of atmospheric water vapor, and (5) as regards the general appearance of the spectral energy curves. These data were therefore carefully recomputed and tabulated. In making up the tables the column of probable errors is omitted in view of the writer's feeling that such data are illusory.

In the present paper the old numbers of the tables are carried in a footnote at the end of each table to facilitate comparison with the data in the previous paper to which reference must be made for details. Series 8, Table 6, is poor owing to various difficulties that were experienced—that is, tremors, bolometric troubles, etc.—so that only the most reliable curves were considered in the present computations.

TABLE 1^a

Series 1912, I.—White Radiating Walls, Fluorite Prism No. 1, Water-Cooled Shutter No. 2.

[Energy curves LXXXVI-XCII]

No.	Form of energy curve	Absolute temperature	Wave length of maximum emission (λ_{maz})				$\lambda_{maz}T$		$C=\alpha\lambda_{maz}T$	
			Mean value		Number of computations		Old	New	Old	New
			Old	New	Old	New				
XCIII ^b	?	1112	μ 2.605	μ 2.601	3	3	2922	2897	14 508	14 384
LXXXVIII.....	P	1209	2.408	2.401	6	6	2910	2903	14 451	14 414
XCI ^b	P	1223	2.374	2.356	4	7	2904	2881	14 418	14 304
LXXXVII.....	PW	1242	2.345	2.332	5	9	2912	2896	14 459	14 379
LXXXVI.....	P	1350	2.154	2.139	6	7	2908	2888	14 438	14 339
LXXXIX ^c	P	1419	2.056	2.034	7	7	2916	2886	14 480	14 329
XC ^b	PW	1427	2.045	2.031	5	3	2918	2898	14 485	14 389
Mean.....							2913	2892	14 463	14 362

^a Table 12 of previous paper.

^b Furnace heated by outer coil only.

^c Poor curve.

TABLE 2 *a*Series 1912, II.—Black (Cr₂O₃) Radiating Walls, Fluorite Prism No. 1, Water-Cooled Shutter No. 2

[Energy curves, XCIII-CVI]

No.	Form of energy curve	Absolute temperature	Wave length of maximum emission (λ_{max})				$\lambda_{max}T$		$C=\alpha\lambda_{max}T$	
			Mean value		Number of computations		Old	New	Old	New
			Old	New	Old	New				
CVI <i>b</i>	P	1025	μ 2.824	μ 2.817	5	5	2895	2887	14 372	14 334
XCIX.....	P	1153	2.507	2.496	4	6	2891	2878	14 356	14 289
XCVIII <i>b</i>	P	1223	2.375	2.343	4	7	2904	2866	14 419	14 230
XCVII.....	P	1301	2.226	2.209	6	8	2896	2874	14 379	14 270
CV.....	P	1338	2.183	2.151	5	6	2921	2878	14 503	14 289
XCVB.....	P	1351	2.145	2.127	4	7	2898	2874	14 388	14 270
XCVI.....	P	1377	2.104	2.092	4	8	2897	2881	14 385	14 304
CIV.....	P	1401	2.080	2.059	6	8	2914	2885	14 470	14 324
XCIV.....	P	1420	2.050	2.026	7	9	2909	2877	14 442	14 285
CII.....	P	1453	2.003	1.992	7	9	2910	2894	14 450	14 368
CIII.....	P	1479	1.973	1.947	5	5	2918	2879	14 491	14 295
XCIII.....	P	1501	1.944	1.918	4	7	2917	2879	14 485	14 295
CI.....	PW	1525	1.908	1.893	5	8	2910	2877	14 447	14 334
C.....	P	1687	1.726	1.717	5	5	2912	2896	14 457	14 378
Mean.....							2907	2881	14 432	14 303

a Table 13 of previous paper.*b* Poor curves.TABLE 3 *a*

Series 1912, III.—New Radiator, White Walls, Fluorite Prism No. 1, Water-Cooled Shutter No. 2

[Energy curves, CVII-CXIII]

No.	Form of energy curve	Absolute temperature	Wave length of maximum emission (λ_{max})				$\lambda_{max}T$		$C=\alpha\lambda_{max}T$	
			Mean value		Number of computations		Old	New	Old	New
			Old	New	Old	New				
CXIII <i>b</i>	?	1151?	μ 2.540	μ	3		2921		14 504	
CX.....	PW	1223	2.383	2.373	6	5	2914	2902	14 469	14 408
CXII.....	P	1276	2.275	2.264	6	9	2903	2889	14 412	14 344
CIX.....	P?	1352	2.155	2.142	7	7	2914	2896	14 466	14 379
CXI.....	P	1379	2.109	2.101	5	7	2911	2897	14 451	14 384
CVIII.....	P	1453	2.003	1.996	5	5	2910	2900	14 450	14 398
CVII.....	P	1527	1.913	1.886	4	5	2921	2880	14 508	14 299
Mean.....							2913	2894	14 466	14 368

a Table 14 of previous paper.*b* Poor curve.

