THE TECHNIQUE OF RECOVERY OF CAUSATIVE SOURCES IN CASE OF STRONG INTERFERENCE OF GRAVITY FIELDS USING THE CONTINUOUS WAVELET TRANSFORM

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Abstract

The technique of processing gravimetric data is offered in this study. Offered technique based on wavelet transform with so-called «native» wavelet basis functions. Distinctive feature of the technique is a close relationship with both direct and inverse problems of gravimetry. It was shown that the peculiarity allows to quite simply and quickly location of causative sources even under of strong interference of gravity fields.

Keywords:

Gravimetry;
Wavelet transform;
Anomaly;
Inverse problem.

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Introduction

In order to localize causative sources of potential fields (e.g. magnetic and gravitational) is often used the idea of scale invariance. In this case, a search of sources is usually implemented either in the time domain [1-4] or in the frequency-time domain of the continuous wavelet transform (CWT) [4],[5-9].

In our opinion, the second approach is better due to scale ordering of the space of wavelet domain. In any case the weak point of both approaches lies in phenomenon of strong interference of real potential fields: a cumulative field caused by many sources don't have the property of homogeneity, inherent for a field of single source.

The technique of recovery of causative sources in the wavelet domain which allows to obtain satisfactory results even under of strong interference of potential fields are offered in this paper.

Mathematical framework

In case of wavelet transform we are free to choose an analyzing wavelet. Let's define a class of wavelet functions for which the wavelet synthesis would be a solution of corresponding geophysical direct problem. Obviously, that in this case the results of wavelet analysis would be a particular solution of the corresponding linear inverse problem and that the dimension of wavelet coefficients would correspond to a geophysical field: density for gravitational field, magnetization for magnetic field, etc.

In order to reach the stated results let's start from a formal solution of the inverse problem in the two-dimensional case. Specifically, we demand that

the formal solution of the 2-D linear inverse gravity problem

$$g(x) = \int_0^\infty dh \int_{-\infty}^\infty \rho(x-y, h) V^{(1)}(y, h) dy \quad (1)$$

would be a product of an action of a linear operator on the gravity field:

$$\rho(x, h) \equiv \frac{1}{f} \int_{-\infty}^\infty l(y, h) g(x-y) dy \quad (2)$$

In formulas (1) and (2): $x, y \in R$ the horizontal coordinates, $h \in R^+$ is vertical coordinate or depth, f – gravitational constant, $\rho(x, h)$ – excessive density, $g(x)$ – the observed gravity field, $V^{(1)}(x, h)$ – the first vertical derivative of the gravitational potential of a two-dimensional point mass:

$$V^{(1)}(x, h) = f \frac{2h}{x^2 + h^2} \quad (3)$$

Function $l(x, h)$ in formula (2) is an unidentified function to be determined. In order to find it, we use Fourier transform. In frequency domain the relations (1) and (2) have the following forms respectively:

$$\hat{g}(\omega) = \sqrt{2\pi} \int_0^\infty \hat{\rho}(\omega, h) \hat{V}^{(1)}(\omega, h) dh \quad (4)$$

$$\hat{\rho}(\omega, h) = \frac{1}{f} \sqrt{2\pi} \hat{g}(\omega) \hat{l}(\omega, h) \quad (5)$$

Here, the upper symbol «^» denotes the Fourier transform of the corresponding function, ω is the spatial frequency. After substituting the function $\hat{\rho}(h, \omega)$ from formula (5) into formula (4), we obtain:

$$\int_0^\infty \hat{l}(\omega, h) \hat{V}^{(1)}(\omega, h) dh = \frac{f}{2\pi} \quad (6)$$

Fourier transform of the relation (3) have the

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following form:

$$\hat{V}^{(1)}(\omega, h) = \frac{f}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \frac{2h}{x^2 + h^2} e^{-i\omega h} dx = f\sqrt{2\pi} e^{-h|\omega|} \quad (7)$$

After substituting the right-hand side of (7) into (6) we get:

$$\int_0^{\infty} \hat{l}(\omega, h) e^{-h|\omega|} dh = \frac{1}{(2\pi)^{3/2}} \quad (8)$$

This equation has plenty of solutions that is reflection of property of inverse problems. Now, we are going to take the next important step: we will seek unknown function $l(x, h)$ as

$$l^{(n)}(x, h) = c_n V^{(n)}(x, h) h^{(n-2)}, \quad (9)$$

where $c_n \in R$; the upper symbol in round brackets «n» corresponds to the order of vertical derivative of the gravitational potential of a two-dimensional point mass $V^{(n)}(x, h)$. The reason for the choice of the function $l(x, h)$ in the form (9) will become clear later. Fourier transform of function (9) is:

$$\hat{l}^{(n)}(\omega, h) = f c_n \sqrt{2\pi} h^{n-2} \omega^{n-1} e^{-h|\omega|} \quad (10)$$

