EFFECTS OF DIFFERENT BLOCK SIZE DISTRIBUTIONS IN PRESSURE TRANSIENT RESPONSE OF NATURALLY FRACTURED RESERVOIRS

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ABSTRACT

This paper presents a model for pressure transient and derivative analysis for naturally fractured reservoirs by a formulation of interporosity flow incorporating variations in matrix block size, which is inversely related to fracture intensity. Geologically realistic Probability Density Functions (PDFs) of matrix block size, such as uniform, bimodal, linear and exponential distributions, are examined and pseudo-steady-state and transient models for interporosity flow are assumed. The results have been physically interpreted, and, despite results obtained by other authors, it was found that the shape of pressure derivative curves for different PDFs are basically identical within some ranges of block size variability, interporosity skin, PDFs parameters and matrix storage capacity. This tool can give an insight on the distribution of block sizes and shapes, together with other sources of information such as Logs and geological observations.

KEYWORDS

block size; probability density function; transient pressure

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1. INTRODUCTION

Block size is considered one of the main parameters of a fractured rock reservoir. In one-phase flow, it controls the transition from the early stages of production to asymptotic behavior, and in two-phase wetting-nonwetting displacement it controls the rate of production by imbibition. In nonwetting-wetting displacement it controls the pressure difference between the extremities of the block, which, provided that the threshold pressure is exceeded, permits displacement of the nonwetting fluid from the blocks into the fractures. It is believed that in two-phase flow the displacing fluid circumvents the blocks in groups rather than singly. In this case, the individual size of the block is not a relevant parameter, even in two-phase flow. Warren and Root (**Warren and Root, 1984**) relate the size of the block to the transient pressure variation from the first to the second straight line of the drawdown/buildup test (**Braester, 1984**). Their model and other dual porosity pressure transient models of naturally fracture reservoirs assume that fracture intensity is constant throughout the reservoir, but the distribution of fracture height is observed in outcrops. The most geologically realistic Probability Density Functions (PDFs) of matrix block size are uniform, bimodal, linear and exponential distributions (**Johns and Jalali, 1991**).

Some fractures are calcite-cemented or mineral-filled, which can restrict flow from the matrix blocks to the fractures. This phenomenon is termed interporosity skin (**Cinco et al, 1985; Moench, 1984**).

The transition curve to the homogeneous equivalent is associated with the block size. The transition time to homogeneous equivalent is relatively short. It also decreases rapidly with the distance outwards from the well. In these circumstances it may be concluded that in one-phase flow the only parameters of interest are those of the homogeneous behavior (**Braester, 1984**).

Cinco et al. (1985) developed a model to predict the pressure transient response for a naturally fractured reservoir having a distribution of different sizes. They found that the shape of the distribution had a significant effect on the shape of the pressure response. In particular, if there is a

broad distribution of block sizes, the classic V shaped minimum in the derivative predicted by the Warren and Root model will not occur. Other authors have conducted field studies of the distribution of fracture and joint spacing. The consensus is that spacing follows an exponential distribution (**Kulatilake et al, 1993**). Also **Spivey and Lee (2000)** examined the consequences of an exponential distribution of fracture spacing on the distribution of block size and interporosity flow coefficient and on the resulting pressure transient response. **Gringarten (1984)** reports a majority of tests that he has personally observed appear to exhibit pseudo steady-state interporosity flow behavior.

2. INTERPOROSITY FLOW MODELS

Most of the models of naturally fractured reservoirs basically involve two approaches to correlate the pressure inside the matrix and that of the fracture system: the pseudo-steady-state interporosity flow and the transient interporosity flow.

2.1 Pseudo-steady-state interporosity flow

The pressure within the matrix and the fracture systems within a representative elementary volume are each assumed to be uniform, but the fluid flow between the two systems occurs under pseudo-steady-state (PSS) conditions. In the Warren and Root model, the shape of the individual matrix blocks has no effect on the pressure response. The pressure transient response for the Warren and Root model exhibits two parallel semilog straight lines: the first corresponding to transient flow in the fracture system alone, before the matrix begins to respond; the second referring to transient flow in the total system, where the matrix and fractures act together as a homogeneous system.