TABLE 4^a

Series 1912, IV.—New Radiator, Black ($\text{Cr}_2\text{O}_3 + 2\text{Co}_2\text{O}_3$) Walls, Fluorite Prism No. 1,
Water-Cooled Shutter No. 2

[Energy curves, CXIV-CXXIII]

No.	Form of energy curve	Absolute temperature	Wave length of maximum emission (λ_{max})				$\lambda_{\text{max}}T$		$C = \alpha\lambda_{\text{max}}T$	
			Mean value		Number of computations		Old	New	Old	New
			Old	New	Old	New				
CXXIII ^b	P?	876	μ 3.312	μ 3.280	3	5	2901	2873	14 405	14 265
CXXII.....	P	1025	2.826	2.818	8	9	2897	2888	14 382	14 339
CXXI.....	P	1077	2.704	2.694	5	6	2912	2901	14 459	14 404
CXVIII.....	P	1154	2.516	2.507	6	7	2903	2893	14 414	14 364
CXX.....	PW	1223	2.376	2.364	5	9	2906	2891	14 428	14 354
CXVII.....	P	1276	2.280	2.270	5	5	2909	2896	14 445	14 379
CXIX.....	P	1353	2.156	2.139	6	7	2917	2894	14 486	14 369
CXVI ^b	P	1377	2.122	2.104	6	3	2922	2897	14 507	14 384
CXV.....	P	1454	2.007	1.996	6	6	2918	2902	14 488	14 408
CXIV ^b	P	1503?	1.954		4		2936		14 580?	
Mean.....							2910	2893	14 446	14 363

^a Table 15 of previous paper.

^b Poor curves.

TABLE 5^a

Series 1912, V.—Blackened Radiator, New Adjustments, Fluorite Prism No. 1,
Water-Cooled Shutter No. 2

[Energy curves, CXXIV-CXL]

No.	Form of energy curve	Absolute temperature	Wave length of maximum emission (λ_{max})				$\lambda_{\text{max}}T$		$C = \alpha\lambda_{\text{max}}T$	
			Mean value		Number of computations		Old	New	Old	New
			Old	New	Old	New				
CXXXV ^b	?	1026	μ 2.831	μ 2.822	5	5	2905	2895	14 423	14 374
CXXXII ^b	P	1077	2.703	2.692	5	7	2911	2899	14 455	14 394
CXXXIV.....	PW	1153	2.520	2.509	6	7	2905	2893	14 425	14 364
CXXXI.....	P	1194	2.441	2.433	4	6	2914	2905	14 469	14 424
CXXXIX.....	P	1224	2.380	2.368	6	6	2913	2898	14 464	14 399
CXXXVI.....	P+	1276	2.284	2.271	9	6	2914	2898	14 469	14 389
CXXXIII.....	P+	1302	2.237	2.226	6	6	2913	2898	14 461	14 389
CXXXVII.....	PW	1302	2.242	2.231	7	6	2920	2904	14 498	14 419
CXXX.....	P	1338	2.182	2.164	5	5	2920	2895	14 497	14 374
CXXXVIII.....	P+	1352	2.160	2.136	4	4	2921	2888	14 501	14 339
CXXV.....	P	1378	2.118	2.096	5	8	2918	2888	14 483	14 339
CXXXVI.....	P	1421?	2.068		5		2939		14 588?	
CXXXVII.....	P+	1454	2.008	1.996	9	5	2920	2902	14 497	14 408
CXXXIV.....	P+	1503	1.961?		4		2948		14 629?	
CXL ^c	P	1627?	1.798	1.794	5		2925	2919	14 524?	14 493?
CXXXVIII ^c	PW	1663	1.756	1.730	5	2	2921	2887	14 500	14 285
CXXXIX ^{b, c}	P+	1796	1.639	1.615	8	5	2945	2900	14 631?	14 358
Mean.....							2915	2895	14 476	14 376

^a Table 16 of previous paper. ^b Poor curves. ^c Alternating current used for furnace heating coils.

