ANALYSIS OF DYNAMIC AND STATIC FILTRATION AND DETERMINATION OF MUD CAKE PARAMETERS

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ABSTRACT

Drilling operations around the world employ a concept called *overbalance*. During this process, it is well known that dynamic and static filtration can occur. Thin filter cakes and low fluid-invasion rates are extremely desirable to promote optimal logging conditions and permeability return. The aim of this work was to compare the different behavior between dynamic and static filtration in drilling wells. To investigate the filtration process of Newtonian suspensions, we built a dynamic and static filtration loop with which we acquired experimental filtration volume data as a function of time. The filtration loop included a tank mixer where a Newtonian aqueous calcium carbonate polydisperse suspension was homogenized. The suspension was pumped through tubes to a dynamic or a static filtration cell. We validated a theoretical model based on Darcy's law and on mass conservation proposed by Ferreira and Massarani (2005). That model predicted mud cake buildup and filtrate flow rate for Newtonian suspensions. Relying on both models and the experimental data, filter cake parameters were calculated. We discuss, based on these parameters, the effects of the filtration configuration in dynamic and static modes. Finally, we generalized Ferreira and Massarani's model (2005) for procedures involving non-Newtonian suspensions. This new model can predict dynamic filtration and fluid invasion for non-Newtonian suspensions as drilling fluids.

KEYWORDS

dynamic and static filtration; Newtonian and non-Newtonian fluids; fluid invasion; mud cake buildup; drilling mud

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1. INTRODUCTION

Dynamic filtration can be used to purify water (van der Bruggen et al., 2003), to process foods (Jiao and Sharma, 1994), to clarify effluents (Ripperger and Altamann, 2002), etc. For example, in water purification, this process is gaining broad use (van der Bruggen et al., 2003).

In petroleum engineering, drilling fluids are specially formulated to be used during perforating operations to control fluid loss and minimize formation damage. They can be formulated to help minimizing fluid invasion when perforating oil or gas bearing zones with an over-balanced fluid. In this condition, dynamic and static filtration and fluid invasion can occur (Outmans, 1963). Drilling fluids are then injected into the drill string, and return to the surface via the annular space between the drill string and the rock formation. Static filtration occurs when fluid pumping is interrupted. The interruption creates a difference between the hydrostatic pressure in the wellbore and that in the reservoir, and from that point on static filtration occurs. Static filtration rates are controlled by continuous thickening of the mud cake. On the other hand, dynamic filtration occurs when drilling fluids are pumped through the well. In this process, the mud cake thickness is determined by dynamic equilibrium of two factors: the amount of solid particles deposited and the erosion rate caused by shear stresses generated by the fluid flow in the wellbore. Thus, the filtration rate to the formation tends to stabilize around a certain value, at which mud cake thickness

becomes constant. In field operations, dynamic filtration causes the permeate flux to decrease over time. This decrease, a result of increasing hydraulic resistance, is caused by the growing thickness of the mud cake on the surface of the rock. As the mud cake thickens, the permeate flux decreases until a steady state is reached. Otherwise, static filtration reaches no steady state and mud cake continues to grow (Hwang et al., 1996). Figure 1 illustrates this dynamic filtration process.

In oil field laboratories, the ability of drilling fluids to prevent invasion is usually measured with API standard static filtration tests. In these tests, the drilling fluid is pressurized against a filter paper. Engineers monitor the volume of fluid that crosses such medium over time. In many circumstances, this kind of test might not be realistic as it is not representative of the borehole configuration where static and dynamic filtrations in cylindrical coordinates occur.

The focus of this work was to study dynamic and static filtration taking in account differences between the properties of mud cakes in both configurations. We investigated the dynamic and static filtration operations both experimentally and theoretically. These two types of filtration also are known as cross flow and dead-end-filtration, respectively.

For this purpose, a loop was built that can operate under both static and dynamic conditions, and the filtration behavior of Newtonian fluids (calcium carbonate in water) was examined. We estimated the mud cake's permeability and thickness as well as the resistance of the filter medium. Moreover, we studied how the axial and cross-flow velocity affects filter cake erosion. In this context, the filtration model for Newtonian suspensions was validated from the literature (Ferreira and Massarani, 2005). This model, which is valid for Newtonian suspensions in cylindrical coordinates, was extended to a model that can predict dynamic filtration for non-Newtonian suspensions. The former is more realistic, since real mud is mostly non-Newtonian. These Newtonian and non-Newtonian models are useful to estimate filtration and fluid invasion's parameters and fluid flow rate. In the design of projects involving drilling fluids, a good model can be a useful tool.