After substitution (10) into (8) we get:

$$c_n \int_0^{\infty} h^{n-2} \omega^{n-1} e^{-2h|\omega|} dh = c_n (n-2)! 2^{1-n} = \frac{1}{f(2\pi)^2}, n > 1 \quad (11)$$

from where

$$c_n = \frac{2^{n-3}}{f\pi^2(n-2)!} \quad (12)$$

And finally,

$$l^{(n)}(x, h) = \frac{2^{n-3}}{f\pi^2(n-2)!} h^{n-2} V^{(n)}(x, h), n > 1 \quad (13)$$

Should be noted that any normalized linear combination of the functions from family (13) is a solution of (8) too, and therefore equation (1). Using of the property of homogeneous function $V^{(n)}$ we are allowed to write

$$V^{(n)}(x, h) = h^{-n} V^{(n)}\left(\frac{x}{h}, 1\right) = h^{2-n} D_h V^{(n)}(x) \quad (14)$$

where D_h is the dilation operator, which is determined as

$$D_h f(x) = \frac{1}{h^2} f\left(\frac{x}{h}\right) \quad (15)$$

Then the formula (12) may be written as

$$l_h^{(n)}(x) = c_n D_h V^{(n)}(x) \quad (16)$$

Functions $l_h^{(n)}(x)$ from (16) of the first three orders is presented below in explicit form:

$$l_h^{(2)}(x) = \frac{1}{h^2 \pi^2} \frac{(1^2 - x^2)}{(1 + x^2)^2} \quad (17)$$

$$l_h^{(3)}(x) = \frac{4}{h^2 \pi^2} \frac{(1 - 3x^2)}{(1 + x^2)^3} \quad (18)$$

$$l_h^{(4)}(x) = \frac{12}{h^2 \pi^2} \frac{(1 + x^4 - 6x^2)}{(1 + x^2)^4} \quad (19)$$

Functions from family (13) (or that is the same, from (16)) up to a normalizing factors is particular case of well known Paul's complex wavelets [10]:

$$\psi(x) = \Gamma(m+1) \frac{i^m}{(1-it)^{m+1}} \quad (20)$$

For instance, the real part of Paul's wavelet for $m = 3$ has form (you can compare with formula (18)):

$$R\psi(x) = -2 \frac{1 - 3x^2}{(x^2 + 1)^3} \quad (21)$$

Therefore, if we adopt the expression (9) then formula (2) will be nothing but the direct continuous wavelet transform:

$$\rho(x, h) = W[l, g](x, h) = \frac{1}{h^2} \int_{-\infty}^{\infty} f(y) l\left(\frac{x-y}{h}\right) dy = (D_h l \cdot g)(x) \quad (22)$$

Let's formulate features of this CWT:

1) for any linear normalized combination of functions from family (13) it's a formal solution of inverse problem of gravimetry (1);

2) the scale parameter of CWT in (22) correspond the depth;

3) the magnitudes of wavelet coefficients have dimension of physical density;

4) a base functions from (13) is vertical derivatives of potential of point source mass.

For the first time the family of wavelets (13) have been used in [11] and was called "native" wavelets.

The «Native» wavelet spectra of the gravity field caused by a single source

In order to obtain formula of wavelet transform of gravity field due to point source we need to put into formula (5) the expression of function $\hat{l}(h, \omega)$ from formula (10) and expression of function $\hat{g}(\omega)$ from formula (7). As a result, we obtain:

$$W[l^{(n)}, V^{(1)}](h, x) = \frac{2^{n-2}}{(n-2)! \pi f} h^{n-2} V^{(n)}(h + h_0, x - x_0) \quad (23)$$

where source has coordinates $\{x_0, h_0\}$ and linear density equal unity. On the figure 1 are demonstrated wavelet transform with using base functions from «native» family (13) with n , equal 2, 3, 4, and 5 for gravity field single source with depth equal 5 km.

The central maximum $\{x^*, h^*\}$ of the function (23) are reached in points with coordinates

$$x^* = x_0, h^* = \left(\frac{n}{2} - 1\right) h_0 \quad (24)$$

If $n = 4$, we get $h^* = h_0$, that is location and value of central maximum of wavelet spectrum correspond location and linear density of source.

Fig.1. Gravity field caused by a two-dimensional point source located 5 km deep and wavelet transform with "native" basis (formula (13)) with order of 2, 3, 4 and 5. (1 mGal = 10⁻⁵ m/c²)

The recovery algorithm in case multiple sources

In case of interference of fields due to several sources we can't use formula (24) because extremes of wavelet spectrum don't indicate on the locations of causative sources. For this complicated cases we have invented special algorithm.

But before moving on to the description of our method we would like to consider the method that have been offered in works [12], [5]. According to this method, the search of causative sources is executed in wavelet domain using geometrical approach. Due to the property of homogeneity of the fields of simplest sources and dilatation qualities of a wavelet transform, a set of points satisfying equation $(\partial^m / \partial h^m)W[l, g](h, x) = 0$, form a cone-like pattern, whose apex lie in negative domain of scale parameter. The location of its apex corresponds the depth of causative source (see fig. 2).