TABLE 6^a

Series 1912, VIII.—Blackened Radiator, New Thermocouple, Fluorite Prism No. 1,
Water-Cooled Shutter No. 2

[Energy curves, CLVI-CLXVII]

No.	Form of energy curve	Absolute temperature	Wave length of maximum emission (λ_{max})				$\lambda_{max}T$		$C = \alpha \lambda_{max}T$	
			Mean value		Number of computations		Old	New	Old	New
			Old	New	Old	New				
		°	μ	μ						
CLIX ^b	?	806	3.610				2910		14 447	
CLX.....	WP	879	3.300	3.286	2	7	2901	2888	14 402	14 339
CLXI ^c	P	953	3.052	3.039	2	7	2909	2896	14 441	14 339
CLVIII.....	P	1077	2.703	2.691	7	7	2911	2898	14 454	14 389
CLVII ^d	P?	1223	2.380	2.368	6	6	2911	2896	14 453	14 379
CLVI.....	P	1277	2.286	2.271	7	5	2920	2900	14 497	14 399
CLXIV ^e	P	1300	2.248		8		2923		14 513?	
CLXVII ^f , ^c	WP	1314	2.236		7		2938?		14 588?	
CLXIII.....	P	1376	2.120	2.103	6	3	2917	2894	14 482	14 369
CLXVI ^c , ^e	P	1377	2.140		5		2947?		14 634?	
CLXII ^g	P+	1452	2.030		10		2948?		14 639?	
CLXV ^e , ^c	P+	1539	1.920		9		2955?		14 671?	
Mean.....							2912	2895	14 461	14 375

^a Table 19 of previous paper.

^b Very poor curve, λ_{max} by inspection.

^c Poor series, tremors.

^d Poor curve.

^e Radiation sensitivity changed.

^f Furnace heated by outer coil only.

^g Bolometer unsteady; low vacuum.

TABLE 7^a

Series 1912, IX.—Blackened Radiator, Fluorite Prism No. 1, Water-Cooled Shutter
No. 1

[Energy curves, CLXVIII-CLXXVI]

No.	Form of energy curve	Absolute temperature	Wave length of maximum emission (λ_{max})				$\lambda_{max}T$		$C = \alpha \lambda_{max}T$	
			Mean value		Number of computations		Old	New	Old	New
			Old	New	Old	New				
		°	μ	μ						
CLXXIV ^b	P?	1175	2.465	2.446	7	7	2896	2874	14 381	14 270
CLXXIII.....	P	1194	2.431	2.416	4	6	2900	2885	14 401	14 324
CLXXV.....	P	1277	2.272	2.257	5	6	2901	2882	14 405	14 309
CLXXII.....	P	1302	2.232	2.216	4	8	2906	2885	14 429	14 324
CLXXVI.....	P	1353	2.153	2.137	7	7	2913	2891	14 462	14 355
CLXXI.....	P	1377	2.114	2.101	7	5	2911	2893	14 456	14 365
CLXX.....	P	1451	2.010	1.996	7	8	2916	2896	14 477	14 379
CLXIX ^b	P	1452	2.007	1.986	2	7	2914	2884	14 467	14 319
CLXVIII ^b	P	1540	1.890	1.871	2	4	2909	2881	14 443	14 304
Mean.....							2909	2886	14 436	14 327

^a Table 20 of previous paper.

^b Poor curves, sensitivity changing.

TABLE 8
Reproducibility of Data Using Different Radiators

Date	Number	Absolute temperature	λ_{max}	$\lambda_{max}T = 2890$	$\Delta\lambda_{max}T$	$C = \alpha\lambda_{max}T$
		°				
February 8, 1912.....	CVI.....	1025	2.817	2887	-3	14 334
February 20, 1912.....	CXXII.....	1025	2.818	2888	-2	14 339
February 17, 1912.....	CXVIII.....	1154	2.507	2893	+3	14 364
February 27, 1912.....	CXXXIV.....	1153	2.509	2893	+3	14 364
February 14, 1912.....	CXII.....	1276	2.264	2889	-1	14 344
February 17, 1912.....	CXVII.....	1276	2.270	2896	+6	14 379
January 29, 1912.....	LXXXVI.....	1350	2.139	2888	-2	14 339
February 13, 1912.....	CIX.....	1352	2.143	2896	+6	14 379
February 24, 1912.....	CXXVIII.....	1352	2.136	2888	-2	14 339
February 19, 1912.....	CXIX.....	1353	2.139	2894	+4	14 369
March 27, 1912.....	CLXXVI.....	1353	2.137	2891	+1	14 355
March 26, 1912.....	CLXXI.....	1377	2.101	2893	+3	14 365
February 23, 1912.....	CXXV.....	1378	2.096	2888	-2	14 339

