Fully Homomorphic Encryption based on Euler's Theorem

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ABSTRACT: Cloud computing enables sharing of resources and services. In the cloud computing the client (user) puts his data in the cloud, and any computations on his stored data will be implemented in the cloud. To produce the effective services, some of the cloud service providers share the information with third parties. The third party can reach to the client private data and modifies the information to make it beneficial to him. Therefore, the security becomes the main thing in cloud computing. The scheme that allows to executing several computations on the encrypted message without decrypting the message is called homomorphic encryption. This paper provides a Fully Homomorphic Encryption Scheme SA that we can apply it for security of a cloud computing. The proposed scheme is based on Euler's theorem, which proved both addition and multiplication operations at the same time on ciphertext without decryption. We compute the time complexity of the encryption and decryption function for the ElGamal and RSA cryptosystems compared to the proposed SA scheme and also compared the execution time among the three schemes with varies sizes of messages and varies lengths of secret key, and finally the security of the three schemes is analyzed. As a result, in calculations of the execution time of ElGamal and RSA cryptosystems compared to the proposed SA scheme, it was observed that the length of the secret key effect on the execution time, in which the execution time increased when the length of the private key was increased. The proposed scheme showed a good security for the stored data on the cloud.

Keywords: Fully Homomorphic Encryption, Secret Key Cryptosystem, Homomorphism, Cloud Computing Security

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1. Introduction

The technology that uses central remote servers and the internet to share the applications and maintain the data is called a cloud computing. It allows client to use an application without installation and he can access to their personal files from any computer just verify internet connection [1]. To protect the data that it's stored in the cloud we must use an encryption system that can

perform computations on the encrypted data. The technique that allows to computation on a ciphertext without previous decryption is called homomorphic encryption HE [2].

HE scheme can be either, symmetric or asymmetric. An encryption scheme contains three algorithms: KeyGen, Encrypt, and Decrypt. In a symmetric (or secret key) encryption scheme, KeyGen utilize λ (is a security parameter that determines the bit length of the keys) to produce a one key that is used in each of encryption and decryption, first transform a message to a ciphertext, and later transform the ciphertext back to the message. In an asymmetric (or public key) encryption scheme, KeyGen utilizes λ to generate two keys: a public encryption key pk , which may be made available to everyone, and a secret decryption key sk . [3]. An encryption technique is called Fully Homomorphic (FHE) if it performs both addition and multiplication at the same time, and can compute any Operation. [4]

The organization of a remainder of the paper: section 2, provides Related Works about HE, Section 3, display Fermat and Euler Theorems. Section 4, offer the proposed scheme, also we illustrate an example about it. Section 5, describes the results and discussion. Finally, our conclusions are mentioned in Section 6.

2. Related Work

The first homomorphism suggested by Rivest, Adleman, and Dertouzos in [5]. The multiplicative homomorphism is given by RSA [6]. A partial homomorphic encryption scheme is suggested by Yao [7], Goldwasser and Micali [8], ElGamal [9] and Paillier [10]. Fontaine & Galand in [11] has presented a survey of homomorphic encryption schemes. Gentry has proposed fully homomorphic encryption in his thesis and paper [4].

Many researchers proposed the variants of Gentry's model with some improvement. Homomorphic encryption on smaller size cipher text is proposed by Smart and Vercauteren [12]. The arithmetic operations over integers are proposed by Dijk, Gentry, Halevi, and Vaikuntanathan [13]. Faster improvement to Gentry's model is proposed by Stehle and Steinfeld [14]. Y Govinda Ramaiah has proposed "Efficient Public Key Homomorphic Encryption over Integer Plaintexts". [15] Coron et al [16] was extended the work of DGHV [13] over the integers for the purpose of batching FHE to a scheme supporting encryption and homomorphic processing of a vector of plaintext bits as a single ciphertext. Emura et al [17] aimed at controlling that who can perform the homomorphic operations, when and where. Because of the malleability property of homomorphic encryption that anyone can perform the operations "freely", it becomes difficult to achieve adaptive chosen ciphertext (CCA) and homomorphic. Kim et al [18] presented a fully homomorphic scheme that generalizes the DGHV [13] scheme and modifies the third proposal of RSA [19] on the Chinese Remainder theorem and ring homomorphism.

