OUTLINE OF DESIGN OF DEFLECTION POTENTIOMETERS, WITH NOTES ON THE DESIGN OF MOVING- COIL GALVANOMETERS

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1. INTRODUCTION

This paper was originally planned as a continuation of an article¹ entitled, "Deflection Potentiometers for Current and Voltage Measurements," to which the reader is referred. The object of the present paper is to outline the principles on which deflection potentiometers are designed. It is thought that the matter here presented will be of interest not only to instrument makers and designers, but also to many users who are interested in the development of the potentiometer method. This method in its purely null form has done and is doing great service in the physical laboratory, where the highest precision must be had, regardless of the outlay of time and labor. It has also found some use in technical applications, such as the laboratories of the larger central stations. In such work, it has always labored under the disadvantages attending any balance method, namely, the large amount of time required, and the necessity for steady sources of current and voltage. The deflection potentiometer combines in any desired proportions the accuracy and reliability of the (null) potentiometer and the quickness of working and independence of fluctuations of a well-damped deflection instrument. It is therefore especially suitable for a large class of work which does not require extreme precision, but in which a high degree of reliability is necessary.

¹ This Bulletin, 8, p. 395; 1911 (Reprint No. 172).

While the essential principle of operation of the deflection potentiometer is very simple,² the requirements of speed of working, accuracy, and the use of a pivoted portable galvanometer demand considerable care in the design. The methods found most suitable by the writer will be considered in detail.

2. OUTLINE OF DESIGN OF DEFLECTION POTENTIOMETERS

In the "Outline of Method of Design" previously given,³ the assumption was made that a volt box was to be used. In the more recent instruments it has been found more convenient to start with assumptions which apply directly to the potentiometer alone, and to consider the volt box later.

As emphasized in the preceding articles, the galvanometer is the central and important feature in the design; its constants determine what can be obtained in the way of sensitiveness of reading, speed of working, resistance of volt box, and resistance of the potentiometer circuits.

The quantity denoted by Σr in the preceding articles is the total resistance in the galvanometer circuit; it should have the proper value for the condition of critical damping.⁴ This is a frequently neglected but very important galvanometer constant. With this value of total resistance the motion of the coil is "dead beat," and when a change of current takes place the coil moves to its new position without oscillation. This condition is assumed in the design of deflection potentiometers, in which the combination of accuracy and speed is the paramount feature. Σr thus denotes the sum of galvanometer resistance (including resistance of springs), the resultant resistance⁵ $r_1 r_s / (r_1 + r_s)$ of the potentiometer wire and rheostats, and the resistance which may be allowed in an accessory (volt box or current shunt). In a former article⁶ it was shown that if $1/p$ is the "volt-box fraction," m the

² This Bulletin, 8, p. 399; 1911 (Reprint No. 172).

³ This Bulletin, 4, p. 298; 1908 (Reprint No. 79).

⁴ This is sometimes called the "total critical resistance."

⁵ The symbol r_s is used here to denote the sum of r_2 and the constant resistance of the shunt rheostat r_6 in parallel with the series rheostat r_3 . (See Fig. 1.)

⁶ This Bulletin, 2, p. 233; 1906 (Reprint No. 33).

number of divisions deflection to be produced by 1 volt change in line voltage, and I the galvanometer current in amperes for a deflection of one division, then

$$\Sigma r = \frac{1}{pmI} \quad (11)$$

Putting $p = 1$ gives a special case in which the volt box becomes unnecessary, for if one were used it would be a single resistance across the source (whose voltage is to be measured) and in parallel with the potentiometer; it is therefore only a superfluous load on the source and does not affect the operation of the potentiometer. Omitting the volt box, the fundamental design equation becomes

$$\Sigma r = \frac{1}{nI}, \text{ or } I\Sigma r = \frac{1}{n} \quad (34)$$

in which n denotes the number of divisions deflection produced by a change of 1 volt at the potentiometer terminals. Equation (34) may be written in the form

$$\frac{1}{\Sigma r} = nI$$

That is, the *conductance* of the whole galvanometer circuit varies directly as the number of divisions to be given per volt, and as the current in amperes for one scale division. For any given galvanometer, Σr and I are fixed,⁷ and hence n is determined. For commercial reasons one would usually try to adapt an existing type of galvanometer to the purpose, using it as a starting point and making changes until the proper values of Σr and I are obtained. In general, the total resistance for critical damping should be from 5 to 10 times the coil resistance, and the period should be short, say about 1 to 1.5 seconds. The combination of

⁷ The resistance for critical damping is not sharply defined, and hence some range of values is permissible for Σr . However, it is better to have Σr slightly greater than the critical value, rather than the reverse, in order that the pointer may pass slightly beyond the position of rest and then return to it. With too low a value of Σr , creeping occurs and the impression is given that friction is present.

these two qualities calls for rather high flux density in the air gap.⁸ The value of n , the number of divisions per volt, would usually be chosen arbitrarily. This fixes the value of the product $I\Sigma r$; the proposed galvanometer is then examined to see how nearly it meets this requirement. Further, it is necessary that Σr should be high enough to permit sufficient resistance in the potentiometer and its adjuncts; it is also necessary that the galvanometer shall have sufficient sensitiveness when used for checking the current in the potentiometer wire by reference to the standard cell.⁹

Assuming that the values of total resistance Σr and the product $I\Sigma r$ are satisfactory,¹⁰ the upper limit of the potentiometer range may be chosen. A range in general use with null potentiometers is 0 to 1.5 volts. This has the advantages of requiring but one storage cell, and of being of the same order of magnitude as the voltage of the standard cell. The total resistance Σr is to be apportioned to the various parts in a somewhat arbitrary manner, according to the judgment of the designer and the requirements to be met. First, we may choose the external resistance; that is, the resistance of the portion of the volt box across which the potentiometer is to be connected. In general, with an average design, about 40 or 50 per cent of Σr would be allotted to this. The resistance assigned ought to have some even and usual value (such as 20, 40, 50, 100 ohms) to avoid the expense of winding and adjusting for odd or special values. The value assigned to this purpose is also the value of the current shunt

⁸ W. P. White, *Physical Review* **23**, p. 385 (1906), gives a table showing the approximate field strengths required for different periods, with various ratios of total resistance to coil resistance.