TABLE 9 ^a
Summary of Data Given in the Preceding Tables

Series	Number of energy curves	Mean value C		Remarks
		Old	New	
1911				
I.....	26	14 469	14 390	Unpainted Marquardt porcelain radiator. (Water-cooled shutter No. 2.)
1912				
I.....	7	14 463	14 362	Unpainted radiator as used in 1911.
II.....	14	14 432	14 303	Radiator painted with chromium oxide.
III.....	6	14 466	14 368	New unpainted Marquardt radiator.
IV.....	9	14 446	14 363	Radiator painted with a mixture of the oxides of cobalt and chromium.
V.....	14	14 476	14 376	Radiator repainted; optical path readjusted.
VIII.....	6	14 461	14 375	New thermocouples; radiator the same as in the preceding series.
IX.....	9	14 436	14 327	New water-cooled shutter, No. 1; the rest of the apparatus is the same as in the preceding series.

New mean value $C = 14\,353$ (94 energy curves; 1912)

$A = 2891$

$C = 14369$

$A = 2894$

} Final value after applying all corrections

^a Table 21 of previous paper.

That conditions were reproducible is indicated (Table 8) by the closeness in the agreement of the values of the radiation constant for various radiators heated to the same temperature. In only a few cases is the error greater than 1°, which is as close as the temperature scale was known at that time. In fact, considering the difficulties involved in making the measurements and the appearance of the spectral energy curves, it is surprising to find so close

an agreement (the maximum range is only about 0.5 per cent) now that the computations have been made and the data assembled for inspection.

In Table 9 are assembled the reliable data which form the basis for obtaining a mean value of the radiation constants. The mean value of all the data including the "series of 1911" is $C = 14\,358$. However, as already stated, the data of 1911 are not as reliable as is desirable, and since there is no way of estimating their value the writer prefers to consider only the "series of 1912," which contains seven sets of observations (65 energy curves), which give a mean value of $C = 14\,353$ micron degrees and $A = 2891$ micron degrees.

If the corrections to the temperature scale (mentioned in the previous paper) are applied, the value is $C = 14\,362$. A further correction ($= +7$) is necessary because the second term in equation 1 was computed using $C = 14\,300$ instead of $C = 14\,350$. Hence, the final corrected value is $C = 14\,369$ and $A = 2894$ micron degrees.

A recent investigation of instruments and methods for evaluating radiant energy in absolute value⁴ gave a new determination of the coefficient, σ , of total radiation⁵ of a black body. The numerical value as published is $\sigma = 5.72 \times 10^{-12}$ watt. cm.⁻² deg.⁻⁴, which from our present knowledge appears to be close to the true value. Hence, if theoretical considerations are to be relied upon, this value of the coefficient, σ , of total radiation indicates that the coefficient of spectral radiation is of the order of $C = 14\,322$, as compared with the experimentally determined value $C = 14\,369$. This is a difference of only 0.3 per cent. The agreement is so close that it must be considered accidental in view of the great experimental difficulties involved in obtaining the data. These data were obtained at different dates, on the same radiator (operated under various conditions), and using the same scale of temperatures. The small outstanding discrepancy is probably as much the result of uncertainties in the calibration curve (refractive indices) of the fluorite prism as it is the result of uncertainties in the temperature scale, and of the inevitable experimental errors.

⁴ This Bulletin, 12, p. 504; 1916.

⁵ This Bulletin, 12, p. 553; 1916. The first issue gave a value of $\sigma = 5.75$, which is incorrect.

III. DISCUSSION OF OTHER DATA

Paschen's Data.—After a lapse of about 18 years it is interesting to turn back to the pioneering work of Paschen and observe how close he came to the truth.

In his first paper Paschen ⁶ used four radiators, heated to 100°, 191°, 304°, and 450°, respectively, and 4 bolometers. He obtained a series of closely agreeing values which gave a mean value (2894, 2892, 2888, 2890) of $\lambda_m T = 2891$. Using the Planck reduction factor (4.9651) instead of the Wien factor (5), this gives a value of $C = 14\,354$. The following year Paschen ⁷ obtained a series of spectral energy curves from radiators operated at high temperatures. He obtained a value of $A = 2921$ as a result of arbitrarily changing his calibration curve as described in the writer's previous paper. He changed also his previous data, increasing the value of A from 2891 to 2915, or about 0.9 per cent. From this it would appear that his later value ($A = 2921$) is 0.9 per cent higher than it should have been; that is, his experimental values were close to $A = 2895$ and $C = 14\,374$. In other words, Paschen's original data fall within the range of the recent and more accurately determined values.