Figure 1. A schematic for the filter cake buildup in dynamic filtration.

Figure 2. Axial and radial flow diagram and filter cake buildup feature.

2. LITERATURE REVIEW

During dynamic and static filtration many factors which are difficult to quantify can affect external and internal mud cake buildup. It can be affected, for instance, by the physical-chemical properties of the solid-fluid suspension, clogging caused by migration of small particles through the filter cake and the history of the process imposed by the fluid flow on the solid matrix. In other words, the characteristics of filtration and mud cake buildup are affected by many factors as: drilling fluid's properties, fluid flow configuration, rock properties and operation conditions (Massarani, 1985).

Mud cake buildup under dynamic filtration is limited by the action of several forces. Many authors developed experimental work showing that filtrate flow rate equilibrium is directly proportional to shear stress on the mud cake surface. According to Dewan and Chenevert (2001), the mud cake thickness is in equilibrium when forces acting to hold particles on the mud cake surface (shear strength) are overlaid by hydrodynamic shear forces (shear rate). Four forces can act on each particle: hydrodynamic tangential force (shear stress) generated by suspension flow, normal force generated by filtrate flow, hydrodynamic lift force, and surface forces. Outmans (1963) proposed that adhesion and detachment of particles from mud cake surface depends on the coefficient of friction between particles and the mud cake surface. Alternatively, Visser (1972) assumed that the tangential force required to remove a particle from a flat surface was proportional to the drag force.

Dynamic filtration is a complex operation that is controlled by many parameters, whereby the flow of filtrate stabilizes during a long operation, and some mechanisms consequently limit mud cake buildup (Massarani, 1985). At the beginning of filtration, filtrate volume increases as a function of the square root of time, and filtration flow rate is a function of time. Later, the filtrate volume increases linearly with time and the filtrate flow rate becomes constant.

2.1 Dynamic filtration model for Newtonian fluids

For procedures involving Newtonian fluids, Ferreira and Massarani (2005) proposed a new filtration model. Figures 2 and 3 show the scheme that corresponds to the situation studied by them. Two equations represent the basis of Ferreira and Massarani's (2005) model: the mass conservation law and Darcy's Law for liquid and solid phases. Based on these laws, the authors proposed Eq. 1, written in cylindrical coordinates. By using this equation, one can estimate the filtrate flow rate as a function of time.

Figure 3. Scheme of the filter cake buildup with thickness $e = r_m - r$.

$$\frac{dt}{dv} = \frac{\mu_F}{(\Delta p)} \left\{ \alpha_c \cdot \varepsilon_c \cdot \rho_s \cdot r_m \ln \left[\frac{r_m}{\left[r_m^2 - \frac{2 \cdot C \cdot \rho_F \cdot r_m \cdot v}{\varepsilon_s \cdot \rho_s} \right]^{1/2}} + R_m \right] \right\} \quad (1)$$

The Initial condition is for $t = 0$, $v = 0$. Where,

$$\alpha_c = \frac{1}{k_c \cdot \rho_s \cdot (1 - \varepsilon_c)} \quad (2)$$

In Eq. 1, t is the filtration time, μ_F is the fluid viscosity, α_c is the filter cake resistivity, ε_c is the filter cake porosity, Δp is the pressure drop in the r direction; C is the suspension concentration; ρ_F and ρ_s are the fluid and solid densities, respectively; r_m is the radius of the filter medium; R_m is the filter medium resistance; and $v(z, t) = dV/dA$ is the filtrate volume per filtrate area given as the derivative of the filtrate volume, V , with respect to the filtration area.

The authors consider the average properties of the filter cake to be a function of the filtration pressure only. Integration of Eq. 1 yields Eq. 3.

$$t = \frac{\mu_F}{(\Delta p)} \left\{ \alpha_c \cdot \varepsilon_c \cdot \rho_s \cdot r_m \cdot \sum_{k=1}^{\infty} \frac{b^k}{2 \cdot k \cdot (k+1) r_m^2} \cdot v^k + R_m \right\} \cdot v \quad (3)$$

$$\text{where, } b = \frac{2C \cdot \rho_F \cdot r_m}{\varepsilon_c \cdot \rho_s} \quad \text{and} \quad v = \frac{1}{2\pi r_m} \cdot \frac{dV}{dz}$$

In this case, Eq. 4 enables the calculation of the total filtrate volume:

$$V(t) = \pi \cdot D_t \int_0^L v(z, t) dz \quad (4)$$

where D_t is the diameter of the tube and L is its length.