But this approach doesn't work in case of strong interference of the fields (see fig. 3). The reason is that the interfering fields don't satisfy to equation of homogeneity. For explanation of our algorithm, we consider simplest case of interference of fields caused by two sources. If are known the parameters of one of them then due to linearity of a wavelet transforms after subtraction of its wavelet spectrum (formula (23)) from cumulative spectrum, the rest wavelet coefficients would correspond to the spectrum of the other source. But even if we don't know the exact location of one source, taking into consideration of its approximate parameters allow us to clarify the parameters of other source. Repeating this procedure we approach step by step toward the exact solution. This principle is also valid for the case of a large number of causal sources. All details of our algorithm is presented on the following block diagram (fig. 4).

Figure 5 shows the results of recovering of parameters of three causative sources. The locations of sources have been recovered with high precision. On the figure 5-B by wide light-gray lines is shown the locus of the sources found on previous steps of iterative process. Figure 6 shows that the algorithm is able to

Fig.2. Geometrical method of determination of source's depth by location of cone-like pattern's apex. The wavelet spectrum is the same as in figure 1, fourth-order. According by [12], [5]

Fig.3. «Native» wavelet-transform (B) of gravity field (A) caused by a three sources and curves satisfying $W[l^{(4)}, g] = 0$ (black curves on figure (B)). Black circles correspond locations of causative sources. We aren't able to determine locations of causal sources as in case shown on figure 2

recover the parameters of the causative sources even when noise is present in the observed data.

As a practical example, consider the results of the determination of causative sources for regional gravimetric latitudinal profile crossing the structural components of the Volga-Kama antecline: Melekesskaya depression in the western part of the profile and the western slope of the southern dome of the Tatar arch in the eastern part of the profile (fig. 7). Figure 8 shows a histogram of the distribution of found sources along depth. As can be seen on this figure, the maximum number of sources is found near the boundary separating the sedimentary cover and crystalline basement at the depth of approximately 2.0 km.

This result was expected, because the boundary between sedimentary cover and crystalline basement is very pronounced due to a variety of factors such as variability of the density and elastic properties of rock, type and value of porosity, and the gap in deposition age.

Fig.5. Reconstruction of three two-dimensional point sources using their total gravity field (A) and wavelet spectrum (B). Locations of causal sources is the same as in figure 4. In (B) circles indicate initial locations of the sources, and black dots – final positions. In figure 5-B by wide light-gray lines is shown the locus of the sources found on the previous steps of iterative process

Fig.6. Reconstruction of three two-dimensional sources using their cumulative gravity field with 5% added noise (A), and its wavelet spectrum (B). The notation is the same as in figure

Fig.7. The Bouguer anomaly along the regional gravimetric profile (A) and the distribution of sources found in the wavelet domain (B)

Fig.8. The histogram of source distribution with depth for gravity model shown in figure 7

Conclusion

The offered class of «native» wavelet-functions has some advantages in processing of geophysical potential data. In addition, these wavelets may be used to execute any transformations of potential fields and creating a new algorithm for solve of inverse problems.

Suggested technique of recovering of sources may be easy adapted for other potential fields as well as may be modified and extended for recovering of sources other types (such as dipole, and for more complicated objects) and from our point of view represents efficient tool for express processing of gravimetric data.

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Метод восстановления аномалиеобразующих источников при сильной интерференции гравитационных полей с помощью непрерывного вейвлет-преобразования

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Реферат

Предлагается метод обработки гравиметрических данных, основанный на вейвлет-преобразовании с использованием так называемых «естественных» базисных вейвлет-функций. Отличительной особенностью метода является тесная связь, как с прямыми, так и с обратными задачами гравиметрии. В работе показано, что данная особенность позволяет достаточно просто и быстро определять параметры аномалиеобразующих источников даже в условиях сильной интерференции гравитационных полей.

Ключевые слова: гравиметрия; вейвлет-преобразование; аномалия; обратная задача.

Fasiləsiz veyvlet çevrilməsinin köməyilə qravitasiya sahələrinin güclü interferensiyası zamanı anomaliya yaradan mənbələrin bərpası metodu

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Xülasə

«Təbii» bazis veyvlet funksiyalarının istifadəsilə veyvlet-çevrilməsinə əsaslanan qravimetrik məlumatların işlənməsi metodu təklif olunur. Metodun fərqli xüsusiyyəti qravimetriyanın həm düz, həm də tərs məsələləri ilə sıx əlaqənin olmasıdır. Məqalədə göstərilir ki, bu xüsusiyyət qravitasiya sahələrinin güclü interferensiyası şəraitində belə anomaliya yaradan mənbələrin parametrlərini kifayət qədər sadə və tez müəyyənləşdirməyə imkan verir.

Açar sözlər: qravimetriya; veyvlet-çevrilmə; anomaliya; tərs məsələ.