The Reichsanstalt Data.—During the past 16 years the Phys.-Tech. Reichsanstalt, at Berlin, has pursued extensive investigations of the laws of radiation and their application to the temperature scale. The first investigation by Lummer and Pringsheim ⁸ gave a mean value of $A = 2879$. In their second paper they obtained a much higher value, viz, $A = 2940$.

In 1906 Holborn and Valentiner ⁹ obtained a value of $C = 14\,200$ as a result of a series of spectrophotometric measurements. This is now admittedly in error, owing to an erroneous temperature scale in which the melting point of palladium was taken to be 1575° instead of about 1550°. Recently Valentiner ¹⁰ corrected this value, raising it to $C = 14\,350$.

During the past seven years these investigations have been pursued by Warburg ¹¹ and his collaborators. In their first commu-

⁶ Paschen, Sitzber. Akad. Wiss., Berlin, 21, p. 403; 1899.

⁷ Paschen, Ann. der Phys., 4, p. 277; 1901.

⁸ Lummer and Pringsheim, Verh. Phys. Ges., 1, pp. 23, 215; 1899.

⁹ Holborn and Valentiner, Am. der Phys. (4), 22, p. 1; 1906.

¹⁰ Valentiner, Am. der Phys. (4), 29, p. 489; 1912.

¹¹ (1) Warburg and Leithauser, Z. S. für Instrk., 30, p. 113, 1910; (2) Warburg and Leithauser, Z. S. für Instrk., 31, p. 124, 1911; (3) Warburg, Hupka, and Müller, Z. S. für Instrk., 32, p. 134, 1912; (4) Warburg, Leithauser, Hupka, and Müller, Sitzber. Akad. Wiss. Berlin I., p. 35, 1913; (5) Warburg, Leithauser, Hupka, and Müller, Ann. de Phys. (4), 40, p. 609, 1913; (6) Hoffmann and Meissner, Z. S. Instrk., 33, p. 157, 1913; (7) Warburg, Hupka, and Müller, Z. S. Instrk., 34, p. 115, 1914; (8) Warburg and Müller, Z. S. Instrk., 35, p. 98, 1915; (9) Warburg and Müller, Ann. der Phys., 48, p. 410, 1915.

nication ¹² they reported a value of $C = 14\,570$. They questioned the temperature scale and proceeded to make their investigations at high temperatures by using the radiation laws to establish their temperature scale, thus avoiding the temperature scale as transferred from the gas thermometer by means of thermocouples. They retain only one temperature fixed point, viz, gold at 1064° (later 1063°). The higher temperatures (1400°) were determined radiometrically.

In their next communication (2) values are reported which varied from $C = 14\,200$ to $14\,600$. In the following report (3) the fluorite prism gave values ranging from $C = 14\,300$ to $14\,600$ and it was discarded. A quartz prism gave a value of $C = 14\,360$; and in a more complete paper (4 and 5) the values of $C = 14\,370$ and $A = 2894$ are given. These values have stood for some years. A subsidiary investigation (6) of the relative brightness of a black body at the melting point of palladium and of gold gave a value of $C = 14\,440$.

Recently further data (8 and 9) were published, in which a value of $C = 14\,250$ was obtained from the temperature scale based on the Stefan-Boltzmann law of total radiation and a value of $C = 14\,300$ or $14\,400$ was obtained by using the Wien displacement law to establish the temperature. The uncertainty in the value of $C = 14\,300$ or $14\,400$ is owing to the uncertainty in their calibration curve (refractive indices) of the quartz prism which is now being investigated. The present status is the adoption ¹³ of the value of $C = 14\,300$, although from the reliability of Paschen's determinations of the refractive indices of quartz the chances are that the higher value ($C = 14\,350$ to $C = 14\,400$) may be found nearer the correct value.

In passing, it is of interest to note that their recent determinations of the absorption coefficients of quartz are in close agreement with those published by the writer,¹⁴ excepting at $2.21\,\mu$, where they find a narrow absorption band. Any errors made in eliminating the effect of this absorption upon the spectral energy curve will affect the temperature scale and hence indirectly the value of the spectral radiation constant.

From this brief review of the experimental data now available it appears that the value of the spectral radiation constant lies between $C = 14\,300$ and $C = 14\,400$.