Figure 4. Scheme of the static filtration.

The filter medium resistance (R_m) is related to the filter medium permeability (k_m) and to the filter medium thickness (l_m), as shown in Eq. 5.

$$R_m = \frac{l_m}{k_m} \quad (5)$$

2.2 Static filtration model for Newtonian fluids

The static filtration model in rectangular coordinates is well established in the literature (**Massarani, 1985**). This model is based on Darcy's Law, which governs static filtration (Fig. 4). Traditionally, it holds for incompressible filter cakes and suspensions with Newtonian behavior. For compressible mud cakes, the approach is to use the simplified theory of filtration. This simplification considers the fluid velocity, for a given time and position, to be independent of filter cake properties. This hypothesis becomes more reasonable as the compressibility of the filter cake decreases, and as the suspension which is subjected to filtration process becomes more diluted (**Massarani, 1985**). In this case, the expression obtained by integrating Darcy's motion equation is represented by Eq. 6.

$$\frac{t}{V} = \frac{\mu}{A(\Delta p)} \left[\frac{\alpha_c \cdot C \rho_l \cdot V}{2A} + R_m \right] \quad (6)$$

where α_c is the filter cake resistivity that is related to filter cake permeability by Eq. 2.

3. MATERIALS AND METHODS

To carry out the experimental work, we built a filtration loop that operated under dynamic or static filtration conditions. We investigated the behavior of Newtonian fluids composed of aqueous calcium carbonate polydisperse suspensions. The filter medium, a type of fabric commonly used in the industry, was in both cases made out of polyester, which was replaced after each experiment. The experiments consisted in measuring directly the filtrate volume as a function of time, and were run at ambient temperature with controlled pressure and fluid flow rate. Mud cake parameters like porosity, thickness, and permeability were determined, after each experiment, using theoretical models (Eq. 1 and Eq. 2 for dynamic filtration, and Eq. 6 for static filtration).

3.1 Static and dynamic filtration loop

Fig. 5 shows the filtration loop designed for the assays where the carbonate suspension, after being pumped through 1-in PVC pipes into a 0.5-m³ tank, is homogenized by a 1.5-HP mixer. The homogenized suspension was moved with a 1.5-HP positive displacement pump. Differential pressure gauges were placed along the surface of the filter medium to register pressure drop. The gauges were in the ranges of 0-36 mmHg, 0-350 mmHg and 0-1800 mmHg, and thermocouples measured the system's temperature. We employed a gravimetric technique to measure the concentration of solids in the suspension in triplicate. During the assays, a computer system

Figure 5. Static and dynamic cross-flow filtration loop.

continuously recorded the weight of the filtrate squeezed into the tank as a function of time, perched atop a balance. In each experiment, the filtrate flow rate was determined as a function of time.

3.1.1 Filtration cells

In the dynamic filtration experiments, the filter medium, a permeable wall made out of industrial filtration fabric, was 1-in across, 1-m long and 0.5-mm thick. The same fabric was used to construct a 7.6-cm diameter static cell. In both cases, an aqueous suspension of different-sized particles of calcium carbonate was filtered (See Fig. 5).

3.1.2 Suspension properties

The Newtonian suspension was prepared by adding solid calcium carbonate, with a density of 2700 kg/m³ (Perry, 1953), to water. The size distribution of the suspension's solid particles, obtained with Malvern Laser Particle Size equipment (MALVERN® 2000), is shown in Fig. 6. Typically, the particle diameters are in the range of 1 to 10 µm.

The main objective in these experiments was to understand the suspension and filter cake characteristics that control filtration flow rate and filter cake thickness. Since the objective was not to investigate the role of the filter permeability in filtration control, the use of a constant permeability medium was acceptable (Dewan and Chenevert, 2001).

