A NEW METHOD FOR THE ABSOLUTE MEASUREMENT OF RESISTANCE.

By Edward B. Rosa.

The method of Lorenz is generally considered the best among the various methods that have been employed for the absolute measurement of resistance. It is ideal in its simplicity, and in its method of directly balancing a constant induced electromotive force against the fall of potential in the resistance to be determined. It has, however, a very serious limitation in the very small electromotive force generated, and in the appreciable thermoelectric forces produced at the sliding contacts. And if one were to attempt to get a tenfold greater precision than has hitherto been obtained in absolute resistance measurements by this method, it would probably be found that these sliding contact troubles would be very serious.

In studying the problem of how to secure an accuracy at least ten times as great as has yet been done (for that is what is now demanded in order to keep pace with the possibilities in the absolute measurement of current) it occurred to me that a revolving coil, or two such coils, could be so disposed in the magnetic field of a pair of fixed coils as to yield an electromotive force which could be compared with the fall of potential through a fixed resistance, by means of a differential galvanometer, and so give the absolute value of the resistance. The advantage of this method would be that the electromotive force generated could be a thousand times greater than in the Lorenz apparatus, while the thermoelectric forces at the sliding contacts would be considerably less; for a revolving coil cuts the lines of force four times in each revolution, and two coils of only 125 turns each would therefore generate a thousand times the electromotive force produced by a disk, supposing the field and speed the same, and the area of the coils equal to that of the disk. The thermo-electric forces due to the brushes would be less because the commutator could be much smaller than the disk. The method of securing this result is not quite so simple as in the Lorenz experiment, but it seems to be free from any serious difficulty, and because of the enormous advantage of having an electromotive force of several volts to work with, instead of several thousandths of one volt, the new method seems to merit a careful trial. We are constructing an apparatus of this kind for use at the Bureau of Standards, and I have thought it worth while to give a brief description of the method in order that it might be considered by others interested in the absolute measurement of resistance.

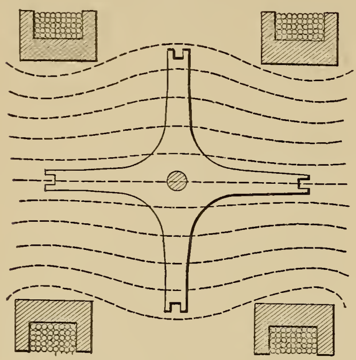

Fig. 1.—Revolving coils and magnetic field of fixed coils.

The two armature coils, at right angles to one another (Fig. 1), rotate in a strong magnetic field produced by two stationary coils, set somewhat farther apart, relatively, than the coils of a Helmholtz galvanometer; the whole constituting a kind of two-phase alternator without iron.

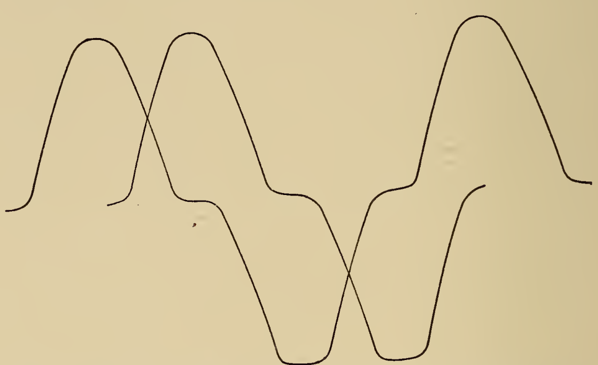

Fig. 2.—Waves produced by the revolving coils of model apparatus.

The wave form of the electromotive force generated is not a sine, as it would be nearly if the armature coils were smaller and the field coils were somewhat nearer together, but has the form shown in Fig. 2. That is, the emf. instead of varying at a maximum rate as it passes through zero, becomes tangential to the axis, permitting the electromotive force to be commutated by means of a two-part commutator, without sensible loss. The lines of force between the field coils swell out as shown in the figure, and the revolving coils slide along the lines very nearly for an appreciable distance at the region of minimum electromotive force, thus giving a very small electromotive force for a considerable angle. It is not practicable to put this commutated electromotive force in series with the constant difference of potential at the terminals of the resistance to be measured, for the commutator would then cut out an uncertain part of the integral emf. of the latter. But they may be compared very accurately by means of a three-circuit differential galvanometer, preferably of the Broca type, as I shall now show.

