REMARKS ON THE QUARTZ COMPENSATING POLARISCOPE WITH ADJUSTABLE SENSIBILITY.

By Frederick Bates.

In an article on a "Quartz Compensating Polariscope with Adjustable Sensibility," which appeared in the Bulletin of the Bureau of Standards (Vol. IV pp. 461-466), the writer discussed the effect on the zero reading of the scale when the Polarization angle a was varied. In general, the intensities of the light emerging from the large and small nicols of a Lippich polarizing system are unequal. Hence, when the analyzing nicol is set for equal intensity of the halves of the field its position is different from what it would have been had the beams been of equal intensity. The angular difference, δ , between the two positions of the analyzing nicol was obtained from the equation

$$\tan \delta = \pm \frac{\sqrt{K} - 1}{\sqrt{K} + 1} \tan \frac{a}{2} \quad (3)$$

$$\text{where } K = \frac{A_2}{A_1}$$

and is the ratio of the intensity of the light emerging from the small nicol of the polarizing system to that emerging from the large nicol.

We have for a Lippich system

$$A_2 = f(A_1 \cos^2 a, M) \quad (5)$$

where $A_1 \cos^2 a$ is the intensity of the light from the small nicol, if there has been no other loss than that due to the partial crossing of the large and small nicols. M is the algebraic sum of the effects of all the remaining factors upon which the relative inten-

sities of A_1 and A_2 depend. These factors are quite numerous. Among them may be mentioned absorption, depolarization, the inclination of the small nicol to the axis of the system, the relative inclination of the faces of the small nicol, the positions of diaphragms, and reflection as the light enters the small nicol. It is evident that M is a constant for a particular instrument, but varies with different polarizing systems. For this reason, and because it is a simple matter for the instrument makers to correct for it, the quantity M was neglected in discussing δ in the paper referred to above.

In "Bemerkungen zu der vorstehenden Batesschen Arbeit,"¹ Dr. Otto Schönrock calls attention to the fact that the writer has not taken the reflection and absorption in the small nicol into consideration in calculating δ from (3), inasmuch as the equation

$$A_1 = A_2 \cos^2 a \quad (6)$$

was used to obtain K . Schönrock modifies (3) in the following manner:

From the well-known Fresnel equations, when light at perpendicular incidence passes from one refracting medium into a differently refracting medium, the fraction

$$B = \left[\frac{n-1}{n+1} \right]^2 \quad (7)$$

is reflected. Applying (7) to the case of air and Iceland spar, where $n = 1.486$, and neglecting (6), Schönrock finds that

$$A_2 = A_1 \left[1 - \left(\frac{n-1}{n+1} \right)^2 \right]^2 \quad (8)$$

$$= A_1 \cdot 0.925 \quad (9)$$

There is therefore a loss of 7.5% by reflection at the faces of the small nicol. A further allowance of 0.005 was arbitrarily made for the loss due to absorption in the small nicol. Thus, he finds, by considering the losses due to the crossing of the large and small nicols and to reflection and absorption, that

¹ Zs. Ver. Zuckerind, Feb., 1908, p. 111.

$$A_2 = 0.92 A_1 \cos^2 a \quad (10)$$

and

$$\tan \delta = \frac{1 - \sqrt{0.92} \cos a}{1 + \sqrt{0.92} \cos a} \tan \frac{a}{2} \quad (11)$$

In Table I is shown a comparison of the values of δ given by (3) and (11).

TABLE I.

a	δ IN SUGAR DEGREES.	
	From Equation (3).	From Equation (11).
5°	0.014	0.164
10°	0.111	0.412
15°	0.378	0.830

The values from (11) are markedly larger than those from (3). It is to be remembered that (11) takes into consideration the effect on δ of the loss in intensity due to reflection and absorption in the small nicol, but neglects all the other factors of $f(M)$. It will be observed for any value of δ that the value from (11) equals the value from (3) increased by $0.03a$.

Hence,

$$\delta = \tan^{-1} \left[\pm \frac{\sqrt{K} - 1}{\sqrt{K} + 1} \tan \frac{a}{2} \right] + 0.0103a \quad (12)$$

$$= \tan^{-1} \left(\tan^3 \frac{a}{2} \right) + 0.0103a \quad (13)$$

Equation (13) holds for all working values of a and is significantly simpler than (11). A consideration of the remaining factors involved in $f(M)$ would result in decreasing the constant, 0.0103 ($=0.03$ for δ in sugar degrees). It is thus evident that if $f(M)$ be taken into account the only effect on the value of δ given by (3) is to change it by $\pm Ca$ where C is a constant for each individual Lippich system.

In Fig. 1 the curve A shows the value of δ (change in the zero reading of the instrument) in sugar degrees, given by (3), for any

Fig. 1.

value of a . Curve B shows the change of δ with a if it were necessary to consider only those factors of $f(M)$ which represent

losses by reflection and absorption in the small nicol. Curve C shows the values of δ given by (13). Its ordinates are the sums of the ordinates of curves A and B. In building a quartz compensating polariscopes with adjustable sensibility, the writer suggested that the analyzing nicol should move through one-half the angular displacement of the large nicol of the polarizing system. The zero point displacements of such an instrument, as a was varied, would approximate curve A with sufficient exactness to give a considerable practical working range of a . The difficulty in displacing the analyzing nicol by the proper amount to eliminate δ as given by curve A is due to the fact that δ is not a linear function of a . It is a simple matter to cause the analyzing nicol to gain or lose on the large nicol, so long as the change is linear. The effect of superimposing on curve A the curve B, which is due to part of $f(M)$, is to make a resulting curve C, which is as nearly linear as A. It will be observed that a straight line can be drawn through curve C, such that for values of a , from 3° to 15° , it will not be distant from the curve at any point by more than 0.05° S.

However, since the value of δ given by (13) does not include any of the factors of $f(M)$, save the reflection and the absorption, it is not the true zero displacement. By considering only the reflection and absorption in the small nicol, Schönrock introduces from $f(M)$ the only factors which could be used to make the apparent zero displacement larger. He has allowed 0.005 for the loss in intensity by absorption in the small nicol, whereas a considerably larger portion of the light lost by first reflection at the surfaces of the small nicol is returned by subsequent reflections to the analyzer. The exact value of δ can best be determined by experiment. An experimental determination with $a = 10^\circ$ for a Lippich system selected at random gave δ at the point W, Fig. I, whereas according to Schönrock it should lie upon the curve C. The curve giving the true values of δ lies between curves A and C.

The change of δ with a is fortunately very nearly linear. As shown above, the only effect of $f(M)$ is to make the change more nearly linear. Hence, in the writer's previous paper $f(M)$ was ignored and only the difficult case was considered when curve A represents approximately the displacement curve of the system.

It is thus entirely practicable to build a quartz compensating polariscope with adjustable sensibility, having a working range of α from 3° to 15° with a negligible zero correction for all ordinary polarizations. It is immaterial whether the true zero point displacement curve approximates curve A or C.

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