CONTROLLING THE PARAMETRIC ROLL OF A CONTAINER SHIP MODEL BY CHANGING THE HEADING ANGLE

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Abstract: Parametric roll on ships is an auto-parametric resonance phenomenon whose onset causes a sudden rise in roll oscillations leading to dangerous situations for the ship, the cargo and the crew. In the paper, we have numerically investigated the effect of modifying the heading angle on the roll amplitudes. We followed three strategies. In the first, we allowed the heading angle to decrease with a constant angular acceleration so that the encounter frequency has left the dangerous region of the resonance. However, this option involves changing the course of the ship in the long run, which is of course a shortcoming. In the second strategy, we changed the heading angle up and down around an average value that generates large roll amplitudes, by using different periodic sinusoidal or triangular profiles. The beneficial effect of this action is to keep the course, even if at the cost of a momentary delay. We noticed that both control techniques listed above generally managed to significantly reduce the roll amplitudes if certain thresholds have been exceeded. As a last idea for decreasing the parametric roll amplitudes, we used the combined effect of ship forward speed and heading angle change.

Key words: parametric roll, control, container ship model.

1. INTRODUCTION

One of the five types of the ship stability failure modes as proposed by IMO, parametric roll resonance is a complex nonlinear hydrodynamic phenomenon often accompanied by large amplitudes of 30 – 40 degrees for ship sailing in following and head seas.

Even though it was known since early 1950's, parametric roll becomes a real threat for the shipping community when some incidents involving large container ships as APL China or Maersk Carolina have been reported in 1990's, with significant damages for millions of dollars. Fishing and passenger vessels or ro-ro ships are also known to be prone to parametric roll in both regular and irregular waves [1 - 3].

The theory behind parametric roll has been developed by many researchers. Their conclusion is that the phenomenon occurs when the following requirements are satisfied: the natural roll period is approximately twice the wave encounter period, the wave length is on the order of the ship length, the wave height exceeds a critical level and the ship damping in roll is low. When these conditions are met and the ship sails in moderate to heavy longitudinal seas, then the wave passage along the hull and the wave excited vertical motions result in a variation of the underwater hull geometry which, in turn, change the roll restoring characteristics. Only a small excitation, likely introduced by a rudder movement, causes a sudden and quick rise of roll oscillations that bring the vessel into conditions dangerous for the cargo, the crew and the hull integrity. For the modern container ships, factors as the exaggerated bow flare and the pronounced overhang stern make the vessel more susceptible to parametric roll [4 - 6].

Apart from the varying restoring arm, other causes for parametric resonance are the coupling between roll, heave and pitch motions. In the last decades, several mathematical models including these couplings have been proposed by scientists [7, 8].

Once the parametric roll has been installed, the stringent problem to be solved by the shipmaster is to find solutions to quickly decrease the roll amplitudes. A good idea is to significantly change the wave encounter frequency, which would mean that the resonance condition would no longer be satisfied. This can be done by changing at least one of the following two human dependent parameters: the ship forward speed and the heading angle.

In the paper, we focused mainly on the second strategy, so we conducted a comprehensive numerical investigation on the effect of changing the ship heading angle. In our approach we used a simplified mathematical model, in which the only degree of freedom was the roll angle. Even if it can

be criticized for simplicity, this model has proven to be able to capture the basic features of the parametric roll resonance [9 - 11].

$$(I_x + \delta I_x) \ddot{\phi} + d_1 \dot{\phi} + d_2 \phi + \rho g \nabla (GM_m + GM_a \cos \omega_e t) \phi + k_3 \phi^3 = 0 \quad (1)$$

with I_x and δI_x denoting the ship and added mass inertia in roll, d_1 and d_2 the linear and quadratic roll damping coefficients, ρ the water density, g the gravitational acceleration, ∇ the water displacement, k_3 the cubic coefficient of the roll restoring moment, GM_m and GM_a the constant and variable parts of the metacentric height, and ω_e the wave encounter frequency.

The last parameter results from

$$\omega_e = \omega_0 - \frac{\omega_0^2}{g} U \cos \beta \quad (2)$$

where ω_0 stands for the wave frequency, U for the ship forward velocity and β for the ship heading angle. The parameters' values have been taken from an experimental research with a container ship model in a towing tank followed by an expansion to a full scale ship [12]. They are presented in Table 1.