This high value of ratio (5 to 10) is necessary for two reasons: First, it makes the reading of the galvanometer (for a given voltage applied to Σr) practically independent of temperature; second, it gives the ability to use higher values of resistance in the potentiometer and in its accessories (volt box or current shunt). By reducing the value of H in the air gap of the galvanometer, Σr being reduced to retain the condition of critical damping, one can increase the volt sensibility, but at the expense of the two important features just mentioned.

⁹ For a discussion of this requirement, see pp. 429-431.

¹⁰ The question of changes to be made in the galvanometer, in case it does not have suitable constants, is treated in another part of this paper (p. 434).

r_m , the internal resistance available for the potentiometer circuits; that is, the constant quantity¹² (see Fig. 1, p. 423).

$$\frac{r_1 \left(r_2 + \frac{r_3 r_6}{r_3 + r_6} \right)}{r_1 + \left(r_2 + \frac{r_3 r_6}{r_3 + r_6} \right)} + r_4$$

If the maximum dial setting is 1 volt or over, in a potentiometer supplied from a single (lead) storage cell, the slider of the main dial reaches or passes the center of the resistance denoted by the denominator of the fraction in the above expression; at the center the resultant resistance of the two halves in parallel is a maximum and is equal to one-fourth of the total. Hence this total may have four times the resistance r_m reserved for use in the internal circuits. As the value of the main dial resistance r_w is rather sharply limited in some cases by the restrictions due to the use of values of r_3 and r_6 which give constant resultant resistance, this point should now be investigated.¹³ If the proposed upper limit of the potentiometer range is not enough below the voltage of the storage cell (or battery) intended to be used, the limits for r_w may be too narrow, and another cell should be added. Having a suitable margin, choose¹⁴ a value for r_w such that each coil is of a convenient (and if possible, a usual) number of ohms, such as one of the following: 1, 2, 5, 10, 20, Having chosen r_w , see how much extension of this is necessary between the upper end of the dial and the rheostats r_3 and r_6 , to provide for the deci-

¹² The significance of this quantity may be best seen by reference to this Bulletin, 4, p. 281 (Reprint No. 79). This quantity is part of the denominator in equation (17), and is discussed.

¹³ This Bulletin, 4, pp. 283 and 284; 1908 (Reprint 79). See discussion of equation (20).

¹⁴ In choosing r_m and r_w , if possible select values such that the maximum resultant resistance r_m is equal to an integral number of steps of the main dial, or to $(n+0.5)$ steps, where n is an integer. This enables a number of ballast coils for the main dial to be dispensed with, thus lowering the cost. For a numerical example see footnote 22, page 429.

As a first approximation, r_w may be taken as $3r_m$ to $3.6r_m$, for potentiometers of 1.5 volts and over. Below 1.5 volts in general a part of r_w will not be in the main dial, but will constitute an extension of this dial. The resistance r_m may then fall short of one-fourth of the resistance $r_w + r_6$; that is, r_w may have a value higher than is given by the general rule.

mal part of the emf of the standard cell. (See Fig. 2.) This increases r_w to an odd value which is used in getting the constant¹⁵

$$r_w(4r_m - r_w)/4ar_m$$

This constant, when multiplied by the successive values of e_1 , the battery emf, gives the successive values of r_3 , the series rheostat. Suitable upper and lower limits of e_1 (per cell) are 2.14 and 1.98 volts; using these, compute the two values of r_3 , and take their difference. This difference is to be divided into a number of equal steps, the number depending on the degree of constancy of Σr desired, since the fine rheostat can not be perfectly compensated. The degree of constancy can, however, be made as perfect as desired. (See pp. 429 and 433.) In the deflection potentiometers so far designed for precision purposes, r_3 increases from its minimum value by 15 equal steps. The above limits for e_1 may be varied a little, so as to get a convenient value per step of r_3 .

Having values of r_3 corresponding to each step, values of r_6 , the resistance in parallel with the potentiometer wire, may be computed from the formula:

$$\begin{aligned} r_6 &= \frac{(4r_m - r_w)r_3}{r_3 - (4r_m - r_w)} \\ &= \frac{kr_3}{r_3 - k} \end{aligned} \quad (22)$$

If values of r_6 at first obtained are extreme, it may be necessary to revise the values of r_w or r_m so as to give workable values of r_6 . In general, the best values are found for r_6 by taking a value for r_w near the mean value given by

$$r_w = \frac{2(a + e_1)r_m}{e_1} \quad (23)$$

where a and e_1 are the lower and upper limits of the battery emf.¹⁶

The ballast coils (r_4) of the main dial are given such values as will make the sum of the ballast resistance and the resultant resistance $r_1 r_8 / (r_1 + r_8)$, a constant, r_m .

¹⁵ This Bulletin, 4, p. 283; 1908 (Reprint No. 79). See equation (21) and context.

¹⁶ This Bulletin, 4, p. 284; 1908 (Reprint No. 79). It is sometimes convenient to solve formula (23) for r_m , and work from an assumed value of r_w which gives a desirable integral value of resistance per step of the main dial to determine whether the resultant value of r_m is attainable. It should be noted that (23) is an arbitrary relation only, being a mean between two impossible extremes.

The fine rheostat demands special consideration, as explained in a previous paper.¹⁷ Since the coarse rheostat accomplishes a given change in the current through the potentiometer wire by the united effect of a change of the series resistance r_3 and a simultaneous change of the shunt resistance r_6 , while the fine rheostat uses series resistance only, an amount of the latter greater than one step of the series rheostat must be used. In a previous article¹⁷ a formula was developed for computing the value of the fine rheostat. In deriving this, the series rheostat r_3 was assumed to have its calculated value for each step, to which the fine rheostat could add any resistance from zero up to its full value. A better plan is to make the values of r_3 low by a constant amount, equal to one-half of the fine rheostat. Then the resultant resistance, $r_3 r_6 / (r_3 + r_6)$, has its normal value when half of the fine rheostat is in circuit, and the departure of this resultant resistance from the normal value, due to adding or subtracting half of the fine rheostat, is but half as great as if r_3 had its normal value and the whole of the fine rheostat were added. This method of keeping down the deviation of the resultant resistance from its normal value was used in a previous form of deflection potentiometer.¹⁸ The value of the fine rheostat is given with all required accuracy by the older formula, so long as the steps of the coarse rheostat are not large. However, a more accurate formula may be obtained, as follows. Assume that the battery voltage is at the upper limit for which the coarse and fine rheostats provide, the coarse rheostat r_3 requiring half of r_8 , the fine rheostat, to give the normal value of resultant resistance of series and shunt rheostats in parallel. Now let the battery voltage drop so that the fine rheostat is all cut out; thus the series rheostat has the resistance $r_3 - r_8/2$, and the shunt resistance has the value r_6 . For this arrangement of circuits, a certain current will flow through the potentiometer wire. To be ready to take care of further drop in battery voltage, we must now cut out one step, Δr_3 , of the coarse rheostat, and throw the fine rheostat all in; the series resistance now has the value $r_3 - \Delta r_3 + r_8/2$, and the shunt resistance has the value $r_6 + \Delta r_6$.