3.2 Evaluation of filter cake porosity and thickness

After each experiment, the porosity was determined by immediately removing a sample of the filter cake and measuring the wet weight. To

Figure 6. The suspension's particle-size distribution.

remove all water, the filter cake was then heated to 105 °C for 24 hours. The porosity is then given by:

$$\varepsilon_c = \frac{\left(\frac{\text{net wet weight}}{\text{net dry weight}} \right)}{\left(\frac{\text{net wet weight} + \frac{\rho_F}{\rho_s}}{\text{net dry weight} + \frac{\rho_F}{\rho_s}} \right)} \quad (7)$$

In this case, $\rho_F = 1000 \text{ kg/m}^3$ and $\rho_s = 2700 \text{ kg/m}^3$. The mean value of the experimental filter cake thickness is computed by Eq. (8), where A is the filtration area.

$$\bar{e} = \frac{\text{net dry weight}}{A \cdot (1 - \varepsilon_c) \cdot \rho_s} \quad (8)$$

4. RESULTS AND DISCUSSION

The experiments were carried out with aqueous suspensions of calcium carbonate ranging in concentration from 2 to 6 % (w/w) and pressures ranging from 1 to 2 kgf/cm^2 . It is quite common, when analyzing filtration under constant pressure drop, to represent the results as “ t/v ” against “ v ”, or dt/dv against dv , as shown in Eq. 1 (Ferreira and Massarani, 2005). Figures 7 and 8 show these results for the filtrate fluid obtained in all filtration experiments in dynamic and static modes, respectively. In both cases, the volume of filtrate is normalized by the filtration area of each filter system. It could be observed that an increase in pressure causes an increase in the filtrate flow in both dynamic and static modes at the same concentration. For a given pressure, however, the increase in the concentration of the suspension caused a decrease in the filtrate flow.

One can also verify that the dynamic filtration curves had two steps. In the first, the filtrate flow rate was a function of time; in the second, it was constant. In dynamic filtration, the first step was extended over a long period, up to 30 minutes, in the most critical experiment (Experiment 2). In this case there was a combination of increased pressure with lower concentration of suspension. In the static filtration, the experimental measurements were initiated 30 seconds after

Figure 7. The t/v ratio measured with time in dynamic filtration mode.

Figure 8. The t/v ratio measured with time in static filtration mode.

Figure 9. Filtration flow rate measured with time in dynamic filtration mode.

each experiment began. Because the flow of filtrate was not stabilized, the second step was no longer visible.

Fig. 9 shows the measured filtrate flow rate as a function of time for the dynamic filtration experiments. One can clearly observe the behavior of the filtration curves. For experiments conducted at 2 kgf/cm^2 , the filtration flow rates were constant after the filter cake thickness stabilized.

Table 1. Values of α and R_m for dynamic and static filtration experiments.

Exp.	$c/(p/p)$	$P/(kgf/cm^2)$	$\alpha_{dynamic}/(cm/g)$	$\alpha_{static}/(cm/g)$	$R_{m,dynamic}/(cm^{-1})$	$R_{m,static}/(cm^{-1})$	$\epsilon_{dynamic}$	ϵ_{static}
1	0.02	1.0	2.30×10^8	4.14×10^9	2.62×10^9	9.42×10^8	0.56	0.56
2	0.02	2.0	3.46×10^8	4.17×10^9	3.86×10^9	9.74×10^8	0.54	0.56
3, 4, 5	0.04	1.5	6.29×10^8	4.79×10^9	2.89×10^9	1.25×10^9	0.55	0.57
6	0.06	1.0	2.58×10^8	5.09×10^9	2.48×10^9	1.31×10^9	0.55	0.58
7	0.06	2.0	6.60×10^7	5.55×10^9	4.56×10^9	1.41×10^9	0.54	0.57
8	0.04	1.0	3.75×10^8	5.27×10^9	2.54×10^9	1.11×10^9	0.57	0.58
9	0.04	2.0	1.36×10^8	6.27×10^9	4.24×10^9	1.12×10^9	0.54	0.56

4.1 Estimation of parameters α_c and R_m

For each experiment of dynamic and static filtration, the values of resistivity of the filter cake (α_c) and the filter medium resistance (R_m) were determined. Table 1 shows the values obtained using Eq. 2 for dynamic filtration and Eq. 11 for static filtration.

The values of α_c found in the dynamic and static filtration modes differ even by order of magnitude of 10. In all cases, the filter cake resistivity in the static filtration is higher than in the dynamic filtration, as reported in the literature (Lawhon et al., 1967). The results show that the configuration of the filter, either in dynamic or static mode, in this case, can affect the filter cake properties leading to different values of α_c .

Three specific experiments were performed in dynamic condition to evaluate the experimental error for each operational condition (experiments 3, 4 and 5). These results indicate an average $\alpha_c = (6.29 \pm 1.18) \times 10^8$ cm/g and an average $R_m = (2.89 \pm 0.18) \times 10^9$ cm⁻¹. For the dynamic filtration, these figures represent, on average, a deviation of 19 % in α_c and 6 % in R_m . Except for Experiments 1 and 2, the filter medium resistances of both systems were similar.