3. Fermat and Euler Theorems

Two important theorems presented the first by Pierre de Fermat and the second by Leonhard Euler. Both theorems related to powers in modular arithmetic.

3.1 Fermat's Little Theorem

Suppose that p is prime and $\gcd(a, p) = 1$ (or a and p are relative prime or p does not divide a).

Then:

$$a^{p-1} \equiv 1 \pmod{p} \quad (1)$$

3.2 Euler's Theorem

Euler's Theorem is a generalize of Fermat's Little Theorem. Suppose n be an arbitrary positive integer, $\phi(n)$ denote the number of integers $1 < a \leq n$ such that if $\gcd(a, n) = 1$, then:

$$a^{\phi(n)} \equiv 1 \pmod{n} \quad (2)$$

Note: When $(n = p)$ is prime, then $\phi(p) = p - 1$, we get Fermat's Little Theorem.

The following facts are true about the Euler ϕ -function.

- $\phi(p) = p - 1$ if p is prime.
- $\phi(p.q) = (p - 1)(q - 1)$ if p and q are prime.

• The general formula of $\phi(n)$ is:

$$\phi(n) = n \prod_{p|n} \left(1 - \frac{1}{p}\right) \quad (3)$$

Where,

The product is over distinct primes dividing (n) [20].

4. The Proposed Scheme: SA

The proposed scheme (SA) is mainly based on Euler's theorem that is mentioned in equation (2). SA is a symmetric (or secret key) encryption scheme and supports both multiplicative and additive homomorphism property that means its Fully Homomorphic Encryption Scheme. The description of SA scheme as follows:

Key Gen: Select two prime numbers p and q

Calculate: $S = p * q$

Encryption: The message M will always be less than $< n$, Select random big integer r :

$$C = M^{r \cdot p - r + 1} \bmod S, \text{ where } C \text{ is a ciphertext.} \quad (4)$$

Evaluate: Apply multiplication and addition on ciphertexts C_i , and then decrypt the result of C_i , we get integer number which is the same as the integer number that is the result of applying multiplication and addition on input M_i .

$$\text{Decryption: } M = C \pmod{p} \quad (5)$$

To proof correctness of the scheme:

$$C = M^{r \cdot p - r + 1} \bmod S$$

$$\text{Dec} = C \bmod p$$

$$= M^{r \cdot p - r + 1} \bmod S \bmod p$$

$$= M^{r \cdot p - r + 1} \bmod p \bmod S$$

We know that $a^{r \cdot p - r + 1} \equiv a \pmod{p}$ (Euler's theorem)

$= M \bmod S = M, M < S$ (proved).

Additive Homomorphism:

The two messages M_1 and M_2 will always be less than $< n$, also addition $(M_1 + M_2)$ should be less than n ,

$$M_1 + M_2 = \text{Dec} [\text{Enc}(M_1) + \text{Enc}(M_2)]$$

(Enc) is Encryption function and (Dec) is Decryption function

Proof:

Select two random big integer r_1 and r_2

$$C_1 = (M_1^{r_1 \cdot p - r_1 + 1} \bmod S), C_2 = (M_2^{r_2 \cdot p - r_2 + 1} \bmod S)$$

$$C_1 + C_2 = (M_1^{r_1 \cdot p - r_1 + 1} \bmod S) + (M_2^{r_2 \cdot p - r_2 + 1} \bmod S)$$

$$\text{Dec}(C_1 + C_2) = (C_1 + C_2) \bmod p$$

$$\begin{aligned}
&= [(M_1^{r_1 \cdot p - r_1 + 1} \bmod S) + (M_2^{r_2 \cdot p - r_2 + 1} \bmod S)] \bmod p \\
&= (M_1^{r_1 \cdot p - r_1 + 1} \bmod S \bmod p) + (M_2^{r_2 \cdot p - r_2 + 1} \bmod S \bmod p) \\
&= (M_1^{r_1 \cdot p - r_1 + 1} \bmod p \bmod S) + (M_2^{r_2 \cdot p - r_2 + 1} \bmod p \bmod S) \\
&\text{As we know } a^{r \cdot p - r + 1} \equiv a \pmod{p} \text{ so} \\
&= (M_1 \bmod S) + (M_2 \bmod S) = M_1 + M_2
\end{aligned} \tag{6}$$