¹⁷ This Bulletin, 4, pp. 284-5; 1908 (Reprint No. 79).

¹⁸ This Bulletin, 4, p. 291 (Reprint No. 79).

If r_s has the correct resistance, the current will be the same as before. The general expression for the current through the potentiometer wire is (when no current flows through the galvanometer)

$$i = \frac{e_1}{r_w + \frac{r_w + r_6}{r_6} r_3} \quad (24)$$

Since e_1 is the same for the two cases just assumed, we may equate the corresponding expressions for the effective resistance, as given by the denominator of the right-hand member of (24); this gives

$$r_w + \frac{r_w + r_6}{r_6} \left(r_3 - \frac{r_8}{2} \right) = r_w + \frac{r_w + r_6 + \Delta r_6}{r_6 + \Delta r_6} \left(r_3 - \Delta r_3 + \frac{r_8}{2} \right)$$

from which

$$r_8 = \Delta r_3 \frac{1}{1 + \frac{r_w \Delta r_6}{2 r_6 (r_w + r_6 + \Delta r_6)}} + \Delta r_6 \frac{r_3 r_w}{r_6 (r_w + r_6 + \Delta r_6) + \frac{r_w \Delta r_6}{2}} \quad (35)$$

This exact formula gives (for Model 3) values of r_8 about 0.7 per cent lower than are given by the older formula, equation (25). Since one would ordinarily allow at least 10 per cent margin above the computed value, in specifying the value to be aimed at by the maker, this small difference is insignificant. However, in special cases, with unusual values of series or shunt resistance, and with fewer steps in the main rheostat, it might be important to use the accurate formula (35). It will be found that different values of r_8 are required for different steps on the coarse rheostat. In the Model 3 potentiometer the value of r_8 required with all of the series rheostat r_3 in circuit is 0.355 ohm, and with but one step of r_3 in use, 0.421 ohm. The higher value should of course be used and a small margin allowed for the maker's convenience. The specifications for Model 3 call for a fine rheostat of nominally 0.5 ohm, the maximum and minimum permissible values being 0.55 and 0.45 ohm.

When the fine rheostat r_8 is in any but its mid-position the resultant resistance $r_3 r_6 / (r_3 + r_6)$ does not have exactly its normal value. The small error in the value of Σr so introduced is greatest when the main dial setting is a maximum and the coarse rheostat

r_3 is a minimum. In a previous paper¹⁹ is given a numerical example of the calculation of the resulting maximum error in the galvanometer deflection (about 0.2 per cent), which would be below the usual limit of reading. This error is compensated (at settings of the main dial near the upper limit) by the use of a fine rheostat in the galvanometer circuit, as shown in the diagram of connections (Fig. 2, p. 430). These two rheostats are combined, a single contact lever running over both spirals.

A numerical example (given in the following seven pages) will illustrate the general method of design.

It is desired to have a fundamental range of 1.5 volts, which must be readable directly to a part in 1,500, and by estimation of tenths of a galvanometer division, to a part in 15,000. One scale division is thus to equal $1.5 \div 1,500 = 0.001$ volt; hence by (34)

$$I\Sigma r = 0.001$$

By modifying an existing type of galvanometer this value of $I\Sigma r$ was obtained. The value of I (current in amperes per scale division) was 0.00001; hence,

$$\Sigma r = \frac{0.001}{0.00001} = 100 \text{ ohms.}$$

The coil was critically damped for this value of the total resistance of the galvanometer circuit, and this total is over 10 times the coil resistance; hence the galvanometer fulfills the requirements. The period is about 1.2 seconds. The only improvement which would be desirable is that of a higher value of Σr , 100 ohms not being as high as is desirable.

Of the 100 ohms, 40 ohms will be assigned to "external" resistance (volt box or shunt); adding 10 ohms for galvanometer resistance and 10 ohms for the calibrating coil gives 60 ohms; $100 - 60 = 40$ ohms is thus available for "internal" resistance. The range being over 1 volt, and a single storage cell being evidently sufficient, the resistance of the potentiometer wire r_w plus the constant resultant of rheostats r_3 and r_6 in parallel will be $4 \times 40 = 160$ ohms. The resistance of the main dial will be from 3 to 3.6 times 40, or 120 to 144 ohms. This raises the question of the resistance of each coil of the main dial. The range of the galvanometer is 30 divisions on each side of the central zero; this covers a total of 0.060 volt. This scale is planned to provide for 0.050 volt, the other 0.005 at each end being left as margin, not ordinarily used. Thus the number of steps in the main dial is $1.5 \div 0.050 = 30$. The value of each coil of the main dial is thus 4 to 4.8 ohms. As 10 per cent is a liberal allowance for the calibrating coil, we may reduce it slightly, and allow more "internal" resistance; it may then be possible to allow 5 ohms per coil, or 150 ohms for the main dial. One hundred ohms

¹⁹ This Bulletin, 4, p. 291; 1908 (Reprint No. 79).

of this dial would have 1 volt drop; to provide for the decimal part of the value of a Weston cell we must add 1.9 ohms to the dial; r_w is thus 151.9 ohms. This must be checked to see whether proper values of series and shunt rheostats (r_3 and r_6) can be obtained. First, use the arbitrary formula (23), solving for r_m ; then substitute numerical values.

$$r_m = \frac{e_1}{2(a + e_1)} r_w = \frac{2.14}{2(1.519 + 2.14)} \times 151.9 = 44.4 \text{ ohms.}$$