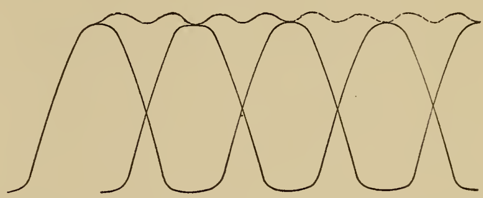

Fig. 3.—The Two Commutated Emf. Waves and their Resultant.

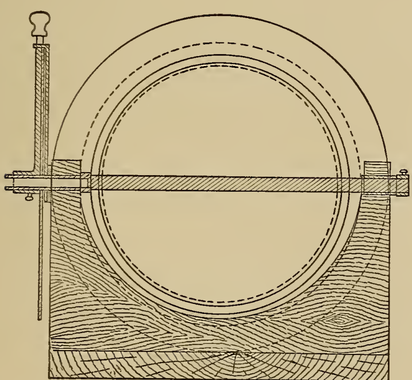

Fig. 4.—Model Apparatus for Studying Wave Form.

If we were to use an ordinary two-circuit differential galvanometer, one circuit carrying a constant current from the terminals A B of the resistance *R*, through which the field current *I* passes, and the other a pulsating current (the alternating current generated by one revolving coil rectified by a two-part commutator) the impulses in the needle would be so great that it would be necessary to use a very high speed in the rotating coil, or a very heavy galvanometer needle to prevent vibration of the needle and an indistinct image on the scale. Moreover, such strong impulses might alter the magnetization of the needle, or perhaps produce a deflection even though the integral current in each circuit (going, of course, in opposite directions) were the same. But by using two rotating coils, at right angles to each other, and so disposing the field coils as to make the electromotive force at 45° either side of the maximum equal to one-half the maximum (as can readily be done), the sum of the two currents is constant within about 3 per cent, and this small fluctuation has a frequency four times that of either component

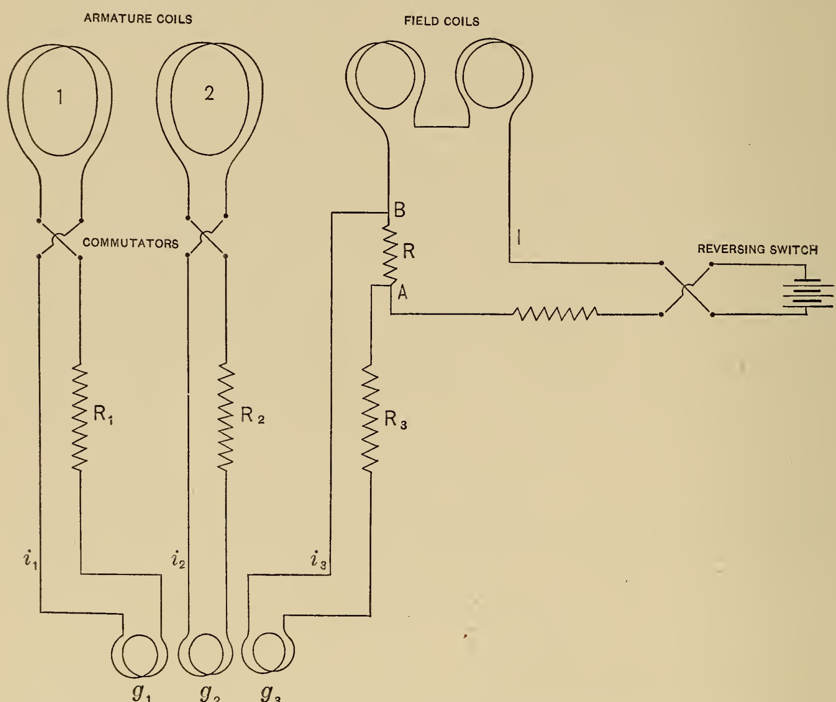

Fig. 5.—The Three Circuits of the Galvanometer with the Connections for measuring the Resistance R.