Multiplicative Homomorphism:

The two messages M_1 and M_2 will always be less than $< n$, also multiplication $(M_1 \times M_2)$ should be less than n ,

$$M_1 \times M_2 = \text{Dec} [\text{Enc} (M_1) \times \text{Enc} (M_2)].$$

Proof:

$$\begin{aligned}
C_1 &= M_1^{r_1 \cdot p - r_1 + 1} \bmod S, \\
C_2 &= M_2^{r_2 \cdot p - r_2 + 1} \bmod S \\
C_1 \times C_2 &= (M_1^{r_1 \cdot p - r_1 + 1} \bmod S) \times (M_2^{r_2 \cdot p - r_2 + 1} \bmod S) \\
\text{Dec} (C_1 \times C_2) &= (C_1 \times C_2) \bmod p \\
&= [(M_1^{r_1 \cdot p - r_1 + 1} \bmod S) \times (M_2^{r_2 \cdot p - r_2 + 1} \bmod S)] \bmod p \\
&= (M_1^{r_1 \cdot p - r_1 + 1} \bmod S \bmod p) \times (M_2^{r_2 \cdot p - r_2 + 1} \bmod S \bmod p) \\
&= (M_1^{r_1 \cdot p - r_1 + 1} \bmod p \bmod S) \times (M_2^{r_2 \cdot p - r_2 + 1} \bmod p \bmod S) \\
&\text{As we know } a^{r \cdot p - r + 1} \equiv a \pmod{p} \text{ so} \\
&= (M_1 \bmod S) \times (M_2 \bmod S) = M_1 \times M_2.
\end{aligned} \tag{7}$$

Algorithm 1: (SA)

1- Key Generation

1-1 generate two prime big integers p and q

1-2 $S = p \cdot q$

1-3 generate random big integer r

1- Encryption

2-1 $C = M^{r \cdot p - r + 1} \bmod S$

2- Decryption

3-1 $M = C \pmod{p}$.

3- Evaluation

4-1 Suppose there are two ciphertexts:

$$C_1 = M_1^{r_1 \cdot p - r_1 + 1} \bmod S$$

$$C_2 = M_2^{r_2 \cdot p - r_2 + 1} \bmod S$$

4-2 $C_3 = C_1 + C_2 \Rightarrow M_3 = \text{Dec} (C_3)$ (or $M_3 = M_1 + M_2$)

4-3 $C_4 = C_1 \cdot C_2 \Rightarrow M_4 = \text{Dec} (C_4)$ (or $M_4 = M_1 \cdot M_2$)

Let $C = f(C_1 \dots C_i)$, such as:

$$C = [(C_1 \cdot C_3) + C_2] \cdot C_4$$

Let $M = f(M_1 \dots M_i)$, such as:

$$M = [(M_1 \cdot M_3) + M_2] \cdot M_4$$

Where f is any function (Addition and Multiplication) applied on the messages or ciphertexts.

$$44 \quad C \pmod{p} \equiv M, \text{ where } M < p, \text{ except that, we must take } (M \pmod{p})$$

Example 1 (SA):-

1- Key Generation

Select two prime numbers $p = 457679$ and $q = 457$,

$$S = p \cdot q \rightarrow S = 457679 \cdot 457 \rightarrow S = 209159303$$

2- Encryption

Now choose two random integer $r_1 = 5$ and $r_2 = 6$, and two messages $M_1 = 97$ and $M_2 = 98$.

(Note: 97 is ASCII code the letter a , and 98 is ASCII code the letter b).

Firstly ensure that: $(M_1 + M_2) \& (M_1 * M_2) < p$,

Now calculate C_1 :

$$\begin{aligned} C_1 &= M_1^{r_1 \cdot p - r_1 + 1} \pmod{S} \\ &= 97^{5 \cdot 457679 - 5 + 1} \pmod{(209159303)} \\ &= 97^{2288391} \pmod{(209159303)} \\ &= 127692538. \end{aligned}$$

Then calculate C_2 :

$$\begin{aligned} C_2 &= M_2^{r_2 \cdot p - r_2 + 1} \pmod{S} \\ &= 98^{6 \cdot 457679 - 6 + 1} \pmod{(209159303)} \\ &= 98^{2746069} \pmod{(209159303)} \\ &= 35699060 \end{aligned}$$