This is close to the desired value (40 ohms), but requires further check. The value of the shunt resistance r_6 is given by the equation ²⁰

$$r_6 = \frac{e_1}{a} \frac{4r_m - r_w}{e_1 - \frac{4r_m}{r_w}} \quad (20)$$

Suitable upper and lower limits of e_1 for a lead storage cell are 2.14 and 1.98 volts; a , the drop on the potentiometer wire plus extension for standard cell, is 1.519 volts. The minimum value of r_6 occurs at the maximum value of e_1 . Substituting,

$$r_6 = \frac{2.14}{1.519} \frac{(4 \times 40) - 151.9}{\frac{2.14}{1.519} - \frac{4 \times 40}{151.9}} = 32.1 \text{ ohms.}$$

The maximum value of r_6 , for $e_1 = 1.98$ volts, = 42.2 ohms. These values are rather low, as with 32 ohms in parallel with the potentiometer wire of 151.9 ohms, the current through the shunt is nearly five times the useful current.²¹ To improve this condition we may reduce the calibrating coil by 2.5 ohms, increasing r_m to 42.5 ohms.²² With this value, the use of formula (20) gives 88 and 128 ohms, respectively, for the lower and upper values of r_6 . These are satisfactory, the average current in the shunt circuit being 1.4 times the current in the potentiometer wire.

At this point we must investigate an important question, namely, the precision with which the auxiliary current through the potentiometer wire can be checked by reference to the standard cell. It is clear that any error in adjusting this current will enter into the null part of the result, which is the main portion. It has been sought (in the instruments so far designed) to have such a sensitiveness that the auxiliary current can be set to 1 part in 10 000. The electromotive force of the standard cell being a little over 1 volt, this requires that the galvanometer shall give a perceptible movement of the pointer for the current due to 0.0001 volt, the resistance in circuit being the sum of the internal resistance of the standard cell, the galvanometer resistance, and the resultant resistance

²⁰ This expression for r_6 should be used only at this preliminary stage; after values of r_3 have been determined, the values of r_6 are more conveniently found by using equation (22). See p. 425.

²¹ A further disadvantage in having r_6 small as compared with r_w is that the fine rheostat must have a value unduly large compared with that of one step of the coarse rheostat r_3 .

²² The value 42.5 satisfies the condition given in footnote 14, p. 424, enabling 13 coils in the ballast resistance r_4 to be dispensed with.

Fig. 2.—Plan of Circuits, Model 3 Potentiometer

of the part of the potentiometer wire around which the drop is taken, shunted by the rest of the potentiometer wire and the (constant) resistance through the rheostats and storage cell. The circuit in question will be apparent from Fig. 2, page 430. The standard cell in series with the galvanometer is connected across the last 100 ohms of the main dial, plus about 2 ohms. The (constant) resistance of the remainder of the main dial and the rheostats is $170 - 102 = 68$ ohms. The resultant resistance of 102 ohms in parallel with 68 ohms is about 41 ohms. Adding 10 ohms for the galvanometer and 200 ohms²³ for the standard cell gives a total resistance of 251 ohms. The current through this resistance due to 0.0001 volt is 0.4 microampere, and the resulting deflection of the galvanometer is 0.04 scale division. While the position of the index can not usually be estimated to better than 0.1 division, a movement of 0.04 division is perceptible when the index is directly over (or close to) a line. Hence a deviation of the auxiliary current from its normal value of 1 in 10 000 can be detected.

Owing to the caking together of the crystals of saturated standard cells, such cells often have very high internal resistance. *They should not be used with the deflection potentiometer.*

It is not always possible to have the condition of critical damping when using the galvanometer in the standard-cell circuit. Thus, in the example just given, the total resistance being 251 ohms, against 100 ohms for critical damping, the motion would be considerably underdamped. As the travel of the index (in this use of the galvanometer) is quite small, this is not of consequence. If (as occurs in Model 5 potentiometer) the total resistance is much below the critical value, when checking against the standard cell, it is advisable to use enough extra resistance in series to give nearly critical damping.

The accuracy in checking the auxiliary current, in the case previously mentioned, could be materially improved if necessary by the use of a special standard cell of larger size and consequently lower resistance. It should be possible to get a cell of, say, 40 or 50 ohms resistance, without undue increase in size. This would give 100 ohms total, and an error of 1 part in 10 000 in the auxiliary current would give 1 microampere through the galvanometer; this would cause a movement of 0.1 division. Except in special cases, however, one would not care to go to the trouble of making up a special standard cell.

Values of r_3 may now be computed, using the formula

$$r_3 = \frac{e_1}{a} \frac{r_w(4r_m - r_w)}{4r_m} = \text{constant} \times e_1 \quad (21)$$

²³ The (European) Weston Co. gives the internal resistance of these cells as about 160 ohms. The values found by the Bureau of Standards range from 130 ohms for a relatively new cell to 260 ohms for one that had been in use for about five or six years. The internal resistance of a given cell can be roughly checked by setting the standard cell switch at the extreme right, adjusting battery current to give no deflection, then throwing the standard cell switch to the extreme left (a change of 0.0001 volt) and observing the deflection. This should be done with a new cell, and the value recorded.

Substituting

$$\begin{aligned} r_3 &= \frac{151.9(170 - 151.9)}{1.519 \times 170} e_1 \\ &= 10.647e_1 \end{aligned}$$

Using the values 1.98 and 2.14 for e_1 , r_3 has the values 21.08 and 22.78 ohms; r_3 will thus consist (neglecting the fine rheostat for the present) of a fixed resistance of 21.08 ohms which is increased by equal steps to 22.78 ohms. The sum of the steps is 1.70 ohms, and as 15 steps have been found a suitable number, we may have 17 steps of 0.1 ohm each.²⁴ In the instrument which is here used for illustration, it was desired to use 15 steps to avoid change of patterns used in a previous model. This was done by narrowing the limits of e_1 to 1.99 and 2.13 volts. The fine rheostat is half in, for the corresponding values of r_3 , and hence by inserting or removing the other half, the battery voltage may be carried a little beyond each of these limits. We may thus assign values for r_3 for all the steps of the coarse rheostat; the next thing is to compute a series of values of r_6 by formula (22), page 425. The lowest value of r_6 is a coil, to which the coarse rheostat adds 15 steps. These steps, unlike those of the series resistance r_3 , will not be equal, the value of a step increasing with increase of r_6 .