¹² Sitzber., 44, p. 225; 1910.

¹³ Warburg, Verh. Phys. Ges., 18, p. 1; January, 1916.

¹⁴ This Bulletin, 11, p. 471; 1914.

IV. SUMMARY

The present paper gives the result of a recomputation of the constants of spectral radiation of a black body, which had been published in a previous paper. This recomputation was necessitated as the result of the adoption of a new and apparently more reliable calibration curve of the fluorite prism used in the work, and as a result of the discovery of a small error which was found in the previous computations. Although these errors are small (and would have been considered negligible four years ago) they happen to be of the same sign and hence have an appreciable effect upon the final result.

The results of the present computations give a mean value of $C = 14\,369$, which is close to the mean value of all the published data.

The data of other investigators are summarized and it is found that they lie close to $C = 14\,350$.

From a consideration of the data now available it appears that the values of the constants of spectral radiation are close to

$$C = 14\,350 \text{ micron degrees.}$$

$$A = 2890 \text{ micron degrees.}$$

and that the coefficient of total radiation is of the order of

$$\sigma = 5.7 \times 10^{-12} \text{ watt cm}^{-2} \text{ deg.}^{-4}$$

This indicates that the constant, h , of the quantum theory is of the order $h = 6.56 \text{ to } 6.57 \times 10^{-27} \text{ erg sec.}$

WASHINGTON, March 24, 1916.

APPENDIX

NOTE ON METHODS OF CALCULATING THE SPECTRAL RADIATION CONSTANTS

Because of the difficulties involved in maintaining a vacuum bolometer at a constant radiation sensitivity or in determining its radiation sensitivity from day to day it has been the custom to observe isothermal spectral energy curves and compute the value of λ_m as an intermediary step in obtaining the value of the constant C .

In the work just described the radiation sensitivity of the vacuum bolometer was determined by using the black body as a standard of radiation. It was therefore not possible to compare data obtained on different days, although the black body served as a convenient standard of radiation for comparison while observing any one isothermal energy curve.

The difficulty in obtaining a value of the constant C by computation from observations on an isothermal spectral energy curve has always been due to the uncertainty as to what formula should be used. If the Wien equation is used, then $\alpha=5$; and if the Planck equation is used, then $\alpha=4.9651$.

In computing his values of λ_m Paschen used the Wien equation, although it did not fit his best data. This gave him a high value of C , as mentioned on a previous page, where a lower and more probable value is given.

From the very beginning of his investigations on black-body radiation the writer found that the Wien equation did not fit his spectral energy curves. The assumption was therefore tentatively made that the observed curves fit the Planck equation; and, at the completion of the investigation, this was found to be true for about 75 per cent of all the observed curves. This conclusion was based upon the uniformity of the values of λ_m which resulted from computation (by the method of equal ordinates $E_{\lambda_1}=E_{\lambda_2}$) of values of λ_1 and λ_2 which were taken far apart, and also close together, on the observed isothermal spectral energy curve.

The method of equal ordinates was used because in this manner it was possible to utilize values of λ_1 and λ_2 which correspond with values of E_{λ_1} and E_{λ_2} which are closely the same in magnitude as originally observed, and hence contain the same errors of observation;¹⁵ also the slit widths are closely the same. This method does not require the most accurate wave lengths and refractive indices (at least not for values of λ_2 greater than 4.5μ) and it per-

¹⁵ The bolometer-galvanometer (zero reading) is always unsteady by 0.1 to 0.3 mm whether the deflection is 5 mm or 200 mm. Hence the parts of the curve having the same sized deflections have the same accuracy.

mits the selection of parts of the spectral energy curves which are free from atmospheric absorption bands. This accounts for the uniformity of the values of the constant C , given on a preceding page, although from the irregular appearance of the complete curves this does not seem possible. (See Fig. 1.) At a temperature of 1000°C . and at $\lambda_2 = 4.5\mu$ to 6μ an error of $\lambda = 0.05\mu$ changes λ_m by 0.007μ to 0.008μ . This amounts to 0.7 per cent in the value of C , and is much greater than would occur in practice.