3- Decryption

$$\begin{aligned} M_1 &= C_1 \pmod{p} \\ &= 127692538 \pmod{457679} \\ &= 97 \end{aligned}$$

$$\begin{aligned} M_2 &= C_2 \pmod{p} \\ &= 35699060 \pmod{457679} \\ &= 98 \end{aligned}$$

4- Evaluation

$$\begin{aligned} C_3 &= (C_1 + C_2) \rightarrow C_3 = 127692538 + 35699060 \\ C_3 &= 163391598 \end{aligned}$$

Now decrypt of C_3 :

$$M_3 = C_3 \pmod{p} \rightarrow M_3 = 163391598 \pmod{457679}$$

$M_3 = 195$, this is equivalent $(M_1 + M_2)$ (i.e. $97 + 98 = 195$)

$$C_4 = C_1 * C_2 \rightarrow C_4 = 127692538 * 35699060$$

$$C_4 = 4558503575614280$$

Now decrypt of C_4 :

$$M_4 = C_4 \pmod{p} \rightarrow M_4 = 4558503575614280 \pmod{457679}$$

$M_4 = 9506$, this is equivalent $(M_1 * M_2)$ (i.e. $97 * 98 = 9506$)

Let $C = f(C_1 \dots C_i)$, such as:

$$\begin{aligned} C &= [((C_1 \cdot C_3) + C_2) \cdot C_4] \pmod{p} \\ &= [((127692538 * 163391598) + 35699060) \cdot 4558503575614280] \pmod{457679} = 412052 \end{aligned}$$

Let $M = f(M_1 \dots M_i)$, such as:

$$\begin{aligned} M &= [((M_1 \cdot M_3) + M_2) \cdot M_4] \pmod{p} \\ &= [97 \cdot 195 + 98] \cdot 9506 \\ &= 180737578 \pmod{457679} \\ &= 412052 \equiv C \text{ (proved).} \end{aligned}$$

5. Results and discussion

In this section we'll calculate the bit complexity of the encryption and decryption function for the Elgamal, RSA Cryptosystems and our proposed SA scheme and also we'll compute the execution time of them.

5.1 Big O Notation (Time Complexity)

The O-notation is very useful in guiding the designers of the algorithms in search for the "best" algorithms for important problem.

The goal of the study of the time complexity of an algorithm is to indicate that its running time is $O(f(N))$ for some function f . [20]

Before performing calculations of the time complexity, we must first analyze the input numbers of the encryption and decryption algorithm, which are either binary integers or decimal digits, where:

The time complexity of a binary integers is $O(n)$, while the time complexity of a decimal digits is $O(\log(n))$, except the constant number, which the time complexity to its $O(1)$, where n is the size of input numbers.

5.1.1 Time Complexity of Elgamal Cryptosystem

Let n is the size of input message that in the type of decimal digits.

Encryption function:

$$K = Y^r \pmod{p}$$

$$C_1 = g^r \pmod{p}$$

$$C_2 = M \cdot K \pmod{p}$$

Then

$$T(K) = O((\log(n))^3), \text{ (Modular exponentiation)}$$

$$T(C_1) = O((\log(n))^3)$$

$$T(C_2) = O(2(\log(n))^2) = O((\log(n))^2)$$

Where, $(\log_2 n)$ is the number of bits of n .

$T(\text{Encryption}) = O((\log(n))^3)$ bit operation, according the most costly operation.

Also,

$T(\text{Encryption}) = O((\log(n))$ arithmetic operation.

Decryption function:

$$K = C_1^x \pmod{p}$$

$$K = K^{-1} \cdot C_2 \pmod{p}$$

Then:

$$T(K) = O((\log(n))^3)$$

$$T(M) = O(2(\log(n))^2) + T(K-1)$$

$$T(K^{-1}) = O((\log(n))^3) \text{ (by extend Euclid's method)}$$

$$T(M) = O(2(\log(n))^2) + O((\log(n))^3)$$

$$T(M) = O((\log(n))^3) \text{ bit operation}$$

$T(\text{Decryption}) = O((\log(n))^3)$ bit operation, according the most costly operation.

Also,

$T(\text{Decryption}) = O(\log(n))$ arithmetic operation.

5.1.2 Time Complexity of RSA Cryptosystem

Let n is the size of input message that in the type of decimal digits.