Table 1. The ship parameters' values requested in equation (1)

Parameter	Value	Unit
I_x	$1.4014 \cdot 10^{10}$	$kg \cdot m^2$
δI_x	$2.17 \cdot 10^9$	$kg \cdot m^2$
d_1	$3.5393 \cdot 10^8$	$kg \cdot m^2 / s$
d_2	$1.8826 \cdot 10^8$	$kg \cdot m^2 / rad$
k_3	$2.974 \cdot 10^9$	$kg \cdot m^2 / s^2 rad^2$
ρ	1000	kg / m^3
∇	76,468	m^3
GM_m	1.91	m
GM_a	0.84	m
ω_0	0.464	rad/s
β	180	$degrees$
g	9.81	m/s^2

The roll natural frequency, ω_ϕ , is provided by

2. THE PARAMETRIC ROLL EQUATION

The equation used throughout the paper for describing the parametric roll motion is written as:

$$\omega_\phi = \sqrt{\frac{\rho g \nabla GM_m}{I_x + \delta I_x}} \quad (3)$$

For the values given in Table 1 and $U = 7.4$ m/s, one obtains $\omega_\phi = 0.2975$ rad/s and $\omega_e = 0.6239$ rad/s, such as $\omega_e / \omega_\phi = 2.0971$.

3. NUMERICAL SIMULATIONS

In this section, we performed an extensive analysis of the ship response in the event that the ship heading angle is changed so that the ratio ω_e / ω_ϕ is removed in the long/short term from the resonant value 2.

It is well known that in achieving the resonance conditions, a mechanical system greatly amplifies any initial perturbation, regardless of its size. The motion described by equation (1) is not an exception.

Fig. 1. (a) The time evolution of roll angle ϕ and of its derivative for the parameters displayed in Table 1; (b) The amplitudes and periods of each oscillation cycle.

The initial conditions were $\phi(0) = 0.1, \dot{\phi}(0) = 0$

Thus, Figure 1(a) shows the time evolution of roll angle ϕ and its derivative for the parameters' values given in Table 1. The initial perturbation of only 0.1 degrees is amplified in about 400 seconds to values that raises serious problems for, at least, the transported goods. To get a better picture of the convergence process to the steady state, Figure 1(b) presents the amplitude, respectively the period, of each oscillation cycle.

Fig. 2. Steady state roll amplitudes for $(U, \beta) \in [3, 10] \times [90, 270]$.

(a) $d_1 = 3.5393 \cdot 10^8$; (b) $d_1 = 7.0786 \cdot 10^8$.

The black point stands for the data included in Table 1

From the Figure 1, it is apparent that the stationary state is reached in a few hundred seconds. The maximum amplitude is associated either with the stationary state or with a pre-run time interval. In the following we chose as parametric roll amplitude the average value of the last ten periods of oscillation for a study time of 800 seconds. In these conditions, to determine how far the ship's forward speed or the heading angle should be changed so the roll amplitude becomes acceptable, we computed these amplitudes for 200 x 200 pairs

$(U, \beta) \in [3, 10] \times [90, 270]$. Our findings are reported in Figure 2, where the beneficial influence of damping's increase is highlighted.

Keeping the ship forward velocity U constant, it is obvious that the ship heading angle will have to be changed to about 110° . Two variables can be involved in this process: the time when the speed control is activated and the angular velocity of the motion. We will analyse the effect of their change in the following.

Case 1: The ship's heading angle is steadily decreased

Let assume that the heading angle control is switched on at 300 s after the parametric roll was installed. The initial excitation of only 0.1° increased during this time to 9.5° . After the control is initiated, the tendency to amplify the roll oscillations is kept for several tens of seconds after which it is reversed and maintained as long as the heading angle is decreased, as shown in Figure 3. When the control is switched off, the oscillations stabilize towards the amplitude indicated in Figure 2(a).

Fig. 3. The time evolution of roll angle for various decreases of ship's heading angle

The heading angle started to lower after 300 s. The black colour is associated with the time intervals in which the heading angle is constant ($\beta = 180^\circ$ or β_{min}) and the red colour for the period when angle β is decreased with constant angular velocity.

The angular velocity $d\beta/dt$ only influences the transient phase. The smaller $d\beta/dt$ is, the closer the transition is to the behaviour of the uncontrolled system, i.e. more dangerous (in the first part).

A delay in activating the heading angle control only worsens the situation of the ship before the control is switched on. In the long run, after initiating the control, the roll amplitudes will decay to zero only if the angle β approaches 110° , as presented in Figure 4.