(Fig. 3). The effect on the needle therefore, due to the two pulsating currents i_1 and i_2, flowing in two of the three-strand windings of the galvanometer, is the same as though these two currents were combined in a single winding, and is equivalent to a direct current of the same average value. Each circuit has its two-part commutator (set at right angles to each other), and each circuit has the same resistance (perhaps several hundred or a thousand ohms) as the third circuit which carries a constant current.

These resistances do not have to be known, but when the galvanometer is balanced, the sum of the average electromotive forces generated by the two coils is equal to the difference of potential at the terminals of the resistance R, which is the resistance to be determined absolutely.

Let M_1 and M_2 be the mutual inductances of the two revolving coils with respect to the field coils, when each is in the position of maximum inductance, R_1 and R_2 the total resistances of the two circuits of which these coils form a part, including the resistances of the respective galvanometer windings, n the number of revolutions per second of the armature coils, and I the field current. Then the average value of the currents in the two circuits will be

$$i_1 = \frac{4n\,M_1\,I}{R_1}$$

$$i_2 = \frac{4n\,M_2\,I}{R_2}$$

if the effect of self-inductance is negligible.

The third circuit carries a steady current i_3, due to the difference of potential at the terminals AB of the resistance R through which flows the main current I, which passes through the field coils. Hence

$$i_3 = \frac{RI}{R_3}$$

R_3 is the total resistance of the circuit, including R. We may suppose the windings of the galvanometer not perfectly balanced, so that the currents in any two windings required to give zero deflection are not quite equal.

Then let

$$i'_3 = f_1 i'_1 = f_2 i'_2$$

f_1 and f_2 are factors very nearly unity.

If two circuits are balanced on the same emf E, for example that on the points A B, we should have

$$i'_1 = \frac{E}{R_1} \qquad i'_3 = \frac{E}{R_3}$$

and
$$f_1 i'_1 = i'_3 = \frac{E}{R_3}$$

$$\therefore\ i'_1 = \frac{E}{R_1} = \frac{E}{f_1 R_3}$$

Therefore when the galvanometer is balanced, $R_1 = f_1 R_3$ and $R_2 = f_2 R_3$. That is, leaving R_3 constant, the resistance R_1 is altered slightly, if necessary, until there is no deflection, and then the second circuit R_2 is put in place of R_1 and balanced in the same way. It is never necessary either to measure R_1, R_2, R_3 or even to compare them with one another except as is done in occasionally balancing the galvanometer in the manner described above. Any variations due to temperature changes or other causes will thus be corrected.

From what precedes

$$\dot{i}_1 = \frac{4nM_1 I}{f_1 R_3}$$

$$\dot{i}_2 = \frac{4nM_2 I}{f_2 R_3}$$

And since $i_3 = f_1 \dot{i}_1 + f_2 \dot{i}_2$ when running regularly, and the constant current i_3 is balancing the pulsating currents in the other two circuits

$$\frac{RI}{R_3} = (4nM_1 + 4nM_2)\frac{I}{R_3}$$

or

$$R = 4n(M_1 + M_2) = 4nM$$

where R is the resistance whose absolute value is to be determined, M is the sum of the maximum values of the mutual inductances of the two revolving coils, with respect to the field coils, and n is the speed, or number of revolutions per second.

(In the Lorenz apparatus the formula is $R = nM$.)