4.2 Simulations for dynamic filtration in cylindrical coordinates

The dynamic model proposed by Ferreira and Massarani (2005), and given by Eq. 1, was not

validated by the authors. In this work, we validate their model by comparing our experimental data with simulation data given by the numerical solution of Eq. 1. This equation was solved using the Euler scheme to move numerical solutions forward in time. With these solutions, we were able to describe the first step on the filtration process, which could not be done using Eq. 2. In this case, the parameters obtained using Eq. 2 and presented in Table 1 were used. Fig. 10 shows that both simulated and experimental results are in good agreement. The initial period of filtration (transient filtration period) is important in predicting the invasion of drilling fluid into the reservoir rock. By using Eq. 1, we studied the effects of pressure and fluid velocity on the filter cake properties. In this case, the average thickness of the filter cake was 1.5 mm; the model fit the experimental thickness with less than 10 % in deviation.

Fig. 11 presents a study of parametric sensitivity of the filter cake resistivity. In this case, the parameter differed from the value obtained experimentally by about 20 %. We observed that the variation in α_c caused a change in the slope of the curve, but the results for the first step were not much altered.

4.3 Filter cake erosion

In dynamic filtration operations, filter cake erosion must be studied when defining the filter cake thickness. Such erosion occurs when the shear stress (τ , in dyn/cm²) imposed by circulating mud

Figure 10. Comparison between experimental and simulated dynamic filtration data.

Figure 11. Parametric sensitivity analysis with respect to the filter cake resistivity (Exp. 1).

on the cake surface exceeds the shear strength (τ_{th} , in dyn/cm^2) of the filter cake at that location. The shear stress imposed by a power law suspension is accounted for in Eq. 9.

$$\tau = -\mu_F \lambda \quad (9)$$

where μ_F is the fluid viscosity and λ is the shear rate at the filter cake surface. Two components define the shear strength of the filter cake (Dewan and Chenevert, 2001):

$$\tau_{th} = \tau_{th,0} + f \cdot \tau_g \quad (10)$$

Figure 12. Experimental shear stress and theoretical shear strength (Eq. 10).

where $\tau_{th,0}$ is the shear strength with zero pressure across the filter cake and $f \tau_g$ is the additional strength provided by the applied pressure. The second term tends to hold the particles together. Parameter f is the coefficient of friction and parameter τ_g is the grain-grain stress resulting from the applied pressure. Outmans (1963) gave one approximation for the grain-grain stress (in dyn/cm^2):

$$\tau_g = 68950 \cdot P_c \cdot \left(\frac{d}{e(z,t)} \right)^{\frac{1}{1-v}} \quad (11)$$

where P_c , in psi, is given by Eq. 12, d is the mean particle diameter, and v is the compressibility exponent.

$$P_c = \frac{q(t) \cdot e(t) \cdot \mu_F}{k_m(t)} \quad (12)$$

The filter cake parameter was determined by non-linear regression using experimental data. The experimental data were obtained in the conditions presented in Table 1. Simulating the filtration plot with data from these nine experiments yielded the following values: $f = 150$, $\tau_{c0} = 200$, and $v = 0.9$. Fig. 12 shows the mean results of experimental shear stress and shear strength (Eq. 10) for all nine experiments presented in Table 1.

As we noted above, erosion occurs when the shear stress generated by the circulating

suspension exceeds the shear strength of the filter cake. The shear rate is a function of the axial fluid velocity and the shear strength is a function of the cross-flow velocity.

5. DYNAMIC FILTRATION MODEL FOR NON-NEWTONIAN FLUIDS

From the model proposed by Ferreira and Massarani (2005), a new model was designed to describe operations involving non-Newtonian fluids. To derive an expression for the dynamic filtration of non-Newtonian fluids, one may begin by analyzing the shear stress imposed by a power-law fluid and its relation with the Darcy's equation. The shear stress by a power-law fluid is given by (Bird et al., 1960):

$$\tau(\lambda) = M \cdot (\lambda^*)^n \quad (13)$$

where M and n are the fluid consistency and behavior indexes, and λ^* is the characteristic shear rate at the filter cake surface as a function of the medium permeability (k) and the filtrate flow rate (q), as shown by Eq. 14 (Massarani, 1985):