2.2 Transient interporosity flow

Kazemi (1969) and **De Swaan (1976)** presented a model similar to that of Warren and Root, but where transient fluid flow was allowed within the matrix. De Swaan provided solutions for the pressure transient response for reservoirs in which the matrix blocks were either slabs or spheres. The

Kazemi-De Swaan model exhibits three semilog straight lines. The first and third semilog straight lines are parallel and correspond to fractured and total system behavior, respectively. The second semilog straight line appears during the transition from fractured behavior to total system behavior. This transition straight line has one half the slope of the first and third semilog straight lines.

3. INTERPOROSITY SKIN

For a naturally fractured reservoir where there is skin damage between the matrix and the fracture, the interporosity flow coefficient is given by (m = matrix, f = fracture):

$$\lambda = \frac{k_d}{k_f} \frac{A_b}{h_d V_b} r_w^2 = 2n \frac{k_d}{k_f} \frac{r_w^2}{h_d h_m} \quad (1)$$

If we assume that the permeability (k_d) and thickness (h_d) of the damaged zone are independent of the block size, and λ_0 is the interporosity flow coefficient for uniform fracture spacing (L_0 , characteristic length), one has:

$$\lambda(h_m) = \lambda_0 \frac{L_0}{h_m} \quad (2)$$

On the other hand, if the thickness of the damage zone is proportional to the block size, then:

$$\lambda = 2n \frac{k_d}{k_f} \frac{r_w^2}{a h_m^2} \text{ or } \lambda(h_m) = \lambda_0 \frac{L_0^2}{h_m^2} \quad (3)$$

which gives the same behavior as the PSS flow in the matrix with no damage skin. Equations 1 and 2 imply that if the matrix-fracture flow is controlled by a PSS flow within the matrix blocks themselves, the interporosity flow coefficient scales as $1/h_m^2$. The same behavior is obtained when matrix-fracture flow is controlled by a skin damage where the thickness of the damaged zone is proportional to the block size.

4. PROBABILITY DENSITY FUNCTION (PDF)

As mentioned above, many authors have

investigated the effects of different PDFs on the pressure response of fracture reservoirs. Here we mention the most important PDFs found in reservoir applications of block height and, at the end of the section, we introduce two linear probability density functions.

For the continuous variation of the matrix blocks (h_{max}), the corresponding result is:

$$\int_{h_{Dmin}}^{h_{Dmax}} f(h_D) dh_D = 1, \text{ with } h_D = h/h_{max} \quad (4)$$

The limits of the integration are $h_{Dmin} = h_{min}/h_{max} = F_h$ and $h_{Dmax} = 1$; $f(h_D)$ is the probability function of the random variable h_D , which describes the distribution of the block sizes (Rodríguez et al., 2001).

4.1 Uniform density function (PDF)

A distribution of interest is the uniform or rectangular distribution where all block sizes (h_{min} to h_{max}) have an equal chance of occurrence. In this case, $f(h) = 1/(h_{max} - h_{min})$, and if $h_D = h/h_{max}$, one has $f(h_D) = 1/(1 - F_h)$, where $F_h = h_{min}/h_{max}$. The applicability of the uniform distribution is possible only when the matrix block size distribution is unknown (Belani, 1988; Johns and Jalali, 1991). Besides, the sum of uniform distributions with a small variance spread must be close to any distribution, and hence the pressure response for other distributions can be obtained (Johns and Jalali, 1991).

4.2 Binodal density function

General tectonic stresses over a region can cause fracturing at a macro scale and associated breaking of the rock at a finer scale. This results in two controlling sets of matrix block sizes, which can be represented by a bimodal distribution. If the two modes of the distribution are equally probable (same height), then:

$$f(h_D) = 1 / \left(1 - \frac{h_1}{h_{max}} + \frac{h_2}{h_{max}} - \frac{h_3}{h_{max}} \right) \quad (5)$$

where $h_1 < h_2 < h_3 < h_{max}$, which corresponds to $\lambda_1 > \lambda_2 > \lambda_3 > \lambda_{min}$ (Belani, 1988).