The values of r_4 (the "ballast" resistance) are such as will make up a total of 42.5 ohms, for any step of the main dial. When the sliding contact of the main dial is at the zero setting (the point A, Fig. 1) the current flowing down through the galvanometer meets no resistance in the potentiometer wire, but flows through r_4 only; for this point r_4 must have its maximum value, 42.5 ohms. When the slider is at the first step to the right of A, the resistance to the galvanometer current is the 5-ohm coil (r_1) shunted by the rest of the 170 ohms (main dial plus rheostats). Hence r_4 must be less than 42.5 ohms by the quantity

$$\frac{5(170 - 5)}{170} = 4.85 \text{ ohms.}$$

Other values of r_4 are found in the same way. At the seventeenth step of the main dial the resistance $r_1 = 85$ ohms, which equals the remainder of main dial plus the rheostats; the resultant of these two 85-ohm paths in parallel is 42.5 ohms, and the value of r_4 is zero. From this maximum point the resultant resistance will decrease as the slider takes positions farther to the right. The resultant resistance will repeat at the eighteenth, nineteenth, thirtieth steps the values it had at the sixteenth, fifteenth, fourth steps. Hence, instead of using a second set of r_4 coils, as shown in the elementary diagram, Fig. 1, we can save 13 coils by using cross connections, as shown in diagram of connections, Fig. 2.

²⁴ Since r_3 varies directly as the battery voltage, it is convenient to give r_3 equal steps, which should be of convenient (round) value.

The fine rheostat is next to be computed. Taking the upper value of r_3 , 22.78 ohms, the corresponding value of r_6 is 88.05 ohms. If r_3 be reduced by 0.1 ohm (Δr_3) the corresponding increase (Δr_6) in r_6 is 1.58 ohms. Substituting in equation (35),

$$\begin{aligned}
 r_8 &= \frac{0.1}{1 + \frac{151.9 \times 1.58}{2 \times 88.05(151.9 + 88.05 + 1.58)}} \\
 &\quad + \frac{1.58 \times 22.78 \times 151.9}{88.05(151.9 + 88.05 + 1.58) + \frac{151.9 \times 1.58}{2}} \\
 &= \frac{0.1}{1 + 0.0056} + \frac{5467}{21267 + 120} \\
 &= 0.0994 + 0.2556 = 0.355 \text{ ohm.}^{25}
 \end{aligned}$$

If a similar calculation be made for the minimum value of r_3 , it will be found that r_8 is larger, namely, 0.418 ohm. Some margin should be allowed beyond this value to avoid the unnecessary expense of adjusting the slide rheostat closely.

To compute the value of the fine rheostat in the galvanometer circuit (see p. 428), we may find the error in Σr for minimum r_3 and the maximum setting on the main dial. For this calculation assume that the battery fine rheostat r_8 , just discussed, is given the value 0.5 ohm. With $r_3 = 21.08$ ohms, $r_6 = 128$ ohms (0.25 ohm of r_8 going to make up r_3), the resultant resistance of the rheostats is

$$\frac{21.08 \times 128}{21.08 + 128} = 18.10 \text{ ohms,}$$

the normal value. Now let the remaining half of r_8 be put in circuit, giving 21.33 ohms in place of 21.08, and increasing the resultant resistance to

$$\frac{21.33 \times 128}{21.33 + 128} = 18.28 \text{ ohms.}$$

In each case, the resistance r_w (151.9 ohms) is in parallel with the rheostats giving (for the two positions of the fine rheostat) final resultant resistances of 16.173 and 16.316 ohms. The difference is 0.14 ohm; a similar calculation for r_3 maximum gives 0.12 ohm. Hence, the galvanometer side of the fine rheostat may have a mean value of about 0.13

²⁵ The small quantities at the right in each of the denominators are absent in the formula given in a previous paper, equation (25), which did not take into account the practice of having the computed values of r_3 include half of the fine rheostat. See discussion of this point, p. 426, and following. It will be seen that for the present design these quantities are negligible.

ohm each side of its central position, or 0.26 ohm total. It may be noted that if this fine rheostat were not provided the error would be less than 0.2 per cent (0.14 in 100) of the deflection part of the result, or 0.04 of a scale division in the maximum deflection. This would be under the usual limit of reading. The fine rheostat is put in the galvanometer circuit on the general principle of avoiding small errors insignificant in themselves, which might add up to an appreciable amount.

The size of wire and length of coil composing the fine (battery) rheostat r_8 should be chosen so as to give a sufficient number of steps. For example, in the preceding design a step on the coarse rheostat corresponds to a change of 0.01 volt in the 2-volt storage cell, or 1 part in 200. If we make the fine rheostat of, say, 100 turns, each turn will be 1 part in 20 000, which is ample fineness of adjustment of the current. The steps on the galvanometer side of the fine rheostat may evidently be very much coarser if desired.

As it is not possible to get as perfect contacts on the fine rheostats as on the coarse, a safety connection is used. (See Fig. 2, p. 430.) This consists simply in connecting the free end of the spiral to the slider. With this connection, the failure of the slider to make contact can at most do no more than introduce the whole of the fine rheostat into the circuit. Without this connection, the failure of the slider to make contact would break the circuit. This is a device used in some dynamo rheostats; it should be more generally known and used. It is not used on the coarse dials of the potentiometer, partly because it is very much less needed, partly because it would have to be disconnected when checking the values of the coils.

The standard-cell dial may have 10 steps of 0.01 ohm each, thus providing for cells of from 1.0180 to 1.0190 volts. (See Fig. 2, p. 430.) The protective coil should be 24 times Σr , or 2400 ohms.²⁶

The grouping of the various dials and the galvanometer should be such as will give speed and convenience of working. The reasons for the general arrangement used in the present models are briefly given in a preceding article.²⁷

3. NOTES ON THE DESIGN OF MOVING-COIL GALVANOMETERS

The complete design of a moving-coil galvanometer of the high grade required for a deflection potentiometer is beyond the limits of the present article. However, some general considerations may be useful to those who wish to adapt an existing type of moving-coil instrument to the purpose.