$$\lambda^* = \frac{q}{\sqrt{k}} \quad (14)$$

The effective viscosity (μ_{ef}) is calculated from the rheology of the suspension:

$$\mu_{ef} = \frac{\tau(\lambda^*)}{\lambda^*} \quad (15)$$

Substituting Eq. 13 and 14 into Eq. 15 gives:

$$\mu_{ef} = M \cdot \left(\frac{q_r}{\sqrt{k}} \right)^{n-1} \quad (16)$$

Eqs. 17 and 18 describe the Darcy's equation of motion for the filtrate flowing through the filter cake and through the filter medium, respectively:

$$-\frac{dP}{dr} = \frac{\mu_{ef}}{k_c} \cdot q_r \quad (17)$$

$$-\frac{dP}{dr} = \frac{\mu_{ef}}{k_m} \cdot q_r \quad (18)$$

where k is the medium permeability; subscripts "c" and "m" denote filter cake and filter medium, respectively. Solid mass balance for fluid and filter cake must be considered in order to establish a correlation between the mud concentration C and the coordinates r and r_m (see Eq. 19, where ε_s is the filter cake porosity, ρ_s is the solid density and L is the filter length). Figure 3 shows that r_m is defined at the surface of the filter medium, and r at the filter cake surface.

$$C = \frac{\varepsilon_s \cdot \rho_s \cdot L \cdot \pi \cdot (r_m^2 - r^2)}{(2 \cdot \pi \cdot r_m \cdot L) \cdot v(z, t) \cdot \rho_F} \quad (19)$$

Eq. 20 gives the relationship between coordinate r and the internal radius of the filter medium:

$$r = \left(r_m^2 - \frac{2 \cdot C \cdot \rho \cdot r_m \cdot v(z, t)}{\varepsilon_s \cdot \rho_s} \right) \quad (20)$$

Since $e(z, t) = r_m - r$, the filter cake thickness can be calculated with Eq. 21:

$$e(z, t) = r_m - \left[r_m^2 - \frac{2 \cdot C \cdot r_m \cdot v(z, t)}{\varepsilon_s \rho_s} \right]^{1/2} \quad (21)$$

The filtrate volume per unit of filtration area with time can be obtained by substituting Eq. 16 into Eq. 17, to yield:

$$-\frac{dP}{dr} = \frac{M}{k_c^{\frac{n+1}{2}}} \cdot q_r^n \quad (22)$$

Defining Eq. 23, and then combining it with Eq. 22, Eq. 24 is obtained.

$$q_r = \frac{dQ_f}{2 \cdot \pi \cdot r \cdot dz} = \frac{1}{2 \cdot \pi \cdot r} \frac{dQ_f}{dz} = \frac{1}{2 \cdot \pi \cdot r} \cdot Q_f^* \quad (23)$$

$$-\frac{dP}{dr} = \frac{M}{k_c^{\frac{n+1}{2}}} \cdot \left(\frac{Q_f^*}{2 \cdot \pi \cdot r} \right)^n \quad (24)$$

Integration of Eq. 25 then yields Eq. 25:

$$P - P_m = \frac{M}{k_c^{\frac{n+1}{2}}} \cdot \left(\frac{Q_f^*}{2 \cdot \pi \cdot r_m^n} \right)^n \cdot r_m^n \cdot \left(\frac{r_m^{1-n} - r^{1-n}}{1-n} \right) \quad (25)$$

By inserting Eq. 20 in Eq. 25, and knowing that $\frac{Q_f^*}{2 \cdot \pi \cdot r_m} = \frac{dv(z,t)}{dt}$ at the internal filter medium wall, the filter cake pressure drop results in:

$$P - P_m = \frac{M}{k_c^{\frac{n+1}{2}}} \left(\frac{dv}{dt} \right)^n \cdot r_m^n \cdot \left(\frac{r_m^{1-n} - \left(r_m^2 - \frac{2 \cdot c \cdot \rho \cdot r_m \cdot v}{\rho_s \cdot \varepsilon_s} \right)^{1-n}}{1-n} \right) \quad (26)$$