If $M_1 = M_2 = 25$ millihenrys, and therefore $M = .050$ henrys, and $n = 25$ per second

$$R = 4 \times 25 \times .050 = 5 \text{ ohms},$$

and if the current I is one ampere, the electromotive force E at the terminals of the resistance A B is 5 volts. It would not be difficult to make M much larger than 50 millihenrys, but in a precision apparatus it is desirable to keep the resistances of the field coils as low as practicable to reduce the heating, and the armature coils should be made of small resistance (relative to R_3) and of relatively few turns to reduce the mean temperature coefficient and the self-inductance of the circuit. If the commutators and brushes are properly made, the thermoelectric forces at the sliding contacts will be wholly negligible in comparison with the electromotive forces generated in each coil, and the variation of resistance at the brushes will be negligible in comparison with 500 or 1000 ohms, the total resistance of each circuit. This is partly due to the fact that the commutators will be of much smaller diameter than the large disc of a Lorenz apparatus, and therefore the surface speed will be much less. The temperature changes of resistance will be very slight, for probably 99 per cent of the resistance can be manganin. The three galvanometer windings will always change together, if the temperature changes, and the two windings on the armature will likewise change together. A third winding of copper, having a resistance equal to that of one of the armature coils, in the i_3 circuit could be employed to balance any slight effect of varying room temperature. But that would probably be a needless refinement, if the room were kept at nearly constant temperature, as it should be for other reasons.

These details are mentioned to show that the use of a differential galvanometer carrying an appreciable current does not introduce sources of error that would be troublesome. It seems as though the manipulation of the apparatus would be simple and straightforward, and that a very high accuracy in the results would depend chiefly on measuring n and M with sufficient precision.

From our experience in previous work, I am confident that the uncertainty in the speed need not exceed one part in 100,000. The use of the direct reading chronograph, which was developed especially for this kind of service, and the same method of maintaining constant speeds that we have employed for several years leaves little to be desired in this respect. The burden of the problem therefore is to determine M, the mutual inductance, with sufficient precision.

It has been the practice in all determinations by the Lorenz method to obtain the mutual inductance by calculation from the dimensions of the apparatus. There are some decided advantages in obtaining it, not directly in this way, but by comparison with a standard of mutual inductance, the latter having its value computed from its dimensions. These advantages are as follows:

1. The resistance machine may thus be more compact, and its field coils may be wound with many layers of wire, thus giving a stronger field, and therefore a higher electromotive force, and the machine will be lighter and less expensive to construct.

2. Being more compact, the stray field of the machine is relatively less, and having a strong field the disturbing effects of the earth's field are reduced.

3. The mutual inductances M_1 and M_2 can be measured under working conditions. If there are any magnetic impurities in the shaft or bearings or any other part of the machine, or in any part of the room, hidden or exposed, which render the permeability of the circuit a little greater than unity, they will be taken into account in measuring M_1 and M_2 by comparison with a standard. That is, no assumption is made as to absence of magnetic impurities from the machine, or that the effect of the iron in the driving motor, or other neighboring apparatus, is zero.

4. The mutual inductance can be redetermined by comparison with the standard every time a run is made, if desired, as it will be but a few moments task. Any change due to changes in the windings or temperature effects will therefore be detected. In other words, the machine, which cannot be supposed to remain as constant as a standard of mutual inductance, will not be assumed to have its windings remain indefinitely of constant dimensions. The standard of mutual inductance may be constructed of pure marble and copper wire, and designed to permit the maximum accuracy in the determination of its inductance

from its dimensions. There are several forms which could be used advantageously. The coaxial solenoids of Fig. 6 constitute one of the best known forms. The lengths of the coils must be accurately known, as well as the diameters, and the winding must be very uniform.

The Campbell form of mutual inductance (Fig. 7) is an improvement over this in one important respect. The primary is a single layer coil in two parts, formed by omitting a section in the center, say one-third or one-half the windings. The secondary is outside

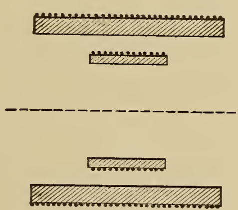

Fig. 6.