In the deflection potentiometers so far designed at this Bureau, pivoted galvanometers only have been used. As much higher

²⁶ The normal maximum deflection of the galvanometer is 25 divisions, and with a preliminary external resistance of 24 times the normal total, the deflection is to be brought to 1 division or less, by manipulating the main dial. If the key be now fully depressed, the reading will not be over 25 divisions.

²⁷ This Bulletin, 8, p. 405, 1911 (Reprint No. 172).

sensibility can be had in the reflecting galvanometer with suspension strips or wires, the question may arise, Why are such galvanometers not used? Briefly, the reasons are as follows:

1. The reflecting galvanometer is harder to read, if a telescope is used, and causes much more fatigue to the operator. If lamp and scale are used, a semidarkening of the room is required.

2. Reflecting galvanometers are far inferior to good pivoted galvanometers in proportionality of scale, repeatability of reading for a given current, and general reliability. This is due largely to the smallness of the controlling force of the suspension in a reflecting galvanometer; disturbing forces, due to magnetic impurities in the coil and to electrostatic action, are relatively much larger and their effect much more pronounced than in good pivoted galvanometers.

3. If the level of a suspended-coil galvanometer is altered, the coil is brought into a different part of the field, and its deflection for the same current will be different. This necessity for accurate leveling makes the suspension galvanometer troublesome when apparatus must be moved.

4. The suspended-coil galvanometer is sensitive to vibration (as that due to running machinery) and will be put out of commission by ordinary handling such as would not affect a good pivoted instrument.

If the magnets of a moving-coil galvanometer be removed, or if we leave the coil on open circuit, no damping frame or coil being present, the coil will have but little damping, that which exists being principally due to air friction. If the coil be displaced from its position of rest, it will oscillate about this position with a period ²⁸ T which is defined by the relation

$$T = 2\pi\sqrt{\frac{K}{U}}$$

where K is the moment of inertia of the coil and its fittings and U is the restoring couple of the spring for unit angle. This expression neglects the small damping due to air friction, which

²⁸ "Period" here refers to a complete oscillation; for example, starting from the position of rest, out to the right, back through the starting point to the left, then back to the starting point.

(in the type of galvanometer considered) does not affect the result appreciably. If K is expressed in $g\text{-cm}^2$ and U in centimeter-dynes per radian, T will be in seconds. The value of K will in general not be subject to modification, since changing the size of wire will not appreciably affect the mass of the coil, unless relatively coarse and fine wires are involved in two coils under consideration. In this case the relatively large space taken by the insulation of the fine wire may reduce the mass of the coil appreciably. However, the moment of inertia of the coil is only part of the total, as that of the pointer will be quite appreciable, due to its length. In general, then, one would regard K as fixed for the type, and would vary U , the spring strength, until a proper value of T (say 1 to 1.5 seconds) was obtained. The value of T may be conveniently found by using a stop watch to measure the time required for several complete oscillations. The value of U may be found²⁹ by turning the instrument so that the axis of rotation is horizontal, and the pointer is also horizontal when in its position of maximum deflection. The pointer is brought to this position by a small weight hung on it, such as a piece of wire. This weight is slid out along the pointer until it balances the torque of the spring and holds the pointer as described. Then the product of the mass of the wire in grams, its distance from the axis in centimeters, and the factor 981, will be the torque of the spring for full deflection, expressed in cm-dynes . The angle corresponding to the given deflection may be measured in any convenient way; the value of U , the torque for one radian ($57^\circ 3'$) may then be computed. The preceding method of torque measurement is subject to some errors (due to shifting of point of contact of pivots in jewels, etc.) which are relatively greater as the spring strength becomes smaller, for a given coil. A pendulum apparatus³⁰ has recently been used at this Bureau for the measurement of spring strength of instruments and the torque of integrating meters. This method has the advantage of keeping the moving coil in its normal position, with its axis of rotation vertical.

²⁹ This method has been used by the writer since 1900; it was described by Friedr. Janus in *Elektrotechnische Zeitschrift*, vol. 26, p. 561; June 15, 1905.

³⁰ P. G. Agnew, this Bulletin, 7, p. 45; 1911 (Reprint No. 145).

In order to avoid errors due to pivot friction, it is necessary that the ratio of torque (for a given angle) to the weight of the entire moving system shall not be allowed to fall below a certain value. According to Heinrich and Bercovitz,³¹ the torque in cm-gm for 90° deflection should not be less than 5 per cent of the weight of the coil in grams. Janus³² gives a considerably higher value, namely, 17 per cent, but does not state what angle; from the context, it is probable that 90° is intended. The quality of the pivots and jewels has much to do with the permissible limit for this ratio, lower values of ratio being allowable for more perfect pivots and jewels.

Assuming that a proper value of T has been obtained, using a spring which satisfies the requirements of torque-weight ratio, we may consider the other fundamental constants of the galvanometer.

With the magnets on the pole pieces, if the coil be connected to a very high external resistance the damping due to induced currents in the coil is inappreciable, and the motion is still periodic, though as at first, the amplitude continually grows less, due to air damping, pivot friction, etc. As the resistance external to the coil is decreased the damping becomes more noticeable and the coil comes to rest sooner. Finally, when the sum of coil resistance and external resistance reaches a particular value (the "critical resistance") the coil if displaced from its position of rest will return to it without oscillation. The time required for this return is roughly equal to the value of T as previously defined. This is the condition under which moving-coil galvanometers should be used in order to save time and to be able to work with unsteady current supply, catching the readings at momentary lulls. We may call this critical resistance R and the coil resistance R' .

The current required to give a deflection of one division may be determined in any convenient way. If this current I be multi-

³¹ Handbuch der Elektrotechnik, vol. 2, pt. 5, p. 14. Published by Hirzel, Leipzig, 1908.