To obtain the average pressure drop in the filter medium, the same steps used to define Eq. 26 were followed, starting by Eq. 18, and generating Eq. 27:

$$P_m - P_{atm} = \frac{M}{k_m^{\frac{n+1}{2}}} \left(\frac{dv}{dt} \right)^n \cdot r_m^n \cdot \left(\frac{r_{ext}^{1-n} - r_m^{1-n}}{1-n} \right) \quad (27)$$

where r_{ext} is the external radius of the filtrate pipe. Adding Eq. 26 to Eq. 27 gives the total pressure drop ($P - P_{atm}$), in Eq. 28:

$$\Delta P_f = \frac{M}{k_c^{\frac{n+1}{2}}} \left(\frac{dv}{dt} \right)^n \cdot r_m^n \cdot \left(\frac{r_m^{1-n} - \left(r_m^2 - \frac{2 \cdot c \cdot \rho \cdot r_m \cdot v}{\rho_s \cdot \varepsilon_s} \right)^{1-n}}{1-n} \right) + \frac{M}{k_m^{\frac{n+1}{2}}} \left(\frac{dv}{dt} \right)^n \cdot r_m^n \cdot \left(\frac{r_{ext}^{1-n} - r_m^{1-n}}{1-n} \right) \quad (28)$$

Finally, rearranging Eq. 28 to derive the filtrate model for a constant pressure filtration of power-law mud, one has:

$$\left(\frac{dv(t)}{dt} \right)^n = \frac{\Delta P}{\left(\frac{M}{k_c^{\frac{n+1}{2}}} \right) \cdot f_c + \left(\frac{M}{k_m^{\frac{n+1}{2}}} \right) \cdot f_m} \quad (29)$$

$$t = 0, v = 0$$

where:

$$f_c(v) = r_m^n \cdot \left(\frac{r_m^{1-n} - \{r(v)\}^{1-n}}{1-n} \right) \quad (30)$$

and

$$f_m = r_m^n \cdot \left(\frac{r_{ext}^{1-n} - r_m^{1-n}}{1-n} \right) \quad (31)$$

5.1 Comparison between Newtonian and non-Newtonian models in cylindrical coordinates

Fig. 13 shows simulation results from the model proposed by Ferreira and Massarani (2005) and from the model proposed in this work. At the same conditions, the procedure using non-Newtonian fluid reduces the volume of filtrate, as verified by

Figure 13. Results generated using Newtonian and non-Newtonian models. In this case, $c = 0.032 \text{ g/g}$, $\varepsilon = 0.63$, $\rho_f = 1 \text{ g/cm}^3$, $\rho_s = 2.91 \text{ g/cm}^3$, $k_c = 2.1 \times 10^{-11} \text{ cm}^2$, $k_m = 4.8 \times 10^{-11} \text{ cm}^2$.

an increase in the slope of the filtration curve. This effect is enhanced as the “n” parameter in the power-law models decreases. When the parameters are set with “n” approaching unity, and $M = 0.01 \text{ g}/(\text{cm} \cdot \text{s}^{2-n})$, the model proposed in this work fits well the data generated by the Newtonian model proposed by Ferreira and Massarani (2005). The non-Newtonian behavior shows that the removal of the condition of Newtonian fluid leads to a decrease in the filtrate loss, as shown by data acquired at 2.2 atm. Moreover, even for a non-Newtonian fluid, for which $n = 0.5$ and $M = 0.8 \text{ g}/(\text{cm} \cdot \text{s}^{2-n})$, the increase in pressure from 0.5 atm to 2.2 atm causes an increase in the filtrate flow rate.

6. CONCLUSIONS

This paper presents experimental data obtained from dynamic and static filtration conditions. We validated the model proposed by Ferreira and Massarani (2005) for cross-flow filtration suspension with Newtonian behavior. Experimental data were used to estimate the values of the filter cake parameters. Our experimental and theoretical filtration curves behaved as expected, that is, increasing pressure for the same concentration causes an increase in the filtrate flow rate. Raising the concentration, for the same pressure, reduces the filtrate flow rate. Under the same experimental conditions, filter cake properties like resistivity varied with the type of filtration (static or dynamic). For values of filter medium resistance, in general we obtained similar results even for different methods of filtration. The calculation of

the shear stress and the shear strength defines when erosion takes place and when the filter cake thickness is stabilized. This analysis proved to be useful in calculating the filter cake thickness. To validate the non-Newtonian model, further experiments will be run with polymeric suspensions. Most drilling fluids behave as non-Newtonian fluids. Therefore, Ferreira and Massarani’s model was generalized for Newtonian fluids in cylindrical coordinates, so that it can be used to describe the filtration of non-Newtonian fluids such as power-law suspensions. Based only on simulation data, we observed that when the power-law parameter “n” tends to unity our model is reduced to the original model. These results are useful tools for calculating the filtration rate and fluid invasion with non-Newtonian muds.