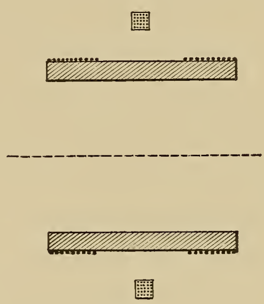

Fig. 7.

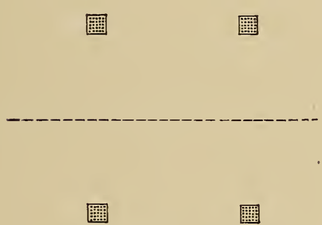

Fig. 8.

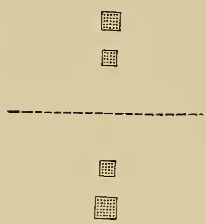

Fig. 9.

Different Forms of Mutual Inductance Standards.

Fig. 6, two coaxial solenoids; Fig. 7, Campbell form; Fig. 8, two equal coaxial coils; Fig. 9, two coaxial coils in the same plane.

the primary and of such diameter that it occupies the neutral region, where the magnetic field is nearly zero and the mutual inductance is a maximum. It results from this that the dimensions of the secondary need not be accurately known, and it may consist of a coil of many layers, and may be subdivided to give several values of the inductance.

A pair of parallel coils of equal radii and rectangular section can be used (Fig. 8). The distance can be very accurately determined, if the precaution be taken to interchange them in use. If they be carefully wound with enameled wire, they can be measured very accurately as wound. But a better method is to compare their radii electrically with a single layer coil, wound on an accurate cylindrical surface. If the number of turns and dimensions be such that the same current can be passed in series through the two coils to neutralize each other's field at the center, the comparison is easily and accurately made.

Another form which I am now having constructed is a pair of coaxial coils in the same plane. (Fig. 9.) The mean radii will be determined electrically by comparison with a standard single layer coil. Being in the same plane, there are no measurements of distance to make. The secondary can be most accurately centered electrically, being in a minimum position with respect to radial displacements, and in a maximum position with respect to axial displacements. The accuracy of the electrical comparisons is probably at least 1 in 100,000, so that the precision of the determinations of radii depend chiefly on the measurements of the standard coil. This can be measured at least as accurately as any single layer winding.

The advantage of this form is that it avoids the measurement of lengths of coils or pitch of windings, and gives a very compact standard which may be of quite large value if desired. The electrical comparisons of radii can easily be repeated, and by winding each coil with a pair of wires which can be joined in series or parallel, one can have three values, as for example 50, 25 and 12.5 millihenrys, merely by changing the connections.

The best way is to build at least two different forms of mutual inductance of the same values, and having the same value as the resistance apparatus, so that they can be compared with one another and with the resistance apparatus by sending the same current in series through their primaries, and connecting their secondaries in opposition through a sensitive galvanometer. One primary may be shunted by a high resistance to secure an exact balance. This comparison can be made very quickly, and with standards of the above values there need not be an error exceeding one part in a million in the comparisons.

By varying the speed from 10 to 25 revolutions per second, and joining the armature windings in series or parallel, various values of R from 1 to 10 ohms can be measured with the values of M mentioned above. With the recent improvements in the construction and comparison of resistance standards, one is now justified in taking the trouble to get absolute measurements of the highest precision.

At the recent International Electrical Conference in London, Lord Rayleigh expressed the hope that the time was not far distant when a resistance could be measured absolutely so conveniently and so accurately that wire standards could be directly standardized against an absolute resistance machine, and the use of mercury ohms as primary standards of resistance eliminated. If that can be done, it seems probable that an agreement could be reached to check the Weston Normal cell from time to time against an absolute current balance, a procedure which would probably be more accurate, as well as more convenient, than undertaking to check them against the silver coulometer. However that may be, the more accurate determination of the absolute value of our legal ohm is a problem of prime interest and importance, and this new revolving coil method is here proposed with the hope that it may be found useful in this connection.

WASHINGTON, February 27, 1909.