4.3 Exponential density function

The exponential density function assumes the following form, proposed by **Johns and Jalali (1991)**:

$$f(h_D) = ae^{-ah_D} / (e^{-aF_D} - e^{-a}) \quad (6)$$

Positive values of the exponential constant (a) represent the range of size distributions from a high fraction of small blocks to a low fraction of big blocks in the reservoir. Alternatively, if the exponential constant is negative the opposite situation is observed. In both cases, good agreement generally occurs. For practical purposes, the exponential constant (a) ranges from -20 to $+20$ (**Rodríguez et al., 2001**). Equation 6 is not defined for $a = 0$, which would correspond to the uniform or rectangular function, where all the block sizes have the same probability of occurrence.

4.4 Linear density function

Johns and Jalali (1991) proposed a linear distribution in the form:

$$f(h_D) = (mh_D + b) / (0.5m[1 - F_h^2] + b[1 - F_h]) \quad (7)$$

where m is the slope and b is the vertical intercept of the Cartesian plot of $f(h_D)$ versus h_D . Here we used two separate modes of linear distribution (positive and negative slope) in the diffusivity equation. Normalized positively skewed and normalized negatively skewed linear distributions are given by Equations 8 and 9, respectively:

$$f(h_D) = 2(h_D - F_D) / (1 - F_D)^2 \quad (8)$$

$$f(h_D) = 2(1 - h_D) / (1 - F_D)^2 \quad (9)$$

A positive slope ($m > 0$) means a low fracture density (big matrix blocks); as m increases, a higher fraction of the pore volume occurs in big blocks. A high fracture density (small matrix blocks) corresponds to negative slope ($m < 0$).

5. MATHEMATICAL MODEL

The classical diffusivity equation from mass balance in a dual-porosity reservoir can be

modified to include the variation in the matrix block size. The classical diffusivity equation for radial flow is given by:

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial P_f}{\partial r} \right) = \frac{\phi_f c_f \mu}{k_f} \frac{\partial P_f}{\partial t} + Q_m \quad (10)$$

For a non-uniformly fractured reservoir with a random distribution of matrix block sizes, the matrix contribution (interporosity flow) can be given by (**Belani and Yazdi, 1988**):

$$Q_m = \int_{h_{\min}}^{h_{\max}} Q(h) f(h) dh \quad (11)$$

Belani and Yazdi (1988) solved Equation 10 for both pseudo-steady-state and transient interporosity flows for two probability density functions (uniform and bimodal), and obtained Equations 12, 13 and 14. For this work, the equations have been tested and proved, but the mathematical procedure has not been included here. Any details can be provided upon request. The well-bore pressure response in the Laplace space, with $x = \sqrt{sg(s)}$ as argument, is:

$$\bar{P}_{wD} = (K_0(x) + S_D x K_1(x)) / (s [C_D s (K_0(x) + S_D x K_1(x)) + x K_1(x)]) \quad (12)$$

For pseudo steady-state interporosity flow, one has:

$$g(s) = \omega_f + \omega_m \int_{F_h}^1 \frac{3\lambda}{\omega_m s + 3\lambda} f(h_D) dh_D \quad (13)$$

Finally, for transient interporosity flow, one has:

$$g(s) = \omega_f + \int_{F_h}^1 \sqrt{\frac{\omega_m \lambda}{s}} \tanh \left(\sqrt{\frac{\omega_m s}{\lambda}} \right) f(h_D) dh_D \quad (14)$$

Eq.14 was numerically solved for the exponential and linear distribution. To find the pressure data in real space, Eq.9 is solved by Stehfest's Algorithm (**Stehfest, 1970**). We found that the best range for K (Number of terms in series) in Stehfest's Algorithm to be optimized ranges from 12 to 16. A FORTRAN program was developed to obtain the dimensionless well-bore pressure response versus dimensionless time.

Despite solving Eq.10 analytically for uniform and bimodal distribution, we also numerically solved that equation in order to analyze the sensitivity of the Eq.10 to numerical methods. Figure 1 shows the plot of P_D vs. T_D and confirms that the numerical solution (Simpson's 1/3 rule for the integration) best fits to the exact solution and Eq.4 is not so sensitive to numerical integration.

Figure 1. Sensitivity of pressure response to the numerical integration.