Fig. 4. The same as in Figure 3 but the heading angle started to lower after 400 s

Case 2: The ship's heading angle is changed periodically

The fact that a change of the heading angle initially produces a decrease of the roll angle can be used to imagine another strategy for controlling the parametric roll. It consists in imposing a periodic ship heading angle regime, having as an average the reference value

$\beta = 180^\circ$. Different options for this profile can be thought of, such as triangular or sinusoidal.

a) Triangular profile

Consider a triangular profile of period T , consisting of three right segments corresponding to a linear decrease of β from 180° to $180^\circ - A_\phi$ for the first quarter of the period, followed by an increase of β to $180^\circ - A_\phi$ in the next half period and finished by a new reduction to the reference value $\beta = 180^\circ$. The relationship between the period T and the constant angular velocity $d\beta/dt$ is $T = 4A_\phi/|d\beta/dt|$. For small angular amplitudes A_ϕ , the use of triangular profile does not allow the attenuation of severe parametric roll oscillations, as shown in Figure 5. For intermediate A_ϕ values, there is a gradual reduction of roll amplitudes, with possible bursts for small $|d\beta/dt|$. If A_ϕ reaches values of $25^\circ - 30^\circ$, then this “serpentine” movement proves to be effective in combating the parametric roll.

Fig. 5. Examples of time series of roll angle for a triangular profile of the heading angle.

The heading angle started to change after 400 s

The influence of the control parameters A_ϕ and $d\beta/dt$ on the reduction of the parametric roll oscillations is more clearly revealed in Figures 6 and 7, where the roll amplitudes for different combinations $(A_\phi, d\beta/dt)$ are presented at the end of five periods T or after 1800 s. Even after half an hour the roll amplitudes continue to be large if the angle A_ϕ does not exceed a threshold of about 10° .

Fig. 6. Steady state roll amplitudes for different pairs $(A_\phi, d\beta/dt)$ after: (a) five periods T ; (b) 1800 seconds. The heading angle started to change in a triangular profile after 400 s

Fig. 7. The roll amplitudes for three different angular velocities at (a) five periods T or (b) 1800 s after the triangular heading angle profile was initiated. The control was switched on after 400 s

If this solution is used, the price paid for suppressing the parametric roll consists in increasing the travel time between two given points. The ship's trajectory in a period T is made up of four identical circular arcs arranged as in Figure 8. Indeed, if one refers the ship movement to the Cartesian frame Oxy having Ox as the initial direction of sailing ($\beta = 180^\circ$), the ship's position at an arbitrary small time t is given by the coordinates:

$$x(t) = \int_0^t v_x(\tau) d\tau = \int_0^t U \cos(\omega\tau) d\tau = \frac{U}{\omega} \sin(\omega t)$$

$$y(t) = \int_0^t v_y(\tau) d\tau = \int_0^t U \sin(\omega\tau) d\tau = \frac{U}{\omega} [1 - \cos(\omega t)]$$

with $\omega = |d\beta/dt|$. Eliminating t , one deduces that the ship moves on the circle of equation $x^2 + (R - y)^2 = R^2$, with $R = U/\omega$. In the first quarter of the period T the ship sails on the arc OA in A_ϕ/ω seconds. In the next three quarters the arcs AB , BC and CD are traversed, respectively. So, $T = 4A_\phi/\omega$. Since the projection length of the AD curve on the Ox axis is $L = 4R \sin A_\phi = 4U \sin A_\phi/\omega$ and the distance travelled in a straight line in the same T would have been $L_0 = U \cdot T = 4UA_\phi/\omega$, one obtains a relative decrease in travel in the Ox direction equal to $\Delta L_{rel} = (1 - \sin A_\phi/A_\phi) \cdot 100\%$. This represents only 0.51% for $A_\phi = 10^\circ$ and 4.51% for $A_\phi = 30^\circ$. The relative deviation ΔD_{rel} from the rectilinear path Ox is also not significant. Noting that $\Delta D_{rel} = (1 - \cos A_\phi)/(2A_\phi) \cdot 100\%$, one gets $\Delta D_{rel} = 4.35\%$ for $A_\phi = 10^\circ$ and $\Delta D_{rel} = 12.79\%$ for $A_\phi = 30^\circ$.

Fig. 8. The trajectory of the ship during a period T of the triangular profile of the heading angle

b) Sinusoidal profile

The second path for the heading angle is chosen to be sinusoidal, namely $\beta = 180^\circ - A_\phi \sin(\tilde{\omega}t)$. The tendencies manifested in the triangular profile's case are maintained for this new “winding” course of the ship, as indicated in Figures 9 to 11. Here, $(d\beta/dt)_{\max} = A_\phi \tilde{\omega}$. There are also some differences. Thus, for small A_ϕ and $(d\beta/dt)_{\max}$, the sinusoidal profile only worsens the situation of the ship, the stationary rolling amplitudes reaching values of 40° - 45° .