Encryption function:

$$C = M^e \pmod{n}$$

Then:

$$T(C) = O((\log(n))^3) \text{ bit operation.}$$

$T(\text{Encryption}) = O((\log(n))^3)$ bit operation, according the most costly operation.

Also,

$T(\text{Encryption}) = O((\log(n))$ arithmetic operation.

Decryption function:

$$M = C^d \pmod{n}$$

Then:

$$T(M) = O((\log(n))^3) \text{ bit operation.}$$

$T(\text{Decryption}) = O((\log(n))^3)$ bit operation, according the most costly operation.

Also,

$T(\text{Decryption}) = O((\log(n))$ arithmetic operation.

5.1.3 Time Complexity of SA scheme

Suppose n is the size of input message in the type of decimal digits.

Encryption function:

$$C = M^{r * p - r + 1} \bmod S$$

Then:

$$T(C) = O((\log(n))^3).$$

Where, $(\log_2 n)$ is the number of bits of n .

$T(\text{Encryption}) = O((\log(n))^3)$ bit operation, according to the most costly operation.

Also,

$T(\text{Encryption}) = O(\log(n))$ arithmetic operation.

Decryption function:

$$M = C \bmod p$$

Then:

$$T(M) = O((\log(n))^2)$$

$T(\text{Decryption}) = O((\log(n))^2)$ bit operation, according to the most costly operation.

Also,

$T(\text{Decryption}) = O(1)$ arithmetic operation.

5.2 Execution Time

We compute the execution time of the ElGamal, RSA Cryptosystems and our proposed SA scheme with five sizes of the message and five lengths of private key as following:

In Table 1: notes that we take five size of the messages begin with (12 byte) and end with (2.5 K Byte) and we use a secret key with length of 32-bit that equivalent (10 digit) to calculate of the execution time of ElGamal, RSA Cryptosystems and our proposed SA scheme as shown in Figure 1.

Size of the message	ElGamal(m.s)	RSA (m.s)	SA (m.s)
12 Byte	15	16	24
1 K Byte	9491	11377	11581
1.5 K Byte	21497	25149	25278
2 K Byte	37816	45866	46186
2.5 K Byte	58699	72900	78159

Table 1. Execution time with Size of Secret key 32- bit

In Table 2: notes that we take also five size of the messages begin with (12 byte) and end with (2.5 K Byte) and we use a secret key with length of 64-bit that equivalent (20 digit) to calculate of the execution time of ElGamal, RSA Cryptosystems and our proposed SA scheme as shown in Figure 2.

In Table 3: notes that we take also five size of the messages begin with (12 byte) and end with (2.5 K Byte) and we use a secret key with length of 128-bit that equivalent (39 digit) to calculate of the execution time of ElGamal, RSA Cryptosystems and our proposed SA scheme as shown in Figure 3.

Figure 1. Execution time with length of Secret key 32- bit

Size of the message	ElGamal(m.s)	RSA (m.s)	SA (m.s)
12 Byte	15	17	20
1 K Byte	15137	15625	15860
1.5 K Byte	33456	34589	35050
2 K Byte	58738	61477	64608
2.5 K Byte	100048	105859	107814

Table 2. Execution time with length of Secret key 64- bit

Figure 2. Execution time with length of Secret key 64- bit

In Table 3: notes that we take also five size of the messages begin with (12 byte) and end with (2.5 K Byte) and we use a secret key with length of 128-bit that equivalent (39 digit) to calculate of the execution time of ElGamal, RSA Cryptosystems and our proposed SA scheme as shown in Figure 3.

Size of the message	ElGamal (m.s)	RSA (m.s)	SA (m.s)
12 Byte	21	22	27
1 K Byte	30751	36431	39041
1.5 K Byte	72440	90424	93189
2 K Byte	140054	163767	169649
2.5 K Byte	217615	263558	258449

Table 3. Execution time with length of Secret key 128- bit

Figure 3. Execution time with length of Secret key 128- bit

In Table 4: notes that we take also five size of the messages begin with (12 byte) and end with (2.5 K Byte) and we use a secret key with length of 256-bit that equivalent (78 digit) to calculate of the execution time of ElGamal, RSA Cryptosystems and our proposed SA scheme as shown in Figure 4.