³² Elektrotechnische Zeitschrift, 26, p. 560; 1905.

plied by the critical resistance R , the product IR is the voltage per scale division, and is one of the principal constants chosen at the start, in designing a deflection potentiometer. (See p. 421, eq. (34).) Another important constant of the galvanometer is H , the strength of the magnetic field in which the coil moves. A knowledge of the numerical value of H is not required if the following conditions hold, the value of T being 1 to 1.5 seconds, or, if necessary, being brought to this value by change of springs: (1) R must be from 5 to 10 times the coil resistance R' ; (2) $R-R'$ must be high enough to allow the desired resistance in potentiometer circuits and external accessories; (3) IR must have the desired value. However, in general, one would not find all these conditions realized, so that it will be necessary to find the value of H and to consider how the galvanometer performance will be affected by change of H . A small test coil of n_1 turns, each inclosing a square centimeters area, may be inserted in the air gap. The test coil, a ballistic galvanometer and the secondary winding of a known mutual inductance are connected in series. The withdrawal of the test coil gives a ballistic throw, d_1 . A direct current is now passed through the primary of the mutual inductance and is adjusted by trial to a value i such that the reversal of the current gives a throw d_2 nearly equal to that given by the withdrawal of the test coil. Then if M is the value of the mutual inductance in henrys, the value of H in the air gap is given by the expression

$$H = \frac{d_1}{d_2} \frac{2}{a} \frac{Mi}{n_1} \times 10^8 \quad (36)$$

If the current in the primary of M be simply broken to get d_2 , the factor 2 must be omitted from the numerator.

If the ballistic galvanometer and mutual inductance are not available, H may be calculated from the dimensions of the coil, the number of turns, and the torque corresponding to a given deflection produced by a current i . Let N = number of turns in the coil, b the mean breadth of the coil (in the direction of the lines of force), and h the height of the coil in the magnetic field,

excluding dead wire at the ends. (Dimensions are to be in centimeters.) Then H is given by the formula ³³

$$H = \frac{10 \times \text{Torque in centimeter-dynes}}{Ni \times bh} \quad (37)$$

The denominator is the product of the ampere-turns by the area of the field enclosed by a mean turn.

To give the proper value of R/R' , namely, 5 to 10, H must be equal to about 1500 to 2000, for the usual form of coil.³⁴ If the original value of H must be increased, a larger magnet ³⁵ may be used, or two magnets may be used in parallel on the same pole pieces. If the area enclosed by the pole pieces is such as to give the usual 90° travel of the coil, the pole pieces may be reduced, as usually such a travel is not required for the present purpose. However, one should not go too far in this direction, as this will fail to give a proportionate increase in H , the leakage flux increasing rapidly after a certain point is reached. The length of the air gap may be reduced, if this can be done without sacrificing proper clearance for the coil. It should be kept in mind that the reduction of area of air gap will tend to impair the permanency of the magnet, unless at the same time the length of the air gap be decreased, or the length of the magnet increased. A formula for this matter is given by Heinrich and Bercovitz,³⁶ as follows: If q_m is the cross section of the magnet, l_m the length of the magnet, q_p the cross section of the air gap, and l_p the length of the air gap, then l_m/q_m must be greater than $100l_p/q_p$. In the usual bipolar construction, l_p is equal to twice the length of each air gap, or, in general, it is the total length of air gap.

It remains to consider what effects are produced by varying H . The critical resistance R for a given coil and springs varies

³³ Friedr. Janus, *Elektrotechnische Zeitschrift*, **26**, p. 560; 1905. This article on "Die Berechnung von Drehspul-Messgeräthen" contains a discussion of the fundamental considerations in the calculation of moving coil measuring instruments, including the calculation of springs to provide a given torque.

³⁴ See note 8, p. 422.

³⁵ Concerning the design of permanent magnets, see J. Busch, *Elektrotechnische Zeitschrift*, **22**, p. 234; 1901.

³⁶ *Handbuch der Elektrotechnik*, vol. 2, pt. 5, p. 69.

directly as the square of H . The complete statement of this relation is given by White in the form ³⁷

$$H = A \sqrt{\frac{R}{R'T}}$$

where A is a nominal constant for a given form of coil, but varies slightly with the amount of inert matter in the coil. The current I per scale division varies inversely as H ; from this fact and the preceding we see that the product IR varies directly as H . Hence if volt sensitiveness (smallness of IR per scale division) were the only requirement, a large value of H would be a disadvantage.

Taking R_2/R'_2 as the desired value of the resistance ratio (to have the value 10, if possible), we must change the original value H_1 to

$$H_2 = H_1 \sqrt{\frac{R_2/R'_2}{R_1/R'_1}} = H_1 \sqrt{\frac{R_2 R'_1}{R'_2 R_1}} \quad (38)$$

where R_1 and R'_1 are the original values. This will raise the original value $I_1 R_1$ to

$$I_1 R_1 \sqrt{\frac{R_2 R'_1}{R'_2 R_1}}$$

In general, this value will not be what is desired, and we must now change the size of wire in the coil in accordance with the principle that IR varies directly as the number of turns.³⁸ This neglects difference of space factors of different sizes of wires, but will give a good first approximation, except in passing to sizes much finer or coarser than that of the existing coil. Let the desired value of IR be denoted by $I_2 R_2$ and the number of turns of wire in the original coil by N_1 ; then a new coil must be substituted having N_2 turns, where

$$N_2 = \frac{I_2 R_2}{I_1 R_1} \sqrt{\frac{R'_2 R_1}{R_2 R'_1}} \times N_1 \quad (39)$$

³⁷ W. P. White, *Every-Day Problems of the Moving Coil Galvanometer*, Physical Review, **23**, p. 384; 1906.

³⁸ If the coil of N turns is rewound to have kN turns, the critical resistance R becomes $k^2 R$, the current per scale division becomes I/k , and their product IR becomes kIR .

If we call the diameter (over insulation) of the wire on the original coil D_1 , then, since the diameter varies inversely as the square root of the number of turns in the coil, the new diameter will be

$$D_2 = \sqrt{\frac{I_1 R_1}{I_2 R_2}} \sqrt{\frac{R_2 R'_1}{R'_2 R_1}} \times D_1 \quad (40)$$

Giving R_2/R'_2 the desired value 10, we may use this formula to compute D_2 ; the computed value must be revised slightly to get the nearest available commercial size (taking account of thickness of insulation). Then substitute the actual number of turns the new coil will have as N_2 in (39), and solve for the value of R_2/R'_2 . Substituting this value in (38) gives the numerical value of H_2 , which will satisfy the requirements of the design. These computed values, D_2 and H_2 , will give the data from which a coil may be made up and the field strength changed; tests may then be made of this "first approximation" coil from which as a new starting point any needed revision of values may be made. The space factor of fine wires varies greatly with the size, and hence the error due to the use of the design equations (38) to (40), which neglect variation of space factor for the sake of simplicity, is greatest when the finest sizes are concerned. Another practical limitation is encountered in the coarser sizes, due to the necessity for an integral number of layers. For example, the size of wire given by (40) may be such that 2.6 layers are required in the theoretical coil; in such a case one would be obliged to use two layers, or enlarge the space in which the coil moves, so as to use three layers.