The data obtained by Warburg¹⁶ and his collaborators were calculated by using E_{λ_1} , E_m , λ_1 , and λ_m . This may introduce complications because E_m often occurs in the region of the spectrum where there are atmospheric absorption bands, and where the slit-width data are difficult to determine. The method requires a more accurate knowledge of the calibration of the prism and greater refinements in the observations. They found that, using a fluorite prism, the value of C computed by using λ_1 on the short wave-length side of λ_m was different from the value obtained by taking λ , on the long wave-length side of λ_m . In the second paper they report that when using a quartz prism (which has about three times the dispersion of fluorite and hence gives greater accuracy in wave lengths, spectral purity, etc.) this difficulty was not found. Of course, this test could not be made in the long wave lengths because of the opacity of the prism. However, it seems quite conclusive that the disagreement in the computations, when using the data obtained with a fluorite prism, is to be attributed to experimental errors rather than to failure of the Planck equation. The writer experienced this same difficulty when using this method of computation, and hence used the method of calculating the constants by taking equal ordinates on the isothermal spectral energy curve.

The method of equal ordinates necessitates reducing the prismatic spectral energy measurements to a normal spectrum, plotting the data upon coordinate paper and drawing in a smooth curve from which the values of λ_1 and λ_2 are read corresponding with the equal ordinates E_{λ_1} and E_{λ_2} . Although it requires but little additional time to plot the data after having made the observations, if one is certain that the observations lie close to the curve, the obviously logical procedure is to compute the spectral radiation constants from any two observed points,¹⁷ E_{λ_1} and E_{λ_2} . This, however, does not shorten the observational work, for, as in previous investigations, it will be necessary to make observations at three adjacent spectrometer settings (separated by the width of the radiometer receiver) in order to determine the spectral purity factor which is required in order to reduce the data to a normal spectrum. After spending months in obtaining the data, the question of saving time in their reduction is of minor importance. It is desirable to draw the complete

¹⁶ *Z. S. für Instr.*, **30**, p. 118, 1910; **31**, p. 124, 1911.

¹⁷ Dellinger, *Proc. Amer. Phys. Soc.*, Washington meeting, Apr. 21-22, 1916; this Bulletin, 1916.

curve and preserve it for future reference. If the observations do not lie close to the smooth curve then this method is slightly arbitrary as to the manner of combining the observations so as to obtain an integration of the whole curve without making more computations than would be used in the "method of equal ordinates."

The appropriate formula for computing the radiation constants from any two points on an isothermal spectral energy curve is easily deduced from Planck's equation, and in its complete form, as shown by Dr. Dellinger, is:

$$C = \frac{\lambda_1 \lambda_2 T}{(\lambda_2 - \lambda_1) \log e} \left[\log \frac{E_{\lambda_2}}{E_{\lambda_1}} + 5 \log \frac{\lambda_2}{\lambda_1} - \log \left(\frac{1 - e^{-c/\lambda_1 T}}{1 - e^{-c/\lambda_2 T}} \right) \right] \quad (2)$$

The term " $\log (1 - e^{-c/\lambda T})$ " may be expanded into a series and usually all the factors can be neglected, except one which is $\log e \cdot e^{-c/\lambda_2 T}$. It is therefore necessary to know the approximate value of C , as was required in the writer's ¹⁸ method of computation.

Dr. Dellinger has made computations on data observed by the writer, using values of E_{λ_1} and E_{λ_2} , which are on opposite sides of E_m , and which are closely the same in magnitude. He obtained values of λ_m , which are in very close agreement with the data obtained by the writer, who used the method of equal ordinates, $E_{\lambda_1} = E_{\lambda_2}$. However, when the computations were made for two points which are on the same side of the maximum and which do not lie close to the smooth curve drawn through all the observations, some of the values of λ_m are discordant, as observed by Warburg. This disagreement, however, was not systematic as observed by Warburg and it was not greater than the experimental errors (2 per cent) that may easily be present in the work. This may seem a little disconcerting to those not familiar with the difficulties in obtaining the observations. It is therefore relevant to say that because of the small dispersion, the large errors of observation, the uncertainties in the dispersion and in the slit-width factors, the possible presence of scattered radiation (which appears at 4.5μ to 5μ in the spectrum produced by a prism containing numerous cleavage surfaces), variations in temperature of the radiator, etc., the writer is of the opinion that the use of equal ordinates ($E_{\lambda_1} = E_{\lambda_2}$), or the use of the general solution of Planck's equation in which E_{λ_1} and E_{λ_2} are closely the same and on opposite sides of E_m , is the most satisfactory method of calculating the spectral radiation constants from isothermal spectral energy curves obtained with a fluorite prism. The general solution is probably a more sensitive test of the validity of Planck's equation, but it requires more accurate experimental data than the writer could obtain four years ago.

¹⁸ This Bulletin, 10, p. 7 (eq. 6); 1913.