Size of the message	ElGamal (m.s)	RSA (m.s)	SA (m.s)
12 Byte	22	24	45
1 K Byte	34541	39965	74470
1.5 K Byte	82953	96749	173277
2 K Byte	155187	172906	325499
2.5 K Byte	241098	276159	488977

Table 4. Execution time with length of Secret key 256- bit

In Table 5: notes that we take also five size of the messages begin with (12 byte) and end with (2.5 K Byte) and we use a secret key with length of 512-bit that equivalent (155 digit) to calculate of the execution time of ElGamal, RSA Cryptosystems and our proposed SA scheme as shown in Figure 5.

Figure 4. Execution time with length of Secret key 256- bit

Size of the message	ElGamal (m.s)	RSA (m.s)	SA (m.s)
12 Byte	35	23	73
1 K Byte	93711	37787	120354
1.5 K Byte	212789	93554	297891
2 K Byte	394844	178951	534314
2.5 K Byte	658823	287563	840937

Table 5. Execution time with length of Secret key 512- bit

Figure 5. Execution time with length of Secret key 512- bit

As a result, noticed that the length of the key was effect on the execution time in which the execution time increased when the length of the key was increase.

5.3 Security

The RSA security depends on the fact that it is easy to multiply two large primes to construct a modulus, the inverse operation of

factoring the modulus into its prime factors can be very difficult, the difficult until we solve the integer factorization problem for the sizes of the numbers involved. Attempting to break RSA by developing an integer factorization solution for the moduli involved is known as a mathematical attack. That is, a mathematical attack on RSA consists of discovering the prime factors p and q of the modulus n . Clearly, knowing p and q , the attacker will be able to discover the private exponent d for decryption. Another way of stating the same as above would be that the attacker would try to discover the totient $\phi(n)$ of the modulus n . But as stated earlier, knowing $\phi(n)$ is equivalent to knowing the factors p and q . If an attacker can somehow figure out $\phi(n)$, the attacker will be able to set up the equation $(p-1)(q-1) = \phi(n)$, that, along with the equation $p \times q = n$, will allow the attacker to determine the values for p and q . Over the years, various mathematical techniques have been developed for solving the integer factorization problem involving large numbers such as Trial Division, Fermat's Factorization Method, Sieve Based Methods, and Pollard- ρ Method.

The security of ElGamal is based on the discrete logarithm problem. To encrypt and decrypt a message, a discrete power is executed. This operation is efficient to compute. An attacker that wants to decrypt an intercepted message may try to recover the private key. To this end a logarithm needs to be computed. No actual method exists for this, given certain requirements on the initial group are met. Under these circumstances, the encryption is secure. ElGamal has the disadvantage that the ciphertext is twice as long as the plaintext. It has the advantage that the same plaintext gives a different ciphertext each time it is encrypted. Today the ElGamal algorithm is used in many cryptographic products.

In brief, the security of the Elgamal and RSA depends on:

- 1) The security of RSA cryptosystem depends on the **difficulty of factoring large integers**.
- 2) The security of ElGamal cryptosystem depends on the **difficulty of computing discrete logs in a large prime modulus**.

The security of the proposed SA scheme depends on the right selection of a private key that must be unknown or uncommonly used, where knowing of the private part makes the decryption easy. If the private key is unknown, it is virtually impossible to decrypt the message in an acceptable time.

6. Conclusion

In this paper, we have a secret key fully homomorphic encryption scheme (SA) essentially based on Euler's theorem and it proved his correctness of supporting Additive and Multiplicative Homomorphism and we offered an example explains the steps of how it works. As a result, The time complexity of the encryption function of Elgamal, RSA cryptosystems and the proposed SA scheme were in the order of $O((\log(n))^3)$ bit operation while the time complexity of the decryption function of Elgamal, RSA cryptosystems were $O((\log(n))^3)$ bit operation and the time complexity of the decryption function of SA scheme were $O((\log(n))^2)$ bit operation which is the better. and also compared the execution time among the three schemes with five sizes of the messages and used five lengths of a secret key, we noticed that the length of the key was effect on the execution time in which the execution time increased when the length of the key was increase. We applied the proposed scheme SA on a cloud computing and it performed both addition and multiplication operations at the same time on ciphertext without decryption. Finally, SA scheme showed a good security for the stored data on the cloud.

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