6. EFFECTS OF BLOCK SIZE DISTRIBUTIONS

6.1 Pseudo-steady-state interporosity flow

Fig.2 shows the PSS pressure derivative response for uniform block size distributions of different variance spread (Lambda ratio= $\lambda_{max}/\lambda_{min}$). λ_{max} is kept constant at 10^{-5} . In the limit λ_{min} approach λ_{max} (or vice-versa), the single block size response is obtained (Warren and Root Model) and the interporosity flow region on the derivative curve shows distinctly the effect of the variation in block size. The 'stretched' transition curve in Fig.3 characterizes the variance of the distribution function as noted in Fig.2 (Belani, 1988).

Figures 4 and 5 illustrate the pressure response for a bimodal distribution with the parameters; $\lambda_1 > \lambda_2 > \lambda_3 > \lambda_4$ ($\lambda_4 = \lambda_{min}$) which correspond to $h_1 < h_2 < h_3 < h_4$ ($h_4 = h_{min}$). Both graphs include uniform distribution with (λ_1, λ_2) and (λ_3, λ_4) to compare with that of the bimodal distribution. When the two modes of the distribution are very close, the response is similar to that of the unimodal uniform distribution corresponding to the larger λ mode.

Figure 2. Uniform distribution (λ Effect) in PSS Flow.

Figure 3. Uniform distribution (ω effect) in PSS Flow.

Beyond a certain degree of separation, the derivative plot behaves like the higher block size. The second drop on the derivative curve is due to depletion of larger fractures in the larger-height block size distribution.

An exponential distribution is represented in Figure 6. A distribution with a positive exponential constant (α) means that a small block size is more likely to occur, and as the positive exponential constant becomes larger the probability of small block size decreases. The interporosity flow region on the derivative curve for negative exponential constants (when a larger block size is more likely to occur) appears later, featuring larger fractures depletion later as well.

Linear distributions with negative slopes are illustrated in Figure 7. In this case, λ_{max} is kept constant and λ_{min} is reduced by an order of 10. Larger lambda ratios ($\lambda_{max}/\lambda_{min}$) mean larger h_{max} . It

Figure 4. Bimodal distribution in PSS flow ($\lambda_{min} = 8 \times 10^{-7}$).

Figure 5. Bimodal distribution in PSS flow ($\lambda_{min} = 8 \times 10^{-9}$).

Figure 6. Exponential distribution in PSS flow.

is quite similar to the uniform distribution (Figure 2). As λ_{min} is decreased with increasing h_{max} , there is a probability of h_{min} to decrease because of PDF characterization, with a plateau in the PDF graph

Figure 7. Linear distribution in PSS flow with negative slope.

Figure 8. Linear distribution in PSS flow with positive slope.

(constant area). In view of this, the distribution would be more similar to a uniform distribution. A linear distribution with a positive slope is illustrated in Figure 8. Here, as λ_{min} decreases, h_{max} increases and the distribution becomes more similar to the Dirac delta function, given by:

$$p(h_D) = \delta(h_D - 1) = \begin{cases} 0 & \text{for } h_D \neq 1 \\ \infty & \text{for } h_D = 1 \end{cases} \quad (15)$$

6.2 Transient interporosity flow

Johns and Jalali (1991) solved the diffusivity equation with consideration of interporosity skin defined as (h_s = interporosity damaged zone thickness):

$$S_{ID} = K_{ma} h_s / K_s h \quad (16)$$

The interporosity skin factor, S_{ID} , is a function of

matrix-block-size distribution and two cases are available. It is constant if h_s/h is constant and S_{ID} is variable if the depth of damage zone, h_s , is variable. The second case can be shown as:

$$S_{ID} = (S_{ID})_{\min} \sqrt{\lambda/\lambda_{\min}} \quad (17)$$

where $(S_{ID})_{\min} = K_{md}h_s/K_s h_{max}$. Therefore, the equation for $g(s)$ in the case of transient interporosity flow would be as follows:

$$g(s) = \omega_f + \omega_m \int_{F_h}^1 \frac{\sqrt{\lambda/\omega_m s} \tanh(\sqrt{\omega_m s/\lambda}) f(h_D)}{1 + (S_{ID})_{\min} \sqrt{\omega_m s/\lambda_{\min}} \tanh(\sqrt{\omega_m s/\lambda})} dh_D \quad (18)$$

Figure 9 shows an example of the effect of interporosity skin. They are all symmetric and the matrix response is more delayed as S_{ID} increases. The effects of F_h are shown in Figure 10. F_h provides an estimate of the matrix-block-size variability. If

Figure 9. Effect of S_{ID} on uniform distribution in transient flow.