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NOMENCLATURE

- A - filtration area
- C - suspension concentration
- D_t - diameter of the tube
- d - mean particle diameter
- $e(z,t)$ - filter cake thickness
- $\bar{e}$ - mean value of the experimental filter cake thickness
- f - coefficient of friction
- k - permeability
- L - length of filtration section
- l_m - porous medium thickness
- M - fluid consistency index
- n - fluid behavior index
- P - pressure
- q - filtrate flow rate
- R_m - porous medium resistivity
- r - radius at filter cake surface
- r_m - pipe (tube) radius
- r_{ext} - external radius pipe
- t - filtration time
- v - volume of filtration volume per filtration area
- V - total filtration volume

Greek Letters

- α - filter cake resistivity
- ΔP - total pressure drop
- ε - porosity

λ^* - characteristic shear rate at filter cake surface
 μ - viscosity
 ν - compressibility exponent
 ρ - density
 τ - shear stress
 τ_g - grain-grain stress

Subscripts

c - filter cake
 ef - effective property
 ext - external
 F - fluid or mud
 g - grain-grain interaction
 m - filter medium
 r - radial direction
 s - solid
 t - tube
 th - shear strength
 th,0 - shear strength with zero pressure

7. REFERENCES

- Bird, R. B.; Stewart, W. E.; Lightfoot, E. N. **Transport Phenomena**. Wiley International Edition, John Wiley & Sons, 1960.
- Dewan, J.T.; Chenevert, M.E. A model for filtration of water-base mud during drilling: determination of filter cake parameters. **Petrophysics**, vol 42, n. 3, p. 237-250, 2001.
- Ferreira, A. S.; Massarani, G. Physical-mathematical modeling of crossflow filtration, **Chemical Engineering Journal**, 111, p.199-204, 2005. [doi:10.1016/j.cej.2005.02.002](https://doi.org/10.1016/j.cej.2005.02.002)
- Hwang, S.J.; Chang, D.J.; Chen, C. H. Steady state permeate flux for particle cross-flow filtration. **The Chemical Engineering Journal**, 61, p.171-178, 1996. [doi:10.1016/0376-7388\(94\)00180-7](https://doi.org/10.1016/0376-7388(94)00180-7)
- Jiao, D.; Sharma, M.M. Mechanism of Cake Buildup in Crossflow Filtration of Colloidal Suspensions, **Journal of Colloid and Interface Science**, 162, p.454-462, 1994. [doi:10.1006/jcis.1994.1060](https://doi.org/10.1006/jcis.1994.1060)
- Lawhon, C. P., Evans, W. M., Simpson, J. P. Laboratory Drilling rate and Filtration Studies of Emulsion Drilling fluids. **Journal of Petroleum Technology**, Reprinted from July 1967, received from SPE 1695.
- Massarani, G. Filtration process. **Brazilian Journal of Chemical Engineering**, special issue September 1985. (In Portuguese).
- Outmans, H. D. Mechanics of static and dynamic filtration in the borehole. **Society of Petroleum Engineering Journal**, vol. 3, n. 3, p. 230, 1963. [doi:10.2118/491-PA](https://doi.org/10.2118/491-PA)
- Perry, J. H. **Chemical Engineer's Handbook**. R 660.2, P 463, 1953.
- Ripperger, S.; Altmann, J. Cross flow microfiltration – state of the art. **Separation Purification Technology**, 26, p.19–31, 2002. [doi:10.1016/S1383-5866\(01\)00113-7](https://doi.org/10.1016/S1383-5866(01)00113-7)
- Van der Bruggen, B.; Vandecasteele, C., Van Gestel, T., Doyen, W., Leysen, R. A review of pressure-driven membrane processes in wastewater treatment and drinking water production. **Environ. Prog.**, 22, p.46–56, 2003.
- Visser, J. On Hamaker constants: A comparison between Hamaker constants and Lifshitz-van der Waals constants. *Adv. Colloid interface science*, Vol.3, n. 4, p.331-363, 1972. [doi:10.1016/0001-8686\(72\)85001-2](https://doi.org/10.1016/0001-8686(72)85001-2) In: Jiao, D.; Sharma, M.M. Mechanism of Cake Buildup in Crossflow Filtration of Colloidal Suspensions. **Journal of Colloid and Interface Science**, 162, p.454-462, 1994. [doi:10.1006/jcis.1994.1060](https://doi.org/10.1006/jcis.1994.1060)