As the error in using any approximate equation is lessened by reducing the relative amount of change which is to be made, it will be of advantage to compute H_2 , then raise the existing value of H to the computed value, and determine the performance of the original coil in the new field. This performance may then be used to compute a revision of H_2 , if necessary, when the performance may again be determined, and used as the basis for calculation of a new coil. It is easy to vary H by changing the size or number of magnets, and by magnetic shunting; or one may use as a trial magnet an electromagnet with suitable means for determining the value of H (see pp. 438-439).

One effect of the poor space factor of fine wires should be specially mentioned. The thickness of (silk) insulation can not be reduced below a certain point, and as the wire becomes finer the amount of metal becomes relatively much less than in a larger size which may have been taken as a starting point for design. Consequently the fine wire coil will have an abnormally high resistance, which will reduce the ratio R/R' , and give less resistance for use in potentiometer and accessories than would be expected from the performance of the same galvanometer with a coarser coil.

It will be realized (by galvanometer makers, at least) that it would be a somewhat difficult and expensive procedure to make galvanometers in quantity, each to have exactly a specified critical resistance, current per scale division and length of scale division. For the deflection potentiometer *this is not necessary*, and we will now consider in what ways the maker can have the latitude which will expedite and cheapen the manufacture without sacrifice of quality.

1. Use tested springs of normal or slightly *under* normal strength.
2. Use the same type of magnet as is used for other standard products made in quantity, so that a good stock of magnets is available from which to select (by test) magnets of nearly the strength desired for galvanometers. Take a magnet whose strength is slightly above normal and reduce its strength by magnetic shunting until the galvanometer shows a critical resistance R equal to or above the normal and a drop per scale division IR within, say, 10 per cent above or below normal, using a trial scale having divisions of standard length.

3. Using the value of total resistance R which gives critical damping for the individual galvanometer, make a scale to fit it. For this purpose the galvanometer plus external resistance may be regarded as a voltmeter, and the scale may be laid out by applying 5, 10, 15 times the standard voltage per division IR , to locate the 5, 10, 15 division points. Care should be taken to make an accurately proportional scale, as there is no way to correct for lack of proportionality.

The reason for selecting the springs of normal strength or below and the magnets of slightly over normal strength will appear from

the following. Let the spring strength (U) be too great by the factor $1 + e$, the moment of inertia of coil and fittings being assumed as not varying from standard.³⁹ To offset the excess of spring strength let the field strength H be changed to $H(1 + f)$ which will give a value of total critical resistance $R(1 + g)$, satisfying the condition of normal voltage IR per scale division. We have from White's equation (20 a) as quoted on page 440, for all quantities normal,

$$\begin{aligned} R &= \frac{H^2}{A^2} R' T = \frac{H^2}{A^2} R' \times 2\pi \frac{\sqrt{K}}{\sqrt{U}} \\ &= H^2 U^{-\frac{1}{2}} \left(\frac{2\pi R' K^{\frac{1}{2}}}{A^2} \right) \\ C_1 H^2 U^{-\frac{1}{2}} \end{aligned} \quad (41)$$

where C_1 is constant for a given coil and springs. For the case of U increased to $U(1 + e)$, etc., as above, we have (making the usual approximations for terms of the form $1 +$ a small quantity)

$$R(1 + g) = C_1 H^2 (1 + f)^2 U^{-\frac{1}{2}} (1 + e)^{-\frac{1}{2}}$$

$$= C_1 H^2 U^{-\frac{1}{2}} \left(1 + 2f - \frac{e}{2} \right)$$

$$R = C_1 H^2 U^{-\frac{1}{2}} \left(1 + 2f - \frac{e}{2} - g \right)$$

From this value of R and equation (41) we have

$$2f - \frac{e}{2} - g = 0 \quad (42)$$

The change of spring strength and change of magnet strength will change the current per scale division to $I(1 + e - f)$. The product of this current and $R(1 + g)$ must give the normal value of IR ; that is

$$I(1 + e - f)R(1 + g) = IR$$

³⁹ As far as the effect on the period is concerned, a given percentage excess in K can be counted as the same percentage deficit in U . However, a change of the latter affects the value of I and IR , while a change in K does not. Hence, in general, K and $1/U$ are not equivalent.

whence $1 + e - f + g = 1$, or $e - f + g = 0$ (43)
 Combining equations (42) and (43),

$$\begin{aligned} f &= -\frac{e}{2} \\ g &= -\frac{3e}{2} \end{aligned} \quad (44)$$

Taking a numerical example: let the springs be 1 per cent too strong (e); then to keep IR normal, by (44) the magnet strength must be 0.5 per cent below normal $\left(-\frac{e}{2}\right)$, and the total critical resistance R must be 1.5 per cent low $\left(-\frac{3e}{2}\right)$. Hence the spring strength ought to be normal or below to keep R normal or above. The current I will be 1 per cent stronger because of the stronger springs, 0.5 per cent stronger because of the weaker magnet; I will thus be 1.5 per cent greater than normal. The product of this greater I by an R 1.5 per cent below normal gives the normal IR . Concerning the question of critical damping, we see from the equation

$$R = C_1 H^2 U^{-\frac{1}{2}} \quad (41)$$

that the right-hand member will be 1 per cent low due to the magnet strength H being 0.5 per cent low, and 0.5 per cent low due to spring strength U being 1 per cent high; hence R will be 1.5 per cent low, as required by the value of I 1.5 per cent greater than normal.

It is evident that other methods are available for adjusting the constants of the galvanometer, such as varying the moment of inertia of the coil by adjustable balance weights, or by adding auxiliary damping coils or rectangles. It is believed, however, that the general method outlined in the preceding discussion is likely to be most economical of time and most satisfactory in the results accomplished.

WASHINGTON, June 23, 1911.