Figure 10. Effect of F_h on uniform distribution in transient flow.

$F_h \rightarrow 1$, a perfectly uniform fracturing is observed, while $F_h \rightarrow 0$ indicates that a perfectly non-uniform fracturing occurs. For $F_h \rightarrow 0$, the response approaches that of a homogeneous reservoir. As long as the fracturing is extremely non-uniform, the response will not exhibit the classic profile of a distinct transition zone separating early and late-time semilog straight lines.

Pressure response with bimodal distribution is plotted in Fig.11. As S_{ID} increases the bimodal response disappear and the curve is more likely similar to the unimodal. Fig.12 shows the response for varying values of (a) in an exponential distribution at constant F_h . For positively increasing values of a , fracture intensity increases and the response approaches the Dirac delta response for a uniform matrix block size h_{min} , (i.e., the response occurs earlier in time). For negatively increasing values of a , fracture intensity decreases and the response approaches the Dirac delta response for a uniform matrix block size h_{max} (i.e., the response occurs later in time). Thus, fracture intensity determines the temporal position of the pressure response. Fracture uniformity, however, affects the shape of the pressure response (Johns and Jalali, 1991). The effect of ω_m and S_{ID} in a linear distribution with a positive slope is shown in Figures 13 and 14. As opposed to the case of pseudo-steady-state, the pressure response with transient flow is asymmetric. In Figure 14, the pressure response becomes symmetric for large values of interporosity skin. The effect of ω_m and S_{ID} in a linear distribution with negative slope is shown in Figures 15 and 16. The interesting result is that there are no measurable differences between linear distribution with positive and negative slopes in transient flow. The pressure response with positive slope is very slightly shifted to the right of the t_D axis and the shapes of the curves are almost the same.

7. TRANSITION PERIOD

In the beginning of a drawdown test, P_{wD} is given by (Cinco et al, 1985):

$$P_{wD}(\omega, t_D) = (1/2) (\ln(t_D/\omega) + 0.80907) \quad (19)$$

This is the flow period dominated by the fractures, where P_{wD} is affected only by the fractures storage ω . At large values of time the

Figure 11. Bimodal distribution in transient flow.

Figure 12. Exponential distribution in transient flow.

 Figure 13. Effect of ω_m on the linear distribution with positive slope in transient flow.

expansion of the total fractured system contributes to fluid production and the flow in the matrix reaches a pseudo-steady state like condition; under this situation P_{wD} can be expressed by (Cinco et al., 1985):

$$P_{wD}(t_D) = (1/2)(\ln t_D + 0.80907) \quad (20)$$

which means that the naturally fractured reservoir behaves like a homogeneous system. A transition exists between the above mentioned flow periods,

 Figure 14. Effect of S_{ID} on the linear distribution with positive slope in transient flow.

 Figure 15. Effect of ω_m on the linear distribution with negative slope in transient flow.

 Figure 16. Effect of S_{ID} on the linear distribution with negative slope in transient flow.

where the pressure response depends on the size and block size distribution, as well as the type of flow between matrix and fractures, i.e., unsteady state or pseudo-steady state.

8. CONCLUSIONS

Recently, two new approaches have been proposed to address the problem of transient pressure testing in naturally fractured reservoirs: using fractals to describe the reservoir geometry, and using numerical simulation to model fluid flow in a fracture network embedded in a matrix. The appearance of a V-shaped minimum on the so called curves is often considered an indication of the presence of a PSS dual porosity system, especially if another evidence of natural fracturing is present. In the case of transient interporosity flow, there is no measurable difference between linear distributions of negative and positive slopes. Numerical inversion of the well-bore pressure in Laplace space is not sensitive to numerical integration in its integration part. Despite results obtained by other authors, it was found that within some ranges of block size variability, interporosity skin, PDFs parameters and matrix storage, the shape of the pressure derivative curves for different PDFs are basically identical. In transient interporosity flow at large S_D values, if at all, the pressure response of the fractured reservoir is similar to that of a pseudo-steady state.

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