FIG. 1.—Distribution of energy in the spectrum of a perfect radiator or so-called black body, observed with a vacuum bolometer, fluorite prism, and mirror spectrometer; also the theoretical curve

In Fig. 1 is shown the observed spectral energy curve (in dotted lines; Series LXXXVI, 1912; see Table 1) of a black body at a temperature of 1077°C . The normal curve is shown by the continuous line. The arrows indicate points at which values of λ_1 and λ_2 were taken in order to compute λ_m by the method of equal ordinates.

This curve is typical of what was obtained when operating the radiator at 800° to 1000°C . From 1.7μ to 3.5μ the curve is irregular and often deeply indented, owing to atmospheric absorption bands of water vapor and carbon dioxide. Part of the indentation at 1.7μ might be caused by errors in the slit-width factors used for reducing the data to a normal spectrum and by absorption in the prism which, although quite free from color, belonged to the green variety of fluorite. An examination¹⁹ of samples of lightly colored green fluorites showed small absorption bands at 1.6 to 3.5μ . Hence no attempt was made to obtain high accuracy in the observations extending from 2μ to 3μ , for it was not intended to use the data in the computations.

The absorption bands of atmospheric water vapor at 5.6μ to 6.2μ were also investigated.²⁰ At 6.25μ there is a narrow region of minimum absorption, amounting to 10 to 12 per cent, followed by absorption bands at 6.3 to 6.7μ .

In Fig. 1 the circles represent the computed emission, using Planck's equation and $C = 14\,350$. In the region of the spectrum from 1μ to 1.6μ and from 3.6μ to 5.5μ there is a complete agreement between the observed and the computed curve. Several observed points deviate from the smooth curve by 1 to 2 per cent, which is better than one can expect, considering the fact that a correction had to be applied for a change in the radiation sensitivity of the bolometer.

An inspection of Fig. 1 and Eq. 2, shows that inconsistent values of C must result from combining data which are above and below the smooth curve and on the same side of λ_m . The computation by the method of equal ordinates suppresses these errors of observation by estimating the value of $E_{\lambda_1} = E_{\lambda_2}$ and by placing more of the burden of proof upon determinations of λ_1 and λ_2 .

In obtaining the experimental data the usual procedure was to begin observations by making a few readings at a few points preceding the wave length (1μ in this series) at which computations were to be made. This was done for the purpose of "warming up the apparatus" and for obtaining the "purity factor" to reduce the data to a normal spectrum. Only three readings were made at 0.92μ , and the temperature of the radiator had not yet become steady. Hence, the writer does not consider the small difference between the observed and the computed value at 0.92μ to be due to a real disagreement between theory and experiment. The same is true for the region of the spectrum beyond 6μ . Correcting

¹⁹ This Bulletin, 9, p. 116, Fig. 12.

²⁰ This Bulletin, 7, p. 635, Fig. 3.

for atmospheric absorption at 6.24μ produces exact coincidence between the observed and the computed values. When one considers that an accuracy higher than 1 per cent was difficult to attain, and that momentary disturbances might introduce a range of 2 to 5 per cent in the galvanometer readings which were less than 2 cm deflection, it is safe to say that the Planck equation fits the experimental data to 6.5μ .

FIG. 2.—Observed and computed energy distribution in the spectrum of a black body

In Fig. 2 is shown the spectral energy distribution at 1323°C . It is typical of the curves obtained at high temperatures. This illustration gives two series of measurements obtained at different dates. The theoretical curve for $T=1596^{\circ}\text{K}$ and $C=14350$ coincides remarkably well with the observations, considering the

possibility of stray light, especially at 5μ where the observations are high. The low values at 0.7μ to 1μ , in Figs. 1 and 2, may be due to improper correction of the absorption by the silver mirrors. The slight shearing of the observed and the computed curves may be due to incorrect temperatures. At any rate it is hardly sufficient to be considered a real departure from the Planck equation; although as mentioned in the previous paper there are some indications that the Planck equation may not fit the observations beyond 7μ .

As mentioned in the previous paper, these data are the first indication that Planck's equation fits the experimental observations in this region of the spectrum. The few discordant experimental data obtained with optical pyrometers are hardly convincing proof that this equation does not apply also to measurements made in the visible spectrum. To make the test more conclusive, an outfit is being constructed for operating the radiator, and spectroradiometric apparatus in a vacuum, in order to eliminate atmospheric absorption. Because of the numerous subsidiary investigations which must be made preliminary to this investigation, some time must necessarily elapse before data can be obtained using all the apparatus in a vacuum.