Fig. 9. Examples of time series of roll angle for a sinusoidal profile of the heading angle
The heading angle started to change after 400 s

Fig. 10. Steady state roll amplitudes for different pairs $(A_\phi, d\beta/dt)_{\max}$ after: (a) five periods T ; (b) 1800 seconds.

Fig. 11. The roll amplitudes for three different angular velocities at (a) five periods T or (b) 1800 s after the sinusoidal heading angle profile was initiated.

For the sinusoidal profile one has

$$\Delta L_{rel} = \left(1 - \frac{1}{\tilde{T}} \int_0^{\tilde{T}} \cos(A_\phi \sin \tilde{\omega} \tau) d\tau\right) \cdot 100\%, \Delta D_{rel} = \frac{1}{\tilde{T}} \int_0^{\tilde{T}/2} \sin(A_\phi \sin \tilde{\omega} \tau) d\tau \cdot 100\%,$$

with $\tilde{T} = 2\pi/\tilde{\omega}$. For $A_\phi = 10^\circ$ one obtains $\Delta L_{rel} = 0.76\%$ and $\Delta D_{rel} = 5.54\%$, while for $A_\phi = 30^\circ$ the results are $\Delta L_{rel} = 6.74\%$ and $\Delta D_{rel} = 16.66\%$.

Case 3: *The ship's forward speed and heading angle are changed*

One can also imagine strategies to combat the parametric roll in which both the forward speed and the heading angle change either simultaneously or sequentially. We propose in the following a variant in which the forward speed is decreased/increased in order to rapidly diminish the roll oscillations to acceptable values and then, keeping the speed unchanged, one impose a “winding” course which has the effect of nullifying the roll amplitudes. Figure 12 shows some results of applying this idea. First, maintaining the ship's course, its speed is lowered from 7.4 m/s to 5 m/s in 120 s and then a sinusoidal profile of the heading angle is used over 5 periods or 1280 s.

Fig. 12. Steady state roll amplitudes. First, the forward speed is decreased in 120 s from 7.4 m/s to 5 m/s and then the heading angle is changed into a sinusoidal profile for: (a) 5 periods T ; (b) 1280 seconds

Although the structure of the panels in Figures 10 and 12 is in general the same, however, the dangerous roll amplitudes associated with areas with small A_ϕ and/or $(dq/dt)_{max}$ have decreased by at least 8 to 10 degrees.

According to Figure 2, before resorting to the winding course of the ship, its speed can be increased. Of course, a substantial amount (somewhere around 10 – 11 m/s) will cause the outing from the dangerous area. Figure 13 shows that even a modest increase of 1 m/s, followed by a few undulations of the trajectory, attenuates to zero the roll oscillations. However, there is a necessary condition, namely the amplitude A_ϕ to exceed a threshold depending on the rate of change of forward speed and on the number of applications of the sinusoidal profile.

Fig. 13. Steady state roll amplitudes. First, the forward speed is increased in 200 s from 7.4 m/s to 8.4 m/s and then the heading angle is changed into a sinusoidal profile for: (a) 5 periods T ; (b) 1280 seconds

4. CONCLUSIONS

In the paper, a control strategy of a ship's parametric roll was analyzed, namely the change of the heading angle. This will affect the encounter wave frequency and, implicitly, one of the necessary conditions for

parametric roll. The ship's heading angle has been modified in two ways:

a) It has been steadily decreased. In order for the parametric roll oscillations to be extinguished, the heading angle had to be lowered below a certain threshold depending on the coefficients of the roll equation. The rate of change of the heading angle seems to influence only to a small extent the steady state oscillations. The main shortcoming of this strategy is the long – term change of the ship's course.

b) It has been changed according to a triangular or a sinusoidal profile. For a sufficiently long time (e.g. 1800 s), the success of this idea is related to exceeding a minimum value ($10^0 - 12^0$) for the slope of the chosen profile. The drawback of this strategy consists in increasing the time interval in which the ship sails between two given points and in recording an instantaneous deviation from the initially established course. However, these deviations are relatively small.

In addition, in the paper a third variant was proposed, in which first the ship's forward speed was decreased/increased (by only 1 – 2 m/s) and then the heading angle was modified according to a sinusoidal profile. Although the time deviation increases even more, this strategy allows the rapid reduction of the parametric roll amplitudes for a wider palette of the involved parameters.

An optimization of a simultaneous forward speed – heading angle change may be the subject